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Robust adaptive prescribed-time stabilization via output feedback for uncertain nonlinear strict-feedback-like systems[☆]

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ABSTRACT

While control design objectives are formulated most commonly in terms of asymptotic behavior (as time goes to infinity) of signals in the closed-loop system, the recently developed notion of “prescribed-time” stabilization considers closed-loop signal behavior over a fixed (prescribed) time interval and addresses the problem of regulating the state to the origin in the prescribed time irrespective of the initial state. While prior results on prescribed-time stabilization considered a chain of integrators with uncertainties matched with the control input (i.e., normal form), we consider here a general class of nonlinear strict-feedback-like systems with state-dependent uncertainties allowed throughout the system dynamics including uncertain parameters (without requirement of any known bounds on the uncertain parameters). Furthermore, we address the output-feedback problem and show that a dynamic observer and controller can be designed based on our dual dynamic high gain scaling based design methodology along with a novel temporal transformation and form of the scaling dynamics with temporal forcing terms to achieve both state estimation and regulation in the prescribed time.

1. Introduction

Control design objectives for continuous-time nonlinear systems are formulated most commonly in terms of ensuring various asymptotic properties (as time goes to infinity) of signals in the closed-loop system, e.g., asymptotic stabilization [5,6,18] wherein the control objective is to ensure convergence of the system state (or output) to a desired value (e.g., the origin) as time t goes to ∞. In contrast, the notion of “finite-time” stabilization [1–4,19–22,27,28] addresses closed-loop signal behavior over finite time intervals, e.g., controller design to achieve desired convergence properties in finite time. Various controller design techniques have been developed in the literature for finite-time stabilization [1–4,19–22,27,28] typically based on feedback using fractional powers of the state variables. Finite-time partial-state-feedback stabilization of high-order nonlinear systems [29] has been addressed in [26] using fractional powers of state variables in the control design. While the finite-time stabilization problem simply requires convergence in some finite time, but allows the achieved finite time to be dependent on the system dynamics and the initial state, the recently introduced notion of “prescribed-time” stabilization [23–25] addresses the more stringent requirement that the terminal time should be a parameter that can be *prescribed* by the control designer independent of the initial condition. In other words, the control designer should be able to arbitrarily pick a finite regulation time T and require that the system state should be made to converge to the origin as $t \to T$ irrespective of the initial condition. This prescribed-time stabilization notion can be viewed in the physical context of controls applications such as missile guidance, autonomous vehicle rendezvous, etc., wherein the state convergence control objective is inherently formulated over a fixed time horizon.

The recent results on prescribed-time stabilization in [23–25] address a system structure comprised of a chain of integrators with uncertainties matched with the control input (i.e., normal form) based on scaling the system state by a function of time that goes to infinity as $t \to T$ and designing a controller to stabilize the system written in terms of the scaled state. In this paper, we consider a general class of nonlinear strict-feedback-like systems with state-dependent nonlinear uncertainties allowed throughout the system dynamics and address furthermore the output-feedback problem. Specifically, we consider a class of

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systems of the following form[1]:

$$\begin{aligned}\dot{x}_i &= \phi_i(x,u,t) + \phi_{(i,i+1)}(x_1)x_{i+1} \quad i=1,\ldots,n-1\\ \dot{x}_n &= \phi_n(x,u,t) + \mu_0(x_1)u\\ y &= x_1 \end{aligned} \tag{1}$$

where $x=[x_1,\ldots,x_n]^T \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}$ is the input, and $y \in \mathcal{R}$ is the output[2]. $\phi_{(i,i+1)}, i=1,\ldots,n-1$, and μ_0, are known scalar real-valued continuous functions of their arguments. $\phi_i, i=1,\ldots,n$, are time-varying scalar real-valued uncertain functions. The system (1) is in a generalized strict-feedback form. Strict-feedback system structures have been studied heavily in the literature [5,6,18] and several mechanical and electromechanical systems can be written in strict-feedback forms (e.g., dynamics of various motors [7]).

We show that a dynamic output-feedback prescribed-time stabilizing controller can be designed for the class of systems (1) based on our dual dynamic high gain scaling based observer-controller design techniques [8,9,11,12,14,15] and introducing a set of modifications to address the prescribed-time stabilization problem instead of the asymptotic stabilization problem addressed in the control designs in [8,9,11–15]. While [23–25] consider state feedback of systems in normal form and utilize a scaling of the state by a function of time, an approach based on state scaling utilizing powers of a dynamic scaling parameter was developed in [16]. In this paper, it is shown that the state-feedback approach in [16] can be extended to the output-feedback case via a control design approach based on (a) introducing a scaling for observer errors and observer state estimates utilizing powers of a dynamic scaling parameter; (b) introducing a time scale transformation to map the prescribed time horizon to the infinite time horizon of a transformed temporal variable; (c) designing the dynamic scaling based observer-controller structure using this temporally scaled system representation; (d) introducing specifically designed time-dependent forcing terms into the dynamics of the scaling parameter and into a gain term appearing in the control design. The class of systems considered here includes state-dependent uncertainties throughout the system dynamics and also allows uncertain parameters (without requirement of any known bounds on the uncertain parameters).

The control objective, assumptions on the system (1), and the statement of the main result of the paper are provided in Section 2. The observer and controller designs are presented in Section 3. The proof of the main result is provided in Section 4. The application of the proposed control design to an example third-order system is presented in Section 5. Concluding remarks are summarized in Section 6.

2. Control objective, assumptions, and statement of main theorem

With $T>0$ being a given constant, the control objective is to design a dynamic output-feedback control law for u so that $x(t) \to 0$ and $u(t) \to 0$ as $t \to T$. The assumptions imposed on the system (1) are summarized below.

Assumption A1 (*lower boundedness away from zero of "upper diagonal" terms $\phi_{(i,i+1)}$ and μ_0*)**.** A constant $\sigma>0$ exists such that[3] $|\phi_{(i,i+1)}(x_1)| \ge \sigma, 1 \le i \le n-1$, and $|\mu_0(x_1)| \ge \sigma$ for all $x_1 \in \mathcal{R}$. Since $\phi_{(i,i+1)}$ and μ_0 are assumed to be continuous functions, this assumption can, without loss of generality, be stated as $\phi_{(i,i+1)}(x_1) \ge \sigma, 1 \le i \le n-1$, and $\mu_0(x_1) \ge \sigma$ with a constant $\sigma>0$.

Assumption A2 (*Bounds on uncertain functions ϕ_i*)**.** The functions $\phi_i, i=1,\ldots,n$, can be bounded as

$$|\phi_i(x,u,t)| \le \Gamma(x_1)\sum_{j=1}^{i}\phi_{(i,j)}(x_1)|x_j| + \theta\beta_i(x_1)|x_1| \tag{2}$$

for all $x \in \mathcal{R}^n$ where $\Gamma(x_1)$, $\phi_{(i,j)}(x_1), i=1,\ldots,n, j=1,\ldots,i$, and $\beta_i(x_1), i=1,\ldots,n$, are known continuous non-negative functions and θ is an unknown non-negative constant.

Assumption A3 (*Bi-directional cascading dominance of "upper diagonal" terms $\phi_{(i,i+1)}, i=2,\ldots,n-1,$*)**.** Positive constants $\overline{\rho}_i, i=3,\ldots,n-1$, and $\underline{\rho}_i, i=3,\ldots,n-1$ exist such that $\forall x_1 \in \mathcal{R}$

$$\phi_{(i,i+1)}(x_1) \ge \overline{\rho}_i\phi_{(i-1,i)}(x_1)\,, \quad i=3,\ldots,n-1 \tag{3}$$

$$\phi_{(i,i+1)}(x_1) \le \underline{\rho}_i\phi_{(i-1,i)}(x_1)\,, \quad i=3,\ldots,n-1. \tag{4}$$

Remark 1. The structure of the assumptions above are analogous to the assumptions introduced for the dual dynamic high gain based output-feedback control design in [8]. Assumption A1 ensures observability, controllability, and uniform relative degree (of x_1 with respect to u). Assumption A2 imposes bounds on uncertain terms in the system dynamics and essentially requires uncertain terms to be bounded linearly in unmeasured state variables with a triangular state dependence structure in the known bounds. Assumption A3 imposes constraints on the relative "sizes" (in a nonlinear function sense) of the upper diagonal terms $\phi_{(i,i+1)}$ and is vital in achieving solvability of a pair of coupled Lyapunov inequalities (Section 3.7). The functions $\phi_{(i,i+1)}$ are referred to as "upper diagonal" terms since if the dynamics (1) were to be written in the form $\dot{x} = A(x_1)x + B(x_1)u + \phi(x)$ with $\phi = [\phi_1,\ldots,\phi_n]^T$, the functions $\phi_{(i,i+1)}$ would appear on the upper diagonal of the matrix $A(x_1)$.

The main result of this paper is summarized as Theorem 1 below. The proof of Theorem 1 will be provided in Section 4 based on the control design developed in Section 3.

Theorem 1. *Given any prescribed time $T>0$, a dynamic output-feedback controller of the form*

$$u = f(\psi, y)\ ; \quad \dot{\psi} = g(\psi, y) \tag{5}$$

can be designed (where ψ is the state of the dynamic controller) for system (1) under Assumptions A1–A3 such that starting from any initial condition for x, the property $\lim_{t\to T}|x(t)| = \lim_{t\to T}|u(t)| = 0$ is satisfied.

