Automatica 58 (2015) 131–138

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive sinusoidal disturbance cancellation for unknown LTI systems despite input delay[☆]

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ARTICLE INFO

Article history:
Received 17 September 2014
Received in revised form
26 January 2015
Accepted 29 April 2015
Available online 2 June 2015

Keywords:
Adaptive control
Delay
Boundary control
Disturbance estimation

ABSTRACT

Solutions exist for the problems of canceling sinusoidal disturbances and compensating input delays. In this paper, two problems are considered simultaneously and an adaptive controller is designed to cancel unknown sinusoidal disturbances forcing an unknown linear time-invariant system in controllable canonical form despite input delay. The design is based on three steps, (1) parametrization of the sinusoidal disturbance as the output of a known feedback system with an unknown output vector that depends on unknown disturbance parameters, (2) representation of the delay as a transport PDE, (3) design of the adaptive controller by using the backstepping boundary control technique for PDEs. It is proven that the equilibrium of the closed loop system is stable and the state of the considered system converges to zero as $t \to \infty$ with perfect disturbance estimation. The effectiveness of the controller is illustrated with a simulation example of a second order system.

1. Introduction

The problem of canceling sinusoidal disturbances in dynamical systems is a fundamental control problem, with many applications such as active noise control (Bodson, Jensen, & Douglas, 2001), rotating mechanisms control (Gentili & Marconi, 2003), and marine vehicles (Basturk & Krstic, 2013b; Basturk, Rosenthal, & Krstic, 2013; Marconi, Isidori, & Serrani, 2002). The common method to approach this problem is the internal model principle for which a general solution is given in Francis and Wonham (1975) and Johnson (1971) in the case of linear systems. In the internal model approach, the disturbance is modeled as the output of a linear dynamic system which is called an exosystem.

The output regulation problem for minimum phase, uncertain nonlinear systems is solved in Serrani and Isidori (2000), Serrani, Isidori, and Marconi (2001), and extended for non-minimum phase plants in Marconi, Isidori, and Serrani (2004). Moreover, designs for nonlinear systems are proposed in Ding (2003), Marino and Santosuosso (2005), Marino and Tomei (2005) and Nikiforov (2001). The regulation of a linear time-varying system for unknown exosystem is considered in Marino and Tomei (2000). Disturbance cancelation and output regulation designs also exist for continuous-time LTI systems (Bobtsov & Pyrkin, 2009; Bodson & Douglas, 1997; Serrani, 2006; Zhang & Serrani, 2006) and discrete-time LTI systems (Guo & Bodson, 2009). Rejection algorithms are also given by state derivative feedback for both known (Basturk & Krstic, 2012a, 2013a) and unknown (Basturk & Krstic, 2012b, 2014) LTI systems.

On the other hand, the input delay causes significant stabilization problems in many dynamical systems including various chemical systems, hydraulically actuated systems, and combustion systems which may be driven by unknown sinusoidal disturbances. Therefore, the design of controllers for the stabilization of systems with delays continues to be an active research area. Controllers for both linear (Artstein, 1982; Bekiaris-Liberis & Krstic, 2010b; Fiagbedzi & Pearson, 1986; Jankovic, 2009, 2010; Kwon & Pearson, 1980; Mondie & Michiels, 2003; Olbrot, 1978; Zhong, 2006a,b) and nonlinear systems (Bresch-Pietri & Krstic, 2014; Krstic, 2008, 2010; Mazenc, Mondie, & Francisco, 2004; Mazenc & Niculescu, 2011) exist in the literature, many of which are based on predictor-like techniques. The solution for time varying delays in nonlinear systems is given in Bekiaris-Liberis and Krstic (2012). Input delays that depend on state in nonlinear systems are considered in Bekiaris-Liberis, Jankovic, and Krstic (2012). Moreover, adaptive control schemes for uncertain plants can be found in Evesque, Annaswamy, Niculescu, and Dowling (2003) and

[☆] The material in this paper will be presented at the 2015 American Control Conference, July 1–3, 2015, Chicago, IL, USA. This paper was recommended for publication in revised form by Associate Editor Akira Kojima under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2015.05.013
0005-1098/