3. Control design

3.1. Observer design

A reduced-order observer with states $\hat{x} = [\hat{x}_2,\ldots,\hat{x}_n]^T$ is designed as

$$\begin{aligned}\dot{\hat{x}}_i &= \phi_{(i,i+1)}(x_1)[\hat{x}_{i+1} + r^i f_{i+1}(x_1)] - r^{i-1}g_i(x_1)[\hat{x}_2 + rf_2(x_1)]\\ &\quad - (i-1)\dot{r}r^{i-2}f_i(x_1), 2 \le i \le n-1\\ \dot{\hat{x}}_n &= \mu_0(x_1)u - r^{n-1}g_n(x_1)[\hat{x}_2 + rf_2(x_1)] - (n-1)\dot{r}r^{n-2}f_n(x_1)\end{aligned} \tag{6}$$

where

- $g_i(x_1)$ are functions that will be designed in Section 3.7

[1] Throughout, a dot above a symbol denotes the derivative with respect to the time t as is the standard notation, e.g., $\dot{x}_1 = \frac{dx_1}{dt}$.

[2] $\mathcal{R}$, $\mathcal{R}^+$, and $\mathcal{R}^k$ denote the set of real numbers, the set of non-negative real numbers, and the set of real k-dimensional column vectors, respectively.

[3] Given a vector a, the notation $|a|$ denotes its Euclidean norm. If a is a scalar, $|a|$ denotes its absolute value.

- $f_i(x_1)$ are functions defined as

$$f_i(x_1) = \int_0^{x_1} \frac{g_i(\pi)}{\phi_{(1,2)}(\pi)} d\pi, 2 \le i \le n \tag{7}$$

- r is a dynamic high-gain scaling parameter whose dynamics will be designed in Section 3.8. The dynamics chosen for r will ensure that $r(t) \geq 1$ for all time $t \geq 0$.

3.2. Observer errors and scaled observer errors

The observer errors are defined as

$$e_i = \hat{x}_i + r^{i-1} f_i(x_1) - x_i, 2 \le i \le n \tag{8}$$

and the scaled observer errors are defined as

$$\epsilon_i = \frac{e_i}{r^{i-1}}, i = 2 \le i \le n \quad ; \quad \epsilon = [\epsilon_2, \ldots, \epsilon_n]^T. \tag{9}$$

The dynamics of ϵ can be written as

$$\dot{\epsilon} = rA_o\epsilon - \frac{\dot{r}}{r} D_o\epsilon + \overline{\Phi} \tag{10}$$

where

- A_o is a $(n-1)\times(n-1)$ matrix with $(i, j)^{th}$ entry

$$A_{o_{(i,1)}}(x_1) = -g_{i+1}(x_1) \quad i = 1, \ldots, n-1$$
$$A_{o_{i,i+1}}(x_1) = \phi_{(i+1,i+2)}(x_1) \quad i = 1, \ldots, n-2 \tag{11}$$

- D_o is a $(n-1)\times(n-1)$ matrix defined as[4] $D_o = \mathrm{diag}(1, 2, \ldots, n-1)$
- $\overline{\Phi}$ is given by

$$\overline{\Phi} = [\overline{\Phi}_2, \ldots, \overline{\Phi}_n]^T; \overline{\Phi}_i = -\frac{\phi_i(x, u, t)}{r^{i-1}} + g_i(x_1)\frac{\phi_1(x_1)}{\phi_{(1,2)}}. \tag{12}$$

3.3. Dynamics of scaled observer estimate signals

Define $\eta_2, \ldots, \eta_n$ as

$$\eta_2 = \frac{\hat{x}_2 + rf_2(x_1) + \zeta(x_1, \hat{\theta})}{r} \quad ; \quad \eta_i = \frac{\hat{x}_i + r^{i-1}f_i(x_1)}{r^{i-1}}, i = 3, \ldots, n \tag{13}$$

where the function ζ is defined to be of the form

$$\zeta(x_1, \hat{\theta}) = \hat{\theta} x_1 \zeta_1(x_1) \tag{14}$$

with $\hat{\theta}$ being a dynamic adaptation parameter (whose dynamics will be designed in Section 3.8) and ζ_1 being a function that will be designed in Section 3.8. The dynamics designed for $\hat{\theta}$ will ensure that $\hat{\theta}(t) \geq 1$ for all $t \geq 0$. The dynamics of η_i are given by[5]

$$\begin{aligned}
\dot{\eta}_2 &= r\phi_{(2,3)}\eta_3 - rg_2\epsilon_2 + \frac{\hat{\theta}[\zeta_1'(x_1)x_1 + \zeta_1(x_1)]}{r} \\
&\quad \times [(r\eta_2 - \zeta - r\epsilon_2)\phi_{(1,2)} + \phi_1] \\
&\quad + g_2\frac{\phi_1}{\phi_{(1,2)}} + \frac{1}{r}\dot{\hat{\theta}} x_1\zeta_1(x_1) - \frac{\dot{r}}{r}\eta_2 \\
\dot{\eta}_i &= r\phi_{(i,i+1)}\eta_{i+1} - rg_i\epsilon_2 + g_i\frac{\phi_1}{\phi_{(1,2)}} - \frac{\dot{r}}{r}(i-1)\eta_i \ , \ i = 3, \ldots, n-1 \\
\dot{\eta}_n &= -rg_n\epsilon_2 + g_n\frac{\phi_1}{\phi_{(1,2)}} - \frac{\dot{r}}{r}(n-1)\eta_n + \frac{1}{r^{n-1}}\mu_0 u
\end{aligned} \tag{15}$$

where $\zeta_1'(x_1)$ denotes the partial derivative of ζ_1 with respect to its argument evaluated at x_1.

[4] The notation $\mathrm{diag}(T_1, \ldots, T_m)$ denotes an $m\times m$ diagonal matrix with diagonal elements $T_1, \ldots, T_m$. I_m denotes the $m\times m$ identity matrix.

[5] For notational convenience, we drop the arguments of functions whenever no confusion will result.

3.4. Design of control input u and dynamics of scaled states

Defining $\eta = [\eta_2, \ldots, \eta_n]^T$, the control input u is designed as

$$u = -\frac{r^n}{\mu_0(x_1)} K_c\eta \tag{16}$$

with $K_c = [k_2, \ldots, k_n]$ where $k_i, i = 2, \ldots, n$, are functions of x_1 that will be designed in Section 3.7. The dynamics of η under the control law (16) are

$$\dot{\eta} = rA_c\eta - \frac{\dot{r}}{r}D_c\eta + \Phi - rG\epsilon_2 + H[\eta_2 - \epsilon_2] + \Xi \tag{17}$$

where A_c is the $(n-1)\times(n-1)$ matrix with $(i, j)^{th}$ element

$$A_{c_{(i,i+1)}}(x_1) = \phi_{(i+1,i+2)}(x_1) \ , \ i = 1, \ldots, n-2$$
$$A_{c_{(n-1,j)}}(x_1) = -k_{j+1}(x_1) \ , \ j = 1, \ldots, n-1 \tag{18}$$

with zeros elsewhere, and

$$D_c = \mathrm{diag}(1, 2, \ldots, n-1) \tag{19}$$

$$G = [g_2, \ldots, g_n]^T \quad ; \quad \Phi = \frac{\phi_1}{\phi_{(1,2)}}G \tag{20}$$

$$H = [\hat{\theta}[\zeta_1'(x_1)x_1 + \zeta_1]\phi_{(1,2)}, 0, \ldots, 0]^T \tag{21}$$

$$\Xi = \left[\frac{(\phi_1 - \zeta\phi_{(1,2)})\hat{\theta}[\zeta_1'(x_1)x_1 + \zeta_1] + \dot{\hat{\theta}}x_1\zeta_1(x_1)}{r}, 0, \ldots, 0\right]^T. \tag{22}$$

3.5. Temporal scale transformation

Let $a(.)$ be a twice continuously differentiable monotonically increasing function over $[0, T)$ that satisfies the following conditions (see Remark 2 below for examples of such functions):

- $a(0) = 0$, $a(T) = \infty$
- $\frac{da}{dt}$ is bounded below by a positive constant over $[0, T)$, i.e., $\frac{da}{dt} \geq a_0$ for $t \in [0, T)$ with a_0 being some positive constant
- Denoting the transformation $\tau = a(t)$ and writing $\frac{da}{dt}$ as a function of τ as $\alpha(\tau) = \frac{da}{dt}$, $\alpha(\tau)$ grows at most polynomially as $\tau\to\infty$, i.e., a polynomial $\overline{\alpha}(\tau)$ exists such that $\alpha(\tau) \leq \overline{\alpha}(\tau)$ for all $\tau \in [0, \infty)$. Also, $\frac{d\alpha}{d\tau}$ grows at most polynomially as $\tau\to\infty$.