Niculescu and Annaswamy (2003). The robustness of the standard adaptive backstepping technique with respect to time delay in input and unmodeled dynamics in the system is studied in Zhou, Wang, and Wen (2009). Adaptive controllers for unknown delays are given in Bekiaris-Liberis and Krstic (2010a) and Bresch-Pietri and Krstic (2010). The designs of trajectory tracking for uncertain linear systems with unknown input delay are given in Bresch-Pietri and Krstic (2009) and Bresch-Pietri, Chauvin, and Petit (2012). Controller designs for canceling unknown sinusoidal disturbances with input delay are considered in Bobtsov and Pyrkin (2008), Bobtsov, Kolyubin, and Pyrkin (2010) and Pyrkin et al. (2010a,b). However, in these references, the disturbance cancellation algorithms are given for systems whose parameters are assumed to be known. We present a method that does not require to know the actual value of the parameters of the system. This is the main contribution of the presented work and provides an important advantage for the application of the result.

In this note, an adaptive controller is designed to estimate and cancel the unknown sinusoidal disturbances forcing general LTI systems in the controllable canonical form with input delay and unknown system parameters by full state feedback. The unknown disturbance is represented in a parameterized form by using the technique given in Nikiforov (2004). The essence of the approach for the compensation of the input delay is predictor feedback which has been shown in Krstic and Symslayev (2008a) to be a form of backstepping boundary control for PDEs (Krstic & Symslayev, 2008b). The results given in Krstic and Symslayev (2008a) and Nikiforov (2004) allow us to reformulate the problem as an adaptive control problem for an uncertain PDE–ODE coupled system. Update laws for uncertain parameters are based on normalized Lyapunov-based tuning which is a similar approach given in Bresch-Pietri and Krstic (2009). Finally, it is proven that the equilibrium of the closed loop system is stable and the state converges to zero as $t \to \infty$ with perfect disturbance estimation.

In Section 2, the problem is introduced. The representation of unknown sinusoidal disturbances is given in Section 3. In Section 4, the main design is presented and the stability theorem is stated. In Section 5, the proof of the stability theorem is given. A simulation example is presented in Section 6.

2. Problem statement

We consider the single-input LTI system

$$\dot{X}(t) = A_0 X(t) + B(\gamma^T X(t) + \nu(t) + bU(t-D)), \tag{1}$$

where $A_0 = \begin{bmatrix} 0_{n-1} & I_{n-1} \\ 0 & 0_{n-1}^T \end{bmatrix}$, $B = \begin{bmatrix} 0_{n-1} \\ 1 \end{bmatrix}$, $\gamma = [a_1, \ldots, a_n]^T$, with $0_{n-1} = [0, \ldots, 0]^T \in \mathbb{R}^{n-1}$, the known input delay $D \in \mathbb{R}$, the state $X = [X_1, \ldots, X_n]^T \in \mathbb{R}^n$, the input $U \in \mathbb{R}$, and the unknown sinusoidal disturbance $\nu \in \mathbb{R}$ given by

$$\nu(t) = g \sin(\omega t + \phi), \tag{2}$$

where $\omega, g, \phi \in \mathbb{R}$.

The sinusoidal disturbance ν can be represented as the output of a linear exosystem,

$$\dot{W}(t) = SW(t), \tag{3}$$

$$\nu(t) = h^T W(t), \tag{4}$$

where $W(t) \in \mathbb{R}^2$, $S = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$, $h^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$.The matrix S depends on the unknown frequency of the sinusoidal disturbance ν, while the uncertainty of amplitude and phase is related to the unknown initial condition of (3).

The system parameters γ and b are unknown, the input delay D is known. The disturbance $\nu(t)$ is not measured. The measurement of the state X is available for feedback.

We make the following assumptions regarding the plant (1) and the exosystem (3)–(4):

Assumption 1. $\omega_{\max} \geq \omega \geq \omega_{\min} > 0$, where $\omega_{\max}$ and $\omega_{\min}$ are known.