With τ defined as the transformation $\tau = a(t)$, we see that when t goes from 0 to T, τ goes from 0 to ∞. Now,

$$d\tau = a'(t)dt \tag{23}$$

where $a'(t)$ denotes $\frac{da}{dt}$. The transformation above is a time scale transformation wherein the interval $[0, T)$ in terms of the time variable t corresponds to the interval $[0, \infty)$ in terms of the time variable τ. To denote a signal $x(t)$ as a function of transformed time variable τ, we use the notation $\breve{x}(\tau)$, i.e., $x(t) = \breve{x}(\tau)$ since both $x(t)$ and $\breve{x}(\tau)$ refer to the value of the same signal at the same physical time point represented as t in the original time axis and τ in the transformed time axis. The conditions on the function a: $[0, T)\to[0, \infty)$ introduced above imply that this function is invertible. Denoting the inverse function by a^{-1}, we have, by definition,

$$x(t) = x(a^{-1}(\tau)) = \breve{x}(\tau) = \breve{x}(a(t)). \tag{24}$$

Remark 2. There are many (in fact, infinite number of) functions that satisfy the conditions defined for the function a above. For example, we can pick a to be one of the following:

$$a(t) = \frac{a_0 t}{1 - \frac{t}{T}} \tag{25}$$

$$a(t)=\frac{a_0 t}{\sqrt{1-(\frac{t}{T})^2}}. \tag{26}$$

For $a(t)$ given in (25), we have:

$$a'(t)=\frac{a_0}{(1-\frac{t}{T})^2} \ \ ; \ \alpha(\tau)=a_0\left(\frac{\tau}{a_0 T}+1\right)^2 \tag{27}$$

For $a(t)$ given in (26), we have:

$$a'(t)=\frac{a_0}{(1-(\frac{t}{T})^2)^{\frac{3}{2}}}. \ \ ; \ \alpha(\tau)=a_0\left(\frac{\tau}{a_0 T}+1\right)^{\frac{3}{2}} \tag{28}$$

Remark 3. To illustrate the motivation for the time scale transformation defined above in the context of prescribed-time stabilization, consider the simple scalar system $\dot{x}=u$. Designing a control law as $u=-kx$ with $k>0$ being a constant would yield exponential convergence of $x(t)$ to 0 as $t\to\infty$. However, with any constant k, the convergence of $x(t)$ to 0 is only asymptotic and $x(t)$ does not go to 0 at any finite time t. Instead, defining $u=-k\gamma(t)x$ analogous to the definition of ζ above where $\gamma(t)$ is a function of time, we get $\dot{x}=-k\gamma(t)x$. Now, defining the time scale transformation $\tau=a(t)$ with $a'(t)=\alpha(\tau)$ as defined above, we have $\frac{dx}{d\tau}=-\frac{k\gamma(a^{-1}(\tau))x}{\alpha(\tau)}$. Hence, defining, for example, $\gamma(t)=\alpha(a(t))$, we obtain $\frac{dx}{d\tau}=-kx$, which implies that the signal x goes to 0 asymptotically as $\tau\to\infty$ or equivalently as $t\to T$, i.e., $x(t)$ converges to 0 at the prescribed time T under the action of the controller given by $u(t)=-k\alpha(a(t))x(t)$.

As discussed in Remark 3 above, the temporal scale transformation defined above yields $dt=\frac{d\tau}{\alpha(\tau)}$. Hence, from (10) and (17), we have:

$$\alpha(\tau)\frac{d\epsilon}{d\tau}=rA_o\epsilon-\frac{\alpha(\tau)}{r}\frac{dr}{d\tau}D_o\epsilon+\overline{\Phi} \tag{29}$$

$$\alpha(\tau)\frac{d\eta}{d\tau}=rA_c\eta-\frac{\alpha(\tau)}{r}\frac{dr}{d\tau}D_c\eta+\Phi-rG\epsilon_2+H[\eta_2-\epsilon_2]+\Xi. \tag{30}$$

3.6. Lyapunov Functions

Define

$$V_o=r\epsilon^T P_o\epsilon \ \ ; \ \ V_c=\frac{1}{2}x_1^2+r\eta^T P_c\eta \ \ ; \ \ V=cV_o+V_c \tag{31}$$

where P_o and P_c are symmetric positive definite matrices to be defined later and c is a positive constant to be picked later. From (29), (30), and (31),

$$\begin{aligned}\frac{dV}{d\tau}=&\frac{1}{\alpha(\tau)}\Big\{cr^2\epsilon^T[P_oA_o+A_o^TP_o]\epsilon+2rc\epsilon^TP_o\overline{\Phi}+\\&x_1[\phi_1+(r\eta_2-\zeta-r\epsilon_2)\phi_{(1,2)}]+r^2\eta^T[P_cA_c+A_c^TP_c]\eta\\&+2r\eta^TP_c(\Phi-rG\epsilon_2+H[\eta_2-\epsilon_2]+\Xi)\Big\}\\&-\frac{dr}{d\tau}\Big\{c\epsilon^T[P_o\tilde{D}_o+\tilde{D}_oP_o]\epsilon+\eta^T[P_c\tilde{D}_c+\tilde{D}_cP_c]\eta\Big\}\end{aligned} \tag{32}$$

where $\tilde{D}_o=D_o-\frac{1}{2}I$ and $\tilde{D}_c=D_c-\frac{1}{2}I$ with I denoting an identity matrix of dimension $(n-1)\times(n-1)$.

3.7. Coupled Lyapunov Inequalities

Assumption A3 is the *cascading dominance* condition introduced in [8] wherein the condition in (3) is the "controller-context" cascading dominance condition and the condition in (4) is the "observer-context" cascading dominance condition. These cascading dominance conditions were shown in [8,10] to be closely related to solvability of pairs of coupled Lyapunov inequalities that appear in the high gain based control design. Specifically, under Assumption A1 and the condition (3) in Assumption A3, it is possible to find (in fact, explicitly construct as in [8,10]) a symmetric positive definite matrix $P_c=P_c^T>0$ and a function $K_c(x_1)=[k_2(x_1),\dots,k_n(x_1)]$ (whose elements appear in the definition of the matrix A_c) such that the following coupled Lyapunov inequalities are satisfied (for all $x_1\in\mathcal{R}$) with some positive constants ν_c, $\underline{\nu}_c$, and $\overline{\nu}_c$:

$$P_cA_c+A_c^TP_c\le-\nu_c\phi_{(2,3)}I \ \ ; \ \ \underline{\nu}_cI\le P_c\tilde{D}_c+\tilde{D}_cP_c\le\overline{\nu}_cI. \tag{33}$$

It is known that under Assumption A1 and condition (4) in Assumption A3, it is possible to find (in fact, explicitly construct as in [8,10]) a matrix $P_o=P_o^T>0$ and a function $G(x_1)=[g_2(x_1),\dots,g_n(x_1)]$ such that the following coupled Lyapunov inequalities are satisfied (for all $x_1\in\mathcal{R}$) with some positive constants ν_o, $\tilde{\nu}_o$, $\underline{\nu}_o$, and $\overline{\nu}_o$:

$$P_oA_o+A_o^TP_o\le-\nu_oI-\tilde{\nu}_o\phi_{(2,3)}C^TC \ \ ; \ \ \underline{\nu}_oI\le P_o\tilde{D}_o+\tilde{D}_oP_o\le\overline{\nu}_oI \tag{34}$$

where $C=[1,0,\dots,0]$. Furthermore, from Theorem 2 in [10], $g_2,\dots,g_n$ can be chosen to be linear constant-coefficient combinations of $\phi_{(2,3)},\dots,\phi_{(n-1,n)}$. Hence, using Assumption A3, a positive constant $\overline{G}$ exists such that

$$\left(\sum_{i=2}^n g_i^2\right)^{\frac{1}{2}}\le\overline{G}\phi_{(2,3)}. \tag{35}$$

3.8. Designs of Function ζ_1, Dynamics of r, and Dynamics of $\hat{\theta}$

The dynamics of r are designed to be of the form[6]

$$\frac{dr}{d\tau}=\lambda(R(x_1,\hat{\theta},\dot{\hat{\theta}})+\alpha(\tau)-r)[\Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}})+\tilde{\alpha}(\tau)] \quad \text{with} \quad r(0)\ge\max\{1,\alpha(0)\} \tag{36}$$

where $\tilde{\alpha}(\tau)$ denotes $\frac{d\alpha}{d\tau}$ and λ, R, and Ω are non-negative functions. $\lambda:\mathcal{R}\to\mathcal{R}^+$ is picked to be any non-negative continuous function such that $\lambda(s)=1$ for $s>0$ and $\lambda(s)=0$ for $s<-\epsilon_r$ with ϵ_r being some positive constant. Hence, from the dynamics above, it is seen that if $r\le R(x_1,\hat{\theta},\dot{\hat{\theta}})+\alpha(\tau)$ at some time instant, then we have $\frac{dr}{d\tau}=\Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}})+\tilde{\alpha}(\tau)$ at that time instant. As in [8], this property of the dynamics of r can be viewed as ensuring that the derivative of r is "large enough" (i.e., $\frac{dr}{d\tau}=\Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}})+\tilde{\alpha}(\tau)$) until r itself becomes "large enough" (i.e., $r\ge R(x_1,\hat{\theta},\dot{\hat{\theta}})+\alpha(\tau)$). From the form of the dynamics above, it is seen that $\frac{dr}{d\tau}\ge0$ for all $\tau\ge0$. It will be seen in Lemma 1 that the form of the dynamics of r given above implies that $r\ge\alpha(\tau)$ for all τ in the maximal interval of existence of solutions.

The dynamic adaptation parameter $\hat{\theta}$ is defined to be comprised of two components, i.e.,

$$\hat{\theta}=\hat{\theta}_1+\hat{\theta}_2 \tag{37}$$

and the dynamics of $\hat{\theta}_1$ and $\hat{\theta}_2$ are designed to be of the form

$$\frac{d\hat{\theta}_1}{d\tau}=\tilde{\alpha}(\tau) \ \ \text{with } \hat{\theta}_1(0)\ge\max\{1,\alpha(0)\} \tag{38}$$

$$\frac{d\hat{\theta}_2}{d\tau}=\frac{c_\theta}{\alpha(\tau)}q_\beta(x_1) \ \ \text{with } \hat{\theta}_2(0)\ge0 \tag{39}$$

[6] The notations $\max(a_1,\dots,a_n)$ and $\min(a_1,\dots,a_n)$ indicate the largest and smallest values, respectively, among the numbers $a_1,\dots,a_n$.

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where c_θ is any positive constant and $q_\beta(x_1)$ is a non-negative function. The motivation for defining $\hat{\theta}$ as comprised of two components, $\hat{\theta}_1$ and $\hat{\theta}_2$, can be seen in the proofs of Lemmas 2 and 3 in Section 4 and Appendix B. From (38) and (39), we see that $\hat{\theta}_1 \geq \alpha(\tau)$ for all $\tau \in [0, \infty)$ and $\hat{\theta}_2 \geq 0$ for all $\tau \in [0, \infty)$. Hence, we also have $\hat{\theta} \geq \alpha(\tau)$ for all $\tau \in [0, \infty)$.

It is shown in Appendix A that positive constants c and δ and functions ζ_1, R, Ω, and q_β can be constructed such that the following inequality (40) holds when either one of the following conditions hold: $r \geq R(x_1, \hat{\theta}, \dot{\hat{\theta}}) + \alpha(\tau)$ or $\dot{r} \geq \Omega(r, x_1, \hat{\theta}, \dot{\hat{\theta}}) + \tilde{\alpha}(\tau)$:

$$\frac{dV}{d\tau} \leq -\frac{\delta}{\alpha(\tau)}\left\{x_1^2\hat{\theta}\zeta_1\phi_{(1,2)} + \nu_o c r^2|\epsilon|^2 + \nu_c\phi_{(2,3)}r^2|\eta|^2\right\} + (\theta^* - \hat{\theta})\frac{1}{\alpha(\tau)}q_\beta(x_1). \tag{40}$$

4. Proof of Theorem 1

In this section, we complete the proof of Theorem 1 by showing that the dynamic control law designed in Section 3 above achieves prescribed-time stabilization in the sense of Theorem 1 given in Section 2. For this purpose, we first summarize several properties of the closed-loop system formed by (1) and the dynamic output-feedback controller designed in Section 3 as the Lemmas below. The proofs of the Lemmas are provided in the Appendix B.

Lemma 1. *The signal r satisfies*[7] *the inequality $\breve{r}(\tau) \geq \alpha(\tau)$ for all τ in the maximal interval of existence of solutions.*

Lemma 2. *At all time instants τ in the maximal interval of existence of solutions, the inequality $\frac{dV}{d\tau} \leq -\kappa V + (\theta^* - \hat{\theta})\chi(x_1, \tau)$ is satisfied with $\chi(x_1, \tau) = \frac{1}{\alpha(\tau)}q_\beta(x_1)$ and with a constant $\kappa > 0$.*

Lemma 3. *V is uniformly bounded over the maximal interval of existence of solutions.*

Lemma 4. *The signals $\hat{\theta}(a^{-1}(\tau))$, $\dot{\hat{\theta}}(a^{-1}(\tau))$, and $r(a^{-1}(\tau))$ grow at most polynomially in τ as $\tau \to \infty$ and solutions to the closed-loop dynamical system formed by the system (1) and the designed dynamic controller exist over the time interval $\tau \in [0, \infty)$.*

Lemma 5. *A finite positive constant τ_0 exists such that for all time instants $\tau \in [\tau_0, \infty)$, the inequality $\frac{dV}{d\tau} \leq -\kappa V$ is satisfied with a constant $\kappa > 0$.*

Lemma 6. *The signals V, x_1, $\sqrt{r}|\epsilon|$, and $\sqrt{r}|\eta|$ go to 0 exponentially as $\tau \to \infty$.*

Lemma 7. *The signals ϵ, η, and u go to zero exponentially as $\tau \to \infty$.*

Proof of Theorem 1. Since $x_2 = r\eta_2 - \zeta - r\epsilon_2$ and $x_i = r^{i-1}(\eta_i - \epsilon_i)$, $i = 3, \ldots, n$, we see from Lemmas 4, 6, and 7 that $x_2, \ldots, x_n$ all go to 0 exponentially as $\tau \to \infty$. Hence, x and u go to 0 exponentially as $\tau \to \infty$. Finally, since $\tau \to \infty$ corresponds to $t \to T$, the above properties hold as $t \to T$. Therefore, x and u go to 0 as $t \to T$, i.e., prescribed-time stabilization is attained.

Also, from the definition of functions f_i in (7), Assumption A1, and the fact that functions g_i can be chosen to be linear constant-coefficient combinations of $\phi_{(2,3)}, \ldots, \phi_{(n-1,n)}$ as noted in Section 3.7, it is seen that $\lim_{x_1 \to 0}\frac{f_i(x_1)}{x_1} = \frac{g_i(0)}{\phi_{(1,2)}(0)}$. Therefore, since x_1 goes to 0 exponentially as $t \to T$ while r grows at most polynomially, it is seen that $r^{i-1}f_i(x_1)$ goes to 0 as $t \to T$ for $i = 2, \ldots, n$. Also, from the definition of the function ζ in (14) and noting that $\hat{\theta}$ grows at most polynomially, it follows that $\zeta(x_1, \hat{\theta})$ goes to 0 as $t \to T$. Hence, from the definition of $\eta_2, \ldots, \eta_n$ in (13), it is seen that the observer state signals $\hat{x}_2, \ldots, \hat{x}_n$ also go to 0 as $t \to T$. Hence, x, u, and $\hat{x} = [\hat{x}_2, \ldots, \hat{x}_n]^T$ all go to 0 as t approaches the prescribed time T. □

Implementation of dynamics of r: From the design of the dynamics of r in (36), it was seen in Lemma 1 that r goes to ∞ as $\tau \to \infty$, which from (36) and (63) implies that $\dot{r}$ can also go to ∞ as $\tau \to \infty$. This can cause numerical difficulties in implementation. While some of the inherent numerical difficulties in implementation of prescribed-time stabilization can be addressed using the techniques discussed in Remark 5, it would be highly desirable to somehow avoid having a controller state variable whose derivative goes to ∞ (since derivative going to ∞ corresponds in numerical integration to requiring progressively smaller step sizes). This can indeed be achieved using a temporal scaling wherein instead of implementing the dynamics of r directly, one implements the dynamics in terms of a variable $\tilde{r}$ defined as $\tilde{r} = rz$ where $z : \mathcal{R} \to \mathcal{R}$ is a function of τ. Then, noting that $\dot{r} = \alpha(\tau)\frac{dr}{d\tau}$ and using (36), we have

$$\dot{\tilde{r}} = \alpha(\tau)\lambda\left(R(x_1, \hat{\theta}, \dot{\hat{\theta}}) + \alpha(\tau) - \frac{\tilde{r}}{z}\right) \times \left[\Omega\left(\frac{\tilde{r}}{z}, x_1, \hat{\theta}, \dot{\hat{\theta}}\right) + \tilde{\alpha}(\tau)\right]z + \tilde{r}\frac{\dot{z}}{z} \tag{41}$$

with initial value $\tilde{r}(0)$ picked such that $\tilde{r}(0) \geq \max\{1, \alpha(0)\}z(0)$. This scaling-based approach to implement the dynamics of r is summarized in the Lemma below (the proof of the Lemma is provided in the Appendix B).

Lemma 8. *A function $z(\tau)$ can be picked such that the signal $\tilde{r} = rz$ is such that $\tilde{r}$ and $\dot{\tilde{r}}$ are uniformly bounded over the time interval $\tau \in [0, \infty)$ and furthermore $\tilde{r}$ converges to 0 as $\tau \to \infty$.*

Remark 4. The designed controller is of dynamic order $(n+2)$ with the controller state comprising of the observer state variables $\hat{x}_2, \ldots, \hat{x}_n$, the dynamic scaling variable r (or equivalently $\tilde{r}$ as discussed above), and the dynamic adaptation state variables $\hat{\theta}_1$ and $\hat{\theta}_2$. The overall controller is given by the observer dynamics (6), the definition of scaled states $\eta_2, \ldots, \eta_n$ in (13), the control law for u in (16), the choice of ζ and ζ_1 in (14) and (61), the dynamics of the adaptation parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ in (38) and (39), and the dynamics of scaling parameter r in (36) or the equivalent implementation in terms of $\tilde{r}$.

Remark 5. The facts that (by definition) the function $\alpha(\tau)$ goes to ∞ as $t \to T$ and that (by construction) $\hat{\theta}$ and r also go to ∞ as $t \to T$ result in unbounded gains as $t \to T$. The characteristic that gains go to ∞ as t approaches the desired prescribed time T is shared with previous results on prescribed-time stabilization as well [23–25], where it is noted that indeed any approach for regulation in finite time (including optimal control with a terminal constraint and sliding mode control based approaches with time-varying gains) will share this characteristic. By the analysis above, we see that the unbounded gains do not result in an unbounded control input u (which indeed converges to 0). However, to alleviate any numerical difficulties in implementation that could be caused by unbounded gains as $t \to T$, a few approaches can be utilized as noted in [24] including adding a dead zone on the state x, a saturation on the control gains, and setting the terminal time in the controller implementation to be a larger value $\overline{T}$ than the desired finite time T. All of these approaches sacrifice asymptotic convergence of x to 0 as $t \to T$ (i.e., x goes not to 0, but to a small neighborhood of 0 as $t \to T$), but facilitate practical implementation by preventing unbounded gains [24].