Assumption 2. $g_{\max} \geq |g|$ where $g_{\max}$ is known.

Assumption 3. $a_i \in [a_{\min}, a_{\max}]$ for $i = 1, \ldots, n$. The bounds $a_{\min}, a_{\max}$ are known and $\bar{a} = \max\{|a_{\max}|, |a_{\min}|\}$.

Assumption 4. The sign of b is known and $b_{\max} \geq |b| \geq b_{\min} > 0$ where $b_{\max}, b_{\min}$ are known.

The main aim is to design an adaptive controller to achieve the stability of the equilibrium of the closed loop system and the convergence of the state $X(t)$ to zero as $t \to \infty$ despite the input delay, unmeasured sinusoidal disturbance and uncertain system parameters.

3. Disturbance representation

The disturbance is parameterized by following (Nikiforov, 2004). Let $G \in \mathbb{R}^{2\times 2}$ be a Hurwitz matrix with distinct eigenvalues and let (G, l) be a controllable pair. Since (h^T, S) is observable and the spectra of S and G are disjoint the unique and nonsingular solution $M \in \mathbb{R}^{2\times 2}$ of the Sylvester equation

$$MS - GM = lh^T \tag{5}$$

is invertible (Chen, 1984). The change of coordinates $Z(t) = MW(t)$ transforms the exosystem (3)–(4) into the form

$$\dot{Z}(t) = GZ(t) + l\nu(t), \tag{6}$$

$$\nu(t) = \theta_s^T Z(t), \tag{7}$$

$$\dot{\nu}(t) = \theta_d^T Z(t), \tag{8}$$

where $\theta_s^T = h^T M^{-1}, \theta_d^T = h^T S M^{-1}$. Since there is a delay in the input, the disturbance $\nu(t)$ needs to be represented with a delayed $Z(t)$ to be able to design an adaptive controller. The following lemma establishes this result.

Lemma 1. *The disturbance can be represented as*

$$\nu(t) = \theta^T Z(t-D), \tag{9}$$

where

$$\theta = \cos(D\omega)\theta_s + \frac{\sin(D\omega)}{\omega}\theta_d. \tag{10}$$

Proof. Using (2), the necessary trigonometric calculations yield

$$\nu(t+D) = \cos(D\omega)\nu(t) + \frac{\sin(D\omega)}{\omega}\dot{\nu}(t). \tag{11}$$

Substituting (7) and (8) into (11), we get (9). ■

Since $\nu(t)$ is not measured, the state $Z(t)$ cannot be used in the design. To overcome this problem, an observer is designed. The following lemma establishes the properties of the observer and the representation of the unknown disturbance.

Lemma 2. *The unknown disturbance $\nu(t)$ can be represented in the form*

$$\nu(t) = \theta^T \Xi_X(t-D) + \sum_{i=1}^{n} \beta_i^T \Xi_i(t-D) + \beta_u^T \Xi_U(t-D) + \theta_\delta^T \delta(t) \tag{12}$$

where $\theta_\delta = e^{-GD}\theta$, $\beta_u = b\theta$, $\beta_i = a_i\theta$ for $i = 1, \ldots, n$.

The observer filters are given by

$$\dot{\Xi}_0(t) = G\left(\Xi_0(t) + NX(t)\right), \tag{13}$$

$$\dot{\Xi}_i(t) = G\Xi_i(t) - lX_i(t), \quad i = 1, \dots, n \tag{14}$$

$$\dot{\Xi}_U(t) = G\Xi_U(t) - lU(t-D), \tag{15}$$

and

$$\Xi_X = \Xi_0(t) + NX(t) \tag{16}$$

with $2 \times n$ *matrix* N *which is given by* $N = \frac{1}{B^T B} lB^T$, *where the given* N *is one of the many solutions of the following equation* $NB = l$.