Remark 6. While the prescribed time T can be any positive value, it is to be noted that the control effort required depends on the

[7] Note that, as defined in (24), the notation $\breve{r}(\tau)$ indicates the value of the signal r at the time instant τ as measured in the transformed time axis, i.e., $r(t) = \breve{r}(\tau)$.

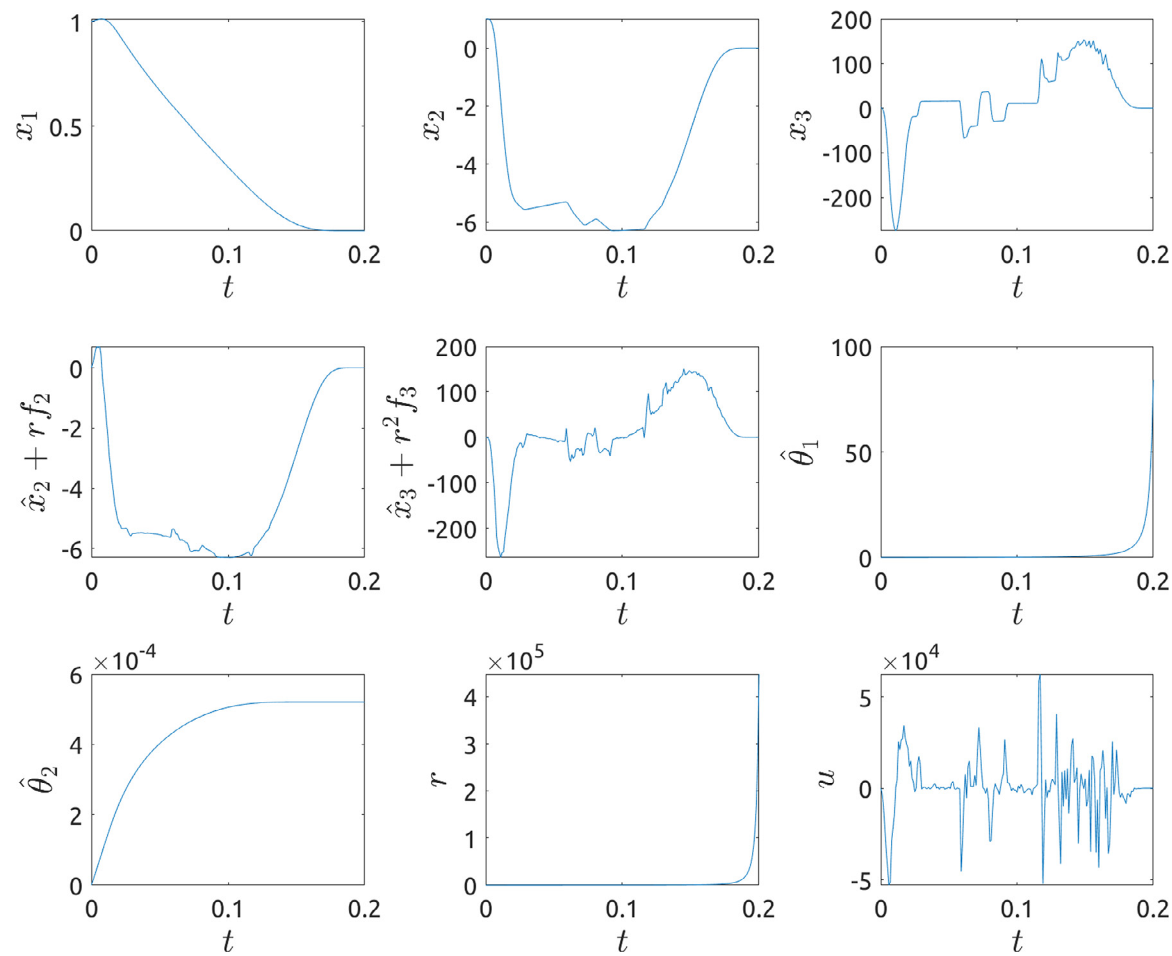

Fig. 1. Simulations for the closed-loop system (system (42) in closed loop with the prescribed-time stabilizing output-feedback controller).

value of T. In particular, making T small (close to zero) will, as could be expected, result in requiring large control effort (required magnitudes of the control input will go to ∞ as $T \to 0$, i.e., impulsive inputs). While the prescribed time T appears at multiple points in the control design, the increase in effective control gains with reducing T can be seen from (16) and the fact that the dynamics of r ensures that r is bigger than $\alpha(\tau)$ for all time τ. It can be seen that $\alpha(\tau)$ itself goes to ∞ as $t \to T$, as seen from the conditions on the functions a and α and the discussion in Remark 2. Hence, making T smaller results in applying larger control gains earlier (i.e., at smaller t), thereby resulting effectively in larger control inputs. This is physically reasonable since requiring the system state to move to zero from an arbitrary non-zero value within a fixed infinitesimal time window will undoubtedly require large control effort. At the other limit, making T larger can be seen to result in smaller control effort and in the limit as $T \to \infty$, the prescribed-time stabilization reduces to asymptotic stabilization.

5. An illustrative example

Consider the third-order system

$$\begin{aligned}
\dot{x}_1 &= (1 + x_1^2)x_2 + \theta_a \cos(x_2 x_3)x_1 \\
\dot{x}_2 &= (1 + x_1^4)x_3 + \theta_b x_1^5 \sin(x_3) + [1 + \cos(tu)]e^{x_1}x_2 \\
\dot{x}_3 &= u + \theta_c e^{x_1}x_1^3 + \sin(x_2 t)x_1^5 x_3 \\
y &= x_1
\end{aligned} \tag{42}$$

where θ_a, θ_b, and θ_c are uncertain parameters (with no known magnitude bounds). Only the output y is assumed to be measured. Here, $\phi_{(1,2)}(x_1) = 1 + x_1^2$, $\phi_{(2,3)}(x_1) = 1 + x_1^4$, and $\mu_0(x_1) = 1$. This system satisfies Assumption A1 with $\sigma = 1$. Assumption A2 is satisfied with $\Gamma(x_1) = 2\max(|x_1|, e^{x_1})$, $\phi_{(1,1)} = \phi_{(2,1)} = \phi_{(3,1)} = \phi_{(3,2)} = 0$, $\phi_{(2,2)} = 1$, $\phi_{(3,3)} = \frac{x_1^4}{2}$, $\beta_1 = c_\beta$, $\beta_2 = c_\beta x_1^4$, $\beta_3 = c_\beta e^{x_1} x_1^2$, and $\theta = \max\{c_\beta\theta_a, c_\beta\theta_b, c_\beta\theta_c\}$ with c_β being any positive constant. Note that the form of the terms $\theta_a \cos(x_2x_3)x_1$, $\theta_b x_1^5 \sin(x_3) + [1 + \cos(tu)]e^{x_1}x_2$, and $\theta_c e^{x_1}x_1^3 + \sin(x_2t)x_1^5x_3$ in the dynamics are not required to be known as long as bounds as in Assumption A2 are known. Assumption A3 is trivially satisfied since $n = 3$. Using the constructive procedure in [8,10], a symmetric positive-definite matrix P_c and functions k_2 and k_3 can be found to satisfy coupled Lyapunov inequalities (33) as $P_c = \tilde{a}_c\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$, $k_2 = 5\phi_{(2,3)}$, and $k_3 = 4\phi_{(2,3)}$, and with $\nu_c = 1.675\tilde{a}_c$, $\underline{\nu}_c = \tilde{a}_c$, and $\overline{\nu}_c = 5\tilde{a}_c$ with $\tilde{a}_c$ being any positive constant. Also, using the constructive procedure in [8,10], a symmetric positive-definite matrix P_o and functions g_2 and g_3 can be found to satisfy coupled Lyapunov inequalities (34) as $P_o = \tilde{a}_o\begin{bmatrix} 30 & -5 \\ -5 & 2.5 \end{bmatrix}$, $g_2 = 12\phi_{(2,3)}$, and $g_3 = 20\phi_{(2,3)}$, and with $\nu_o = 6.675\tilde{a}_o$, $\tilde{\nu}_o = 32.070\tilde{a}_o$, $\underline{\nu}_o = 3.698\tilde{a}_o$, and $\overline{\nu}_o = 33.802\tilde{a}_o$ with $\tilde{a}_o$ being any positive constant. With this choice of g_2 and g_3, the inequality (35) is satisfied with $\overline{G} = 23.324$. As in Section 3, the functions f_2 and f_3 are defined as $f_2(x_1) = 12\int_0^{x_1} \frac{(1+x_1^4)}{1+x_1^2} d\pi$ and $f_3(x_1) = 20\int_0^{x_1} \frac{(1+x_1^4)}{1+x_1^2} d\pi$. The required integral can be evaluated in closed form as $\int_0^{x_1} \frac{(1+x_1^4)}{1+x_1^2} d\pi = \frac{x_1^3}{3} - x_1 + 2\tan^{-1}(x_1)$. A

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reduced-order observer is designed as

$$\dot{\hat{x}}_2 = (1+x_1^4)[\hat{x}_3 + r^2 f_3(x_1)] - r g_2(x_1)[\hat{x}_2 + r f_2(x_1)] - \dot{r} f_2(x_1) \quad (43)$$

$$\dot{\hat{x}}_3 = u - r^2 g_3(x_1)[\hat{x}_2 + r f_2(x_1)] - 2\dot{r} r f_3(x_1). \quad (44)$$

Then, as in Section 3, defining $\eta_2 = \frac{\hat{x}_2 + r f_2(x_1) + \zeta(x_1,\hat{\theta})}{r}$ and $\eta_3 = \frac{\hat{x}_3 + r^2 f_3(x_1)}{r^2}$, the control input is designed as $u = -r^3[k_2\eta_2 + k_3\eta_3]$. The function α can be picked for instance as in (27) with any $a_0 > 0$. The functions R and Ω can be computed following the procedure in Appendix A and using sharper bounds taking the specific system structure into account and noting that several terms in the upper bounds vanish since $\phi_{(1,1)}$, etc., are zero for this system. The dynamics of r are then given by (36). The dynamic adaptation parameter $\hat{\theta}$ is defined as the combination $\hat{\theta}_1 + \hat{\theta}_2$ as in (37) and the dynamics of $\hat{\theta}_1$ and $\hat{\theta}_2$ are defined as in (38) and (39).