The estimation error $\delta \in R^2$ *obeys the equation*

$$\dot{\delta}(t) = G\delta(t). \tag{17}$$

Proof. Following (Nikiforov, 2004), the estimation error $\delta(t)$ is given by

$$\delta(t) = Z(t) - \left(\Xi_X(t) + \sum_{i=1}^{n} a_i \Xi_i(t) + b\Xi_u(t)\right). \tag{18}$$

Using (18) and the fact that $NA_0 = 0$ and $NB = l$, the time derivative of (18) in view of (1), (13)–(15) yields (17). Representing $Z(t-D)$ by using (18) and substituting it into (9), we get (12). ∎

The representation (12) established with Lemmas 1 and 2, allows us to represent a time-varying unknown sinusoidal disturbance $\nu(t)$ as a constant unknown vector multiplied by a known regressor which is delayed, plus an unknown exponentially decaying disturbance $\delta(t)$.

Defining the bound of the unknown vector θ is necessary for the update law. The parametric solution of (5) is given by $m_1(\omega) = -\frac{(g_1 l_1 + g_2 l_2)\omega^2 + g_1 g_4^2 l_1 + g_2^2 g_3 l_2}{\sigma_m} - \frac{g_2 g_4 (g_1 l_2 + g_3 l_1)}{\sigma_m}$, $m_2(\omega) = -\frac{l_1 \omega^3 + \omega(g_4^2 l_1 - g_1 g_2 l_2 + g_2 g_3 l_1 - g_2 g_4 l_2)}{\sigma_m}$, $m_3(\omega) = -\frac{\omega^2 (g_3 l_1 + g_4 l_2) + g_2 g_3^2 l_1 + g_1^2 g_4 l_2}{\sigma_m} - \frac{g_1 g_3 (g_2 l_2 + g_4 l_1)}{\sigma_m}$, $m_4(\omega) = -\frac{l_2 \omega^3 + \omega(g_1^2 l_2 - g_1 g_3 l_1 + g_2 g_3 l_2 - g_3 g_4 l_1)}{\sigma_m}$, with $\sigma_m = \omega^4 + \omega^2\left(g_1^2 + 2g_2 g_3 + g_4^2\right) + (g_1 g_4 - g_2 g_3)^2$ where $M = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix}$, $G = \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix}$, $l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$.Then using (10) and $\theta_s^T = h^T M^{-1}$, $\theta_d^T = h^T S M^{-1}$, θ is written as

$$\theta^T(\omega) = [\theta_1(\omega), \theta_2(\omega)] \tag{19}$$

where $\theta_1(\omega) = \frac{\cos(D\omega) m_4(\omega) - \sin(D\omega) m_3(\omega)}{\det\{M(\omega)\}}$ and $\theta_1(\omega) = \frac{-\cos(D\omega) m_2(\omega) + \sin(D\omega) m_1(\omega)}{\det\{M(\omega)\}}$. The extreme values of $\theta_1(\omega)$ and $\theta_2(\omega)$ for $\omega \in [\omega_{\min}, \omega_{\max}]$ are defined by taking the first derivative with respect to ω and finding the roots. Define the sets $\Omega_i = \{\omega \in \mathbb{R} | \omega_{\min} \geq \omega \geq \omega_{\max}$ and $\frac{d\theta_i(\omega)}{d\omega} = 0\}$ for $i = 1, 2$. If $\Omega_i = 0$, then it is defined as $\Omega_i = \{\omega_{\min}, \omega_{\max}\}$. The maximum value of $|\theta_1|, |\theta_2|$ can be defined as follows

$$\overline{\theta} = \left[\max_{\omega \in \Omega_1} |\theta_1(\omega)|, \max_{\omega \in \Omega_2} |\theta_2(\omega)|\right]^T. \tag{20}$$