The performance of the prescribed-time stabilizing output-feedback controller is illustrated in Fig. 1. The terminal time is specified as $T = 0.2$ s. To avoid numerical issues, the effective terminal time $\overline{T}$ in implementation is defined as $\overline{T} = 0.205$ s. Also, $a_0 = \tilde{a}_c = 0.05$, $\tilde{a}_o = 1$, $\zeta_0 = 0.1$, and $c_\beta = c_\theta = 10^{-3}$. The initial condition for the system state vector $[x_1, x_2, x_3]^T$ is specified as $[1, 1, 1]^T$. Since the initial conditions for x_2 and x_3 are not known, the initial conditions for $\hat{x}_2$ and $\hat{x}_3$ are picked simply as the values that make the initial values of the estimates for x_2 and x_3 zero, i.e., such that $\hat{x}_2 + r f_2(x_1)$ and $\hat{x}_3 + r^2 f_3(x_1)$ are zero at time $t = 0$. Hence, the initial condition for $[\hat{x}_2, \hat{x}_3]^T$ is $[-10.85, -18.08]^T$. $\hat{\theta}_1$, $\hat{\theta}_2$, and r are initialized to be 0.05, 0, and 1, respectively. The values of the uncertain parameters θ_a, θ_b, and θ_c are picked for simulations as $\theta_a = \theta_b = \theta_c = 1$. The closed-loop trajectories and the control input signal are shown in Fig. 1.

6. Conclusion

A dynamic output-feedback prescribed-time stabilizing controller was developed for a general class of uncertain nonlinear strict-feedback-like systems. It was shown that given any desired convergence time, the proposed control design enables regulation of the state to the origin in the prescribed time irrespective of the initial state. The control design was based on our dual dynamic observer-controller design methodology and it was shown that while the underlying control design methodology was previously developed in the context of asymptotic stabilization, the design techniques can be applied to the prescribed-time setting by designing a novel temporal scale transformation, designing the scaling parameter dynamics using the transformed time axis, and incorporating temporal forcing terms in the scaling parameter dynamics. It was shown that the control input and observer state variables also converge to the origin in the prescribed time. Hence, the proposed control design provides both prescribed-time state estimation and prescribed-time state regulation for the considered class of uncertain nonlinear systems. A topic of on-going work is to determine if the types of modifications proposed here to the dual dynamic observer-controller design procedure to obtain prescribed-time results instead of asymptotic results can be applied to the various other classes of nonlinear systems to which the original dual dynamic observer-controller design procedures are applicable (e.g., feedforward systems, non-triangular systems, systems with time delays and uncertain appended dynamics).

Appendix A. Construction of positive constants c and δ and functions ζ_1, R, and Ω

Using (33) and (34), (32) reduces to

$$\frac{dV}{d\tau} = \frac{1}{\alpha(\tau)}\Big\{ - c r^2 \nu_o |\epsilon|^2 - c r^2 \tilde{\nu}_o \phi_{(2,3)} \epsilon_2^2 - r^2 \nu_c \phi_{(2,3)} |\eta|^2 + 2rc\epsilon^T P_o \overline{\Phi} + x_1[\phi_1 + (r\eta_2 - \zeta - r\epsilon_2)\phi_{(1,2)}] + 2r\eta^T P_c(\Phi - rG\epsilon_2 + H[\eta_2 - \epsilon_2] + \Xi)\Big\} - c\frac{dr}{d\tau}\underline{\nu}_o|\epsilon|^2 - \frac{dr}{d\tau}\underline{\nu}_c|\eta|^2 \quad (45)$$

Using Assumption A2, it is seen that the functions $\overline{\Phi}_i, i = 2, \ldots, n$ defined in (12) satisfy the inequality

$$|\overline{\Phi}_i| \le \frac{\Gamma(x_1)}{r^{i-1}}[\phi_{(i,1)}(x_1)|x_1| + \phi_{(i,2)}(x_1)|\zeta|] + \Gamma(x_1)|g_i(x_1)x_1|\frac{\phi_{(1,1)}(x_1)}{\phi_{(1,2)}(x_1)} + \theta\Big[\frac{\beta_i(x_1)}{r^{i-1}}|x_1| + |g_i(x_1)x_1|\frac{\beta_1(x_1)}{\phi_{(1,2)}(x_1)}\Big] + \frac{\Gamma(x_1)}{r^{i-1}}\sum_{j=2}^{i}\phi_{(i,j)}(x_1)r^{j-1}|\eta_j - \epsilon_j|. \quad (46)$$

Hence, using (14) and (35) and the property that $r \ge 1$,

$$|\overline{\Phi}| \le \frac{\Gamma(x_1)|x_1|}{r}[|\tilde{\phi}_1| + |\hat{\theta}\zeta_1(x_1)||\tilde{\phi}_2|] + \Gamma(x_1)||\tilde{A}(x_1)||(|\eta| + |\epsilon|) + \Gamma(x_1)|x_1|\frac{\phi_{(1,1)}(x_1)}{\phi_{(1,2)}(x_1)}\overline{G}\phi_{(2,3)}(x_1) + \theta\Big[\frac{|\beta||x_1|}{r} + |x_1|\frac{\beta_1(x_1)}{\phi_{(1,2)}(x_1)}\overline{G}\phi_{(2,3)}(x_1)\Big] \quad (47)$$

where $\tilde{\phi}_1 = [\phi_{(2,1)}, \phi_{(3,1)}, \ldots, \phi_{(n,1)}]^T$, $\tilde{\phi}_2 = [\phi_{(2,2)}, \phi_{(3,2)}, \ldots, \phi_{(n,2)}]^T$, $\beta = [\beta_2, \ldots, \beta_n]$, $||.||$ denotes the Frobenius norm of a matrix, and $\tilde{A}$ denotes the $(n-1)\times(n-1)$ matrix with $(i, j)^{th}$ element $\phi_{(i+1,j+1)}$ at locations on and below the diagonal and zeros everywhere else. Note that $|\tilde{\phi}_1| = \sqrt{\sum_{i=2}^n \phi_{(i,1)}^2(x_1)}$, $|\tilde{\phi}_2| = \sqrt{\sum_{i=2}^n \phi_{(i,2)}^2(x_1)}$, and $|\beta| = \sqrt{\sum_{i=2}^n \beta_i^2(x_1)}$.

Therefore (with some conservative overbounding for algebraic simplicity), the term $2rc\epsilon^T P_o\overline{\Phi}$ can be upper bounded as[8]

$$2rc\epsilon^T P_o\overline{\Phi} \le (1+\theta^2)\zeta_0\phi_{(1,2)}x_1^2 + 3rc\lambda_{max}(P_o)\Gamma||\tilde{A}||(|\eta|^2 + |\epsilon|^2) + \frac{c^2\lambda_{max}^2(P_o)|\epsilon|^2}{\zeta_0\phi_{(1,2)}}\Gamma^2[|\tilde{\phi}_1| + |\hat{\theta}\zeta_1||\tilde{\phi}_2|]^2 + \frac{1}{\zeta_0\phi_{(1,2)}}c^2|\epsilon|^2\lambda_{max}^2(P_o)|\beta|^2 + 8\frac{c}{\nu_0}\lambda_{max}^2(P_o)x_1^2\frac{[\Gamma^2\phi_{(1,1)}^2 + \theta^2\beta_1^2]}{\phi_{(1,2)}^2}\overline{G}^2\phi_{(2,3)}^2 + \frac{c}{4}r^2\nu_o|\epsilon|^2 \quad (48)$$

where $\zeta_0 > 0$ is any constant. Using Assumptions A1–A3 and the property $r \ge 1$, the other uncertain terms appearing in (32) can also be upper bounded as (with some conservative overbounding for algebraic simplicity)

$$x_1\phi_1 \le x_1^2\Gamma(x_1)\phi_{(1,1)}(x_1) + \theta\beta_1 x_1^2 \quad (49)$$

$$x_1 r(\eta_2 - \epsilon_2)\phi_{(1,2)} \le \frac{\nu_c}{4}r^2\phi_{(2,3)}|\eta|^2 + \frac{1}{\nu_c}x_1^2\frac{\phi_{(1,2)}^2}{\phi_{(2,3)}} + cr^2\frac{\tilde{\nu}_o}{4}\phi_{(2,3)}\epsilon_2^2 + \frac{1}{c\tilde{\nu}_o}x_1^2\frac{\phi_{(1,2)}^2}{\phi_{(2,3)}} \quad (50)$$

$$2r\eta^T P_c\Phi \le \frac{8}{\nu_c}\lambda_{max}^2(P_c)\overline{G}^2\frac{\phi_{(2,3)}}{\phi_{(1,2)}^2}[\Gamma^2\phi_{(1,1)}^2 + \theta^2\beta_1^2]x_1^2 + \frac{\nu_c}{4}r^2\phi_{(2,3)}|\eta|^2 \quad (51)$$

[8] Given a symmetric positive-definite matrix P, $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote its maximum and minimum eigenvalues, respectively.

$$-2r^2\eta^T P_c G\epsilon_2 \le \frac{8}{\nu_c}\phi_{(2,3)} r^2 \lambda_{max}^2(P_c)\overline{G}^2\epsilon_2^2 + r^2\frac{\nu_c}{8}\phi_{(2,3)}|\eta|^2 \tag{52}$$

$$2r\eta^T P_c H(\eta_2 - \epsilon_2) \le 3r\lambda_{max}(P_c)\hat{\theta}\phi_{(1,2)}|\zeta_1' x_1 + \zeta_1|[|\eta|^2 + |\epsilon|^2] \tag{53}$$

$$2r\eta^T P_c \Xi \le (1+\theta^2)\zeta_0\phi_{(1,2)}x_1^2 + \frac{2}{\zeta_0\phi_{(1,2)}}\lambda_{max}^2(P_c)|\eta|^2\Big[\dot{\hat{\theta}}^2\zeta_1^2 + \Big((\Gamma\phi_{(1,1)} + |\zeta_1|\hat{\theta}\phi_{(1,2)})^2 + \frac{\beta_1^2}{2}\Big)(\zeta_1' x_1 + \zeta_1)^2\hat{\theta}^2\Big]. \tag{54}$$

Picking $c > 0$ such that

$$c \ge \frac{32\lambda_{max}^2(P_c)\overline{G}^2}{3\tilde{\nu}_o\nu_c}, \tag{55}$$

the inequality (52) reduces to

$$-2r^2\eta^T P_c G\epsilon_2 \le r^2\frac{\nu_c}{8}\phi_{(2,3)}|\eta|^2 + \frac{3}{4}c\tilde{\nu}_o\phi_{(2,3)}r^2\epsilon_2^2. \tag{56}$$

Using the inequalities in (45) and (48)–(56) yields

$$\begin{aligned}\frac{dV}{d\tau} \le \frac{1}{\alpha(\tau)}\Big\{ & -x_1\zeta\phi_{(1,2)} + q_1(x_1)\phi_{(1,2)}x_1^2 + \theta^* q_2(x_1)\phi_{(1,2)}x_1^2 \\ & - \frac{3}{4}\nu_o c r^2|\epsilon|^2 - \frac{3}{8}\nu_c\phi_{(2,3)}r^2|\eta|^2 \\ & + r w_1(x_1,\hat{\theta},\dot{\hat{\theta}})\phi_{(1,2)}[|\eta|^2 + |\epsilon|^2]\Big\} \\ & - c\underline{\nu}_o\frac{dr}{d\tau}|\epsilon|^2 - \underline{\nu}_c\frac{dr}{d\tau}|\eta|^2\end{aligned} \tag{57}$$

where $\theta^* = 1 + \theta + \theta^2$ and

$$\begin{aligned}q_1(x_1) = 2\zeta_0 &+ \Gamma(x_1)\frac{\phi_{(1,1)}(x_1)}{\phi_{(1,2)}(x_1)} \\ &+ 8\frac{c}{\nu_o}\lambda_{max}^2(P_o)\Gamma^2(x_1)\frac{\phi_{(1,1)}^2(x_1)}{\phi_{(1,2)}^3(x_1)}\overline{G}^2\phi_{(2,3)}^2(x_1) \\ &+ \frac{1}{\nu_c}\frac{\phi_{(1,2)}(x_1)}{\phi_{(2,3)}(x_1)} + \frac{1}{c\tilde{\nu}_o}\frac{\phi_{(1,2)}(x_1)}{\phi_{(2,3)}(x_1)} \\ &+ \frac{8}{\nu_c}\lambda_{max}^2(P_c)\overline{G}^2\frac{\phi_{2,3}(x_1)}{\phi_{(1,2)}^3(x_1)}\Gamma^2(x_1)\phi_{(1,1)}^2(x_1)\end{aligned} \tag{58}$$

$$q_2(x_1) = 2\zeta_0 + \frac{\beta_1(x_1)}{\phi_{(1,2)}(x_1)} + \Big[\frac{8c}{\nu_o}\lambda_{max}^2(P_o)\phi_{(2,3)}(x_1) + \frac{8}{\nu_c}\lambda_{max}^2(P_c)\Big] \times \frac{\beta_1^2(x_1)\overline{G}^2\phi_{(2,3)}(x_1)}{\phi_{(1,2)}^3(x_1)} \tag{59}$$

$$\begin{aligned}w_1(x_1,\hat{\theta},\dot{\hat{\theta}}) = &\frac{c^2\lambda_{max}^2(P_o)}{\zeta_0\phi_{(1,2)}^2(x_1)}\Gamma^2(x_1)[|\tilde{\phi}_1(x_1)| + |\hat{\theta}\zeta_1(x_1)||\tilde{\phi}_2(x_1)|]^2 \\ &+ 3c\frac{\lambda_{max}(P_o)}{\phi_{(1,2)}(x_1)}\Gamma(x_1)||\tilde{A}(x_1)|| + \frac{2}{\zeta_0\phi_{(1,2)}^2(x_1)}\lambda_{max}^2(P_c)\Big[\dot{\hat{\theta}}^2\zeta_1^2(x_1) \\ &+ \Big((\Gamma(x_1)\phi_{(1,1)}(x_1) + |\zeta_1(x_1)|\hat{\theta}\phi_{(1,2)}(x_1))^2 + \frac{\beta_1^2(x_1)}{2}\Big) \\ &\times (\zeta_1'(x_1)x_1 + \zeta_1(x_1))^2\hat{\theta}^2\Big] + 3\lambda_{max}(P_c)\hat{\theta}|\zeta_1'(x_1)x_1 + \zeta_1(x_1)| \\ &+ \frac{1}{\zeta_0\phi_{(1,2)}^2(x_1)}c^2\lambda_{max}^2(P_o)|\beta(x_1)|^2\end{aligned} \tag{60}$$

Note that the functions $q_1(x_1)$, $q_2(x_1)$, and $w_1(x_1,\hat{\theta},\dot{\hat{\theta}})$ involve only known functions and quantities.

In the equations above, note that $\dot{\hat{\theta}}$ denotes $\frac{d\hat{\theta}}{dt}$. We have $\frac{d\hat{\theta}}{d\tau} = \frac{1}{\alpha(\tau)}\frac{d\hat{\theta}}{dt}$.

Design the function ζ_1 such that

$$\frac{1}{4}\zeta_1(x_1) = \max\Big\{\underline{\zeta}, q_1(x_1) + q_2(x_1)\Big\} \tag{61}$$

with $\underline{\zeta}$ being any positive constant.

Pick the functions R and Ω as

$$R(x_1,\hat{\theta},\dot{\hat{\theta}}) = \max\Big\{1, \frac{4w_1(x_1,\hat{\theta},\dot{\hat{\theta}})\phi_{(1,2)}(x_1)}{\nu_c\phi_{(2,3)}(x_1)}, \frac{2w_1(x_1,\hat{\theta},\dot{\hat{\theta}})\phi_{(1,2)}(x_1)}{\nu_o c}\Big\} \tag{62}$$

$$\Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}}) = \frac{r w_1(x_1,\hat{\theta},\dot{\hat{\theta}})\phi_{(1,2)}(x_1)}{a_0}\max\Big\{\frac{1}{c\underline{\nu}_o}, \frac{1}{\underline{\nu}_c}\Big\}. \tag{63}$$

Pick the function q_β as

$$q_\beta(x_1) = q_2(x_1)\phi_{(1,2)}(x_1)x_1^2. \tag{64}$$

If $r \ge R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau)$, then we note from (62) that

$$\frac{dV}{d\tau} \le \frac{1}{\alpha(\tau)}\Big\{-\frac{3}{4}x_1^2\hat{\theta}\zeta_1\phi_{(1,2)} - \frac{1}{4}\nu_o c r^2|\epsilon|^2 - \frac{1}{8}\nu_c\phi_{(2,3)}r^2|\eta|^2\Big\} + (\theta^* - \hat{\theta})\chi(x_1,\tau). \tag{65}$$

If $\frac{dr}{d\tau} \ge \Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}}) + \tilde{\alpha}(\tau)$, then using (61) and (63), we have

$$\frac{dV}{d\tau} \le \frac{1}{\alpha(\tau)}\Big\{-\frac{3}{4}x_1^2\hat{\theta}\zeta_1\phi_{(1,2)} - \frac{3}{4}\nu_o c r^2|\epsilon|^2 - \frac{3}{8}\nu_c\phi_{(2,3)}r^2|\eta|^2\Big\} + (\theta^* - \hat{\theta})\chi(x_1,\tau). \tag{66}$$

Therefore, it is seen that (40) is satisfied with $\delta = \frac{1}{8}$ when either one of the following conditions hold: $r \ge R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau)$ or $\dot{r} \ge \Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}}) + \tilde{\alpha}(\tau)$.