4. Main result-design and stability statement

The delayed signals are represented as the boundary of a transport PDE to prepare the system for the design. This idea for delay systems is given and discussed in Krstic and Symslayev (2008a). Substituting (12) into (1) and representing the delayed signals as the boundary of a transport PDE, the ODE–PDE coupled system is written as

$$\begin{aligned} \dot{X}(t) &= A_0 X(t) + B\Big(\gamma^T X(t) + \theta^T \xi^X(0,t) + bu(0,t) \\ &\quad + \beta_u^T \xi^U(0,t) + \sum_{i=1}^{n} \beta_i^T \xi^{a_i}(0,t) + \theta_\delta^T \delta(t)\Big), \end{aligned} \tag{21}$$

$$u_t(x,t) = u_x(x,t), \tag{22}$$

$$u(D,t) = U(t), \tag{23}$$

$$\xi^X{}_t(x,t) = \xi^X{}_x(x,t), \tag{24}$$

$$\xi^X(D,t) = \Xi_X(t), \tag{25}$$

$$\xi^{a_i}{}_t(x,t) = \xi^{a_i}{}_x(x,t), \quad i = 1, \dots, n \tag{26}$$

$$\xi^{a_i}(D,t) = \Xi_i(t), \tag{27}$$

$$\xi^U{}_t(x,t) = \xi^U{}_x(x,t), \tag{28}$$

$$\xi^U(D,t) = \Xi_U(t). \tag{29}$$

The solutions of the transport PDEs are given by $u(x,t) = U(t+x-D)$, $\xi^X(x,t) = \xi^X(t+x-D)$, $\xi^{a_i}(x,t) = \xi^{a_i}(t+x-D)$, $\xi^U(x,t) = \xi^U(t+x-D)$. The adaptive controller for the system (21)–(29) is given by

$$\begin{aligned} U(t) &= \frac{1}{\hat{b}}\Bigg(-\hat{\theta}(t)^T \Xi_X(t) - \sum_{i=1}^{n} \hat{\beta}_i(t)^T \Xi_i(t) - \hat{\beta}_u(t)^T \Xi_U(t) \\ &\quad + (K - \hat{\gamma}^T)\Big(\int_0^D e^{(A_0 + B\hat{\gamma})(D-y)} B\zeta(y,t) dy \\ &\quad + (K - \hat{\gamma}^T) e^{(A_0 + B\hat{\gamma})D} X(t)\Big)\Bigg) \end{aligned} \tag{30}$$

where the control gain $K \in R^{1 \times n}$ is chosen so that $\left(A_0^T + BK\right)$ is Hurwitz and the positive definite matrices P and P_G are solutions of the matrix equations

$$(A_0^T + BK)^T P + P(A_0^T + BK) = -Q, \tag{31}$$

$$G^T P_G + P_G G = -Q_G \tag{32}$$

with $\lambda_{\min}\{Q_G\} > 1 + 2\max\{c_0, c_\gamma\}$, $\lambda_{\min}\{Q\} > 2 + \lambda_{\max}\{P_G G l l^T G^T P_G\} + n\lambda_{\max}\{P_G l l^T P_G\} + 2c_0 \lambda_{\max}\{N^T N\} > 0$ where $c_0, c_\gamma > 0$ and

$$\zeta(x,t) = u_{\beta_u}(x,t) + \hat{\theta}^T \xi^X(x,t) + \sum_{i=1}^{n} \hat{\beta}_i^T \xi^{a_i}(x,t), \tag{33}$$

where

$$u_{\beta_u}(x,t) = \hat{b}u(x,t) + \hat{\beta}_u \xi^U(x,t). \tag{34}$$

The update laws are given by

$$\dot{\hat{b}}(t) = \kappa_b \mathrm{Proj}_{\Pi_b}\{u(0)\tau_c(t)\}, \tag{35}$$

$$\dot{\hat{\gamma}}(t) = \kappa_\gamma \mathrm{Proj}_{\Pi_\gamma}\{X(t)\tau_c(t)\}, \tag{36}$$

$$\dot{\hat{\theta}}(t) = \kappa_\theta \mathrm{Proj}_{\Pi_\theta}\left\{\xi^X(0)\tau_c(t)\right\}, \tag{37}$$

$$\dot{\hat{\beta}}_i(t) = \kappa_{\beta_i} \mathrm{Proj}_{\Pi_{\beta_i}}\left\{\xi^{a_i}(0)\tau_c(t)\right\}, \quad i = 1, \dots, n \tag{38}$$