Appendix B

Proof of Lemma 1. From (36), we note that $\frac{dr}{d\tau} \ge \tilde{\alpha}(\tau)$ at any time instant at which $r \ge R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau)$ is not satisfied. Since $R(x_1,\hat{\theta},\dot{\hat{\theta}}) > 0$, this implies that any time instant at which $r > \alpha(\tau)$ is not satisfied, we definitely have $\frac{dr}{d\tau} \ge \tilde{\alpha}(\tau)$. If the claim in Lemma 1 is not satisfied, there should exist some time instants τ at which $r(\tau) < \alpha(\tau)$. Taking the infimum τ_{min} of all such time instants, we note that since $\check{r}(0) \ge \alpha(0)$ from (36), we should have $\tau_{min} > 0$ and such that $\check{r}(\tau_{min}) = \alpha(\tau_{min})$ and with some τ in an infinitesimal open interval after τ_{min} such that $r(\tau) < \alpha(\tau)$. However, $\check{r}(\tau_{min}) = \alpha(\tau_{min})$ implies $\frac{dr}{d\tau} \ge \tilde{\alpha}(\tau) = \frac{d\alpha}{d\tau}$. Hence, we should have $\check{r}(\tau) \ge \alpha(\tau)$ in an infinitesimal open interval after τ_{min}, thus leading to a contradiction implying that the claim of Lemma 1 is satisfied. □

Proof of Lemma 2. Consider two cases: (a) $r < R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau)$; (b) $r \ge R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau)$. From the design of λ, we see that under Case (a), $\lambda(R(x_1,\hat{\theta},\dot{\hat{\theta}}) + \alpha(\tau) - r) = 1$ and therefore $\frac{dr}{d\tau} \ge \Omega(r,x_1,\hat{\theta},\dot{\hat{\theta}}) + \tilde{\alpha}(\tau)$. Hence, from the construction in Appendix A, it is seen that (40) holds under both the cases (a) and (b). Since one of these cases should definitely hold at all times, we see that (40) holds for all times in the maximal interval of existence of solutions. From Lemma 1 and the property $\hat{\theta} \ge \alpha(\tau)$, which followed from the design of the dynamics of $\hat{\theta}_1$ and $\hat{\theta}_2$ in (38) and (39), we see that

$$\frac{dV}{d\tau} \le -\delta\left\{x_1^2\zeta_1\phi_{(1,2)} + \nu_o cr|\epsilon|^2 + \nu_c\phi_{(2,3)}r|\eta|^2\right\} + (\theta^* - \hat{\theta})\chi(x_1,\tau). \quad (67)$$

From the definition of V from (31), this implies that

$$\frac{dV}{d\tau} \le -\kappa V + (\theta^* - \hat{\theta})\chi(x_1,\tau) \quad (68)$$

where $\kappa = \min\left\{2\delta\underline{\zeta}\sigma, \frac{\delta\nu_o}{\lambda_{max}(P_o)}, \frac{\delta\nu_c\sigma}{\lambda_{max}(P_c)}\right\}$. □

Proof of Lemma 3. Define $\overline{V} = V + \frac{1}{c_\theta}(\hat{\theta}_2 - \theta^*)^2$. Noting from (39) that we have $\dot{\hat{\theta}}_2 = c_\theta\chi(x_1,\tau)$ where $\chi(x_1,\tau)$ is as defined in Lemma 2, and noting that $\hat{\theta}_1 \ge 0$ for all τ and that $\chi(x_1,\tau) \ge 0$ for all x_1 and τ, it is seen from Lemma 2 that $\dot{\overline{V}} \le -\kappa V \le 0$. Hence, $\overline{V}$ and therefore V are uniformly bounded over the maximal interval of existence of solutions. □

Proof of Lemma 4. By the design of the dynamics of $\hat{\theta}_1$ in (38) and the conditions imposed on the function $\alpha(\tau)$, we see that $\hat{\theta}_1(a^{-1}(\tau))$ and $\dot{\hat{\theta}}_1(a^{-1}(\tau))$ are polynomially upper bounded in τ. From Lemma 3 and the definition of V in (31), it is seen that the signal x_1 is uniformly bounded over the maximal interval of existence of solutions. Hence, from the dynamics of $\hat{\theta}_2$ in (39), it follows that $\hat{\theta}_2(a^{-1}(\tau))$ and $\dot{\hat{\theta}}_2(a^{-1}(\tau))$ are also polynomially upper bounded in τ. Noting that $\hat{\theta}$ and $\dot{\hat{\theta}}$ appear polynomially in the definition of w_1, it follows that $R(x_1, \hat{\theta}, \dot{\hat{\theta}})$ grows at most polynomially in τ. From the dynamics of r in (36), it is seen that at each time τ, we either have $r \le R(x_1, \hat{\theta}, \dot{\hat{\theta}}) + \alpha(\tau) + \epsilon_r$ or $\dot{r} = 0$. Note that $\alpha(\tau)$ and $\tilde{\alpha}(\tau)$ are polynomially upper bounded in τ due to the conditions imposed on $\alpha(\tau)$ in Section 3.5. Hence, $R(x_1, \hat{\theta}, \dot{\hat{\theta}}) + \alpha(\tau) + \epsilon_r$ and therefore $r(a^{-1}(\tau))$ grow at most polynomially as a function of time τ. Noting from Lemma 3 that V (and therefore x_1, $\sqrt{r}\epsilon$, and $\sqrt{r}\eta$) remains bounded over the maximal interval of existence of solutions, it follows that solutions to the closed-loop dynamical system exist over the time interval $\tau \in [0, \infty)$. □

Proof of Lemma 5. By the dynamics of $\hat{\theta}_1$ and $\hat{\theta}_2$ in (38) and (39), it is seen that $\hat{\theta}$ is monotonically increasing with τ and goes to ∞ as $\tau \to \infty$. Hence, a finite positive constant τ_0 exists such that $\hat{\theta} \ge \theta^*$ for all $\tau \ge \tau_0$. Hence, using Lemma 2, it follows that for all $\tau \ge \tau_0$, the inequality $\frac{dV}{d\tau} \le -\kappa V$ is satisfied. □

Proof of Lemma 6. From Lemma 5, V goes to 0 exponentially as $\tau \to \infty$. From the definition of V from (31), it follows that x_1, $\sqrt{r}|\epsilon|$, and $\sqrt{r}|\eta|$ go to 0 exponentially as $\tau \to \infty$. □

Proof of Lemma 7. From Lemma 6, $\sqrt{r}|\epsilon|$ and $\sqrt{r}|\eta|$ go to 0 exponentially as $\tau \to \infty$ while from Lemma 4, r grows at most polynomially in τ. Hence, we see that $|\epsilon|$ and $|\eta|$ go to 0 exponentially as $\tau \to \infty$. From the form of u in (16), this implies that u goes to 0 exponentially as $\tau \to \infty$. □

Proof of Lemma 8. Pick, for example, $z(\tau) = e^{-k_z\tau}$ with $k_z > 0$ being any constant. Hence, we have

$$\dot{\tilde{r}} = \alpha(\tau)\lambda(R(x_1, \hat{\theta}, \dot{\hat{\theta}}) + \alpha(\tau) - e^{k_z\tau}\tilde{r}) \times [\Omega(e^{k_z\tau}\tilde{r}, x_1, \hat{\theta}, \dot{\hat{\theta}}) + \tilde{\alpha}(\tau)]e^{-k_z\tau} - k_z\alpha(\tau)\tilde{r}. \quad (69)$$

From Lemma 4, r grows at most polynomially in τ. Hence, it follows that $\tilde{r} = rz$ is uniformly bounded over the time interval $\tau \in [0, \infty)$ since $z(\tau) = e^{-k_z\tau}$ goes to 0 exponentially as $\tau \to \infty$. Furthermore, $\tilde{r}$ asymptotically goes to 0 as $\tau \to \infty$. Since $\alpha(\tau)$, $\tilde{\alpha}(\tau)$, $\hat{\theta}$, $\dot{\hat{\theta}}$, and r grow at most polynomially in τ while x goes to 0 as $\tau \to \infty$, it follows from the definition of Ω in (63) and the definition of w_1 in (60) that $\Omega(r, x_1, \hat{\theta}, \dot{\hat{\theta}})$ grows at most polynomially in τ. Also, since $\tilde{\alpha}(\tau)$ and r grow at most polynomially in τ while $z(\tau)$ goes to 0 exponentially as $\tau \to \infty$, it follows that $\alpha(\tau)\tilde{r} = \alpha(\tau)z(\tau)r$ goes to 0 exponentially as $\tau \to \infty$. Also, from the fact that $\lambda(.)$ is, by definition, constrained to be in the interval [0,1] and the fact that $\alpha(\tau)$, $\tilde{\alpha}(\tau)$, and $\Omega(r, x_1, \hat{\theta}, \dot{\hat{\theta}})$ grow at most polynomially in τ, it is seen that $\tilde{r}$ is uniformly bounded over the time interval $[0, \infty)$. □

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejcon.2019.09.005.

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