$$\dot{\hat{\beta}}_u(t) = \kappa_{\beta_u} \mathrm{Proj}_{\Pi_{\beta_u}}\left\{\xi^U(0)\tau_c(t)\right\}, \tag{39}$$

where $\kappa \in [0, \kappa^*[$ (κ^* is given in the analysis) and

$$\begin{aligned} \tau_c(t) &= \frac{\frac{2}{c_w} B^T P X(t)}{N_L(t)} \\ &\quad - \frac{(K - \hat{\gamma}^T)\int_0^D (1+x) e^{(A_0 + B\hat{\gamma})x} Bw(x,t) dx}{N_L(t)} \end{aligned} \tag{40}$$

with

$$N_L(t) = 1 + X(t)^T PX(t) + \Xi_0(t)^T P_G \Xi_0(t) + \sum_{i=1}^{n} \Xi_i^T P_G \Xi_i + \int_0^D (1+x)\Big(c_w w(x,t)^2 + c_0 \xi^X(x,t)^T \xi^X(x,t) + c_\gamma \sum_{i=1}^{n} \xi^{a_i}(x,t)^T \xi^{a_i}(x,t)\Big)dx \quad (41)$$

where $c_w > \frac{1}{b_{\min}^2}\lambda_{\max}\{PBB^TP\}$ and

$$w(x,t) = \zeta(x,t) - (K - \hat{\gamma}^T)\int_0^x e^{(A_0+B\hat{\gamma})(x-y)} B\zeta(y,t)dy - (K-\hat{\gamma}^T)e^{(A_0+B\hat{\gamma})x}X(t). \quad (42)$$

Using Assumptions 1–4 and (20) with $\beta_u = b\theta$, $\beta_i = a_i\theta$ for $i = 1, \ldots, n$. the sets of parameters are given by

$$\Pi_b = \{b \in \mathbb{R} | b_{\max} > |b| > b_{\min} > 0\}, \quad (43)$$

$$\Pi_\gamma = \Big\{\gamma = [a_i, \ldots, a_n]^T \in \mathbb{R}^n | a_{\max} > a_i > a_{\min}, \ i = 1, \ldots, n\Big\}, \quad (44)$$

$$\Pi_\theta = \Big\{\theta = [\theta_1, \theta_2]^T \in \mathbb{R}^2 | \bar{\theta}_i > \theta_i > -\bar{\theta}_i, \ i = 1, 2\Big\}, \quad (45)$$

$$\Pi_{\beta_i} = \Big\{\beta_i = [\beta_{i1}, \beta_{i2}]^T \in \mathbb{R}^2 | \bar{a}\bar{\theta}_j > \beta_{ij} > -\bar{a}\bar{\theta}_j, \ j = 1, 2\Big\}, \quad (46)$$

$$\Pi_{\beta_u} = \Big\{\beta_u = [\beta_{u1}, \beta_{u2}]^T \in \mathbb{R}^2 | b_{\max}\bar{\theta}_i > \beta_{ui} > -b_{\min}\bar{\theta}_i, \ i = 1, 2\Big\}. \quad (47)$$

The standard projection operators are given by

$$\mathrm{Proj}_{\Pi_b}\{\tau_b(t)\} = \begin{cases} 0, & \mathrm{sign}(b)\hat{b} = b_{\max} \text{ and } \tau_b > 0 \\ 0, & \mathrm{sign}(b)\hat{b} = b_{\min} \text{ and } \tau_b < 0 \\ \tau_b, & \text{else,} \end{cases} \quad (48)$$

where $\tau_b(t) = u(0)\tau_c(t)$ and

$$\mathrm{Proj}_{\Pi_k}\{f_k\tau_c(t)\} = \begin{cases} 0, & \hat{k} = \bar{k} \text{ and } f_k\tau_c(t) > 0 \\ 0, & \hat{k} = \underline{k} \text{ and } f_k\tau_c(t) < 0 \\ f_k\tau_c(t), & \text{else,} \end{cases} \quad (49)$$

for $f_k = X(t), \xi^X(0), \xi^{a_i}(0), \xi^U(0)$, $k = \gamma, \theta, \beta_i, \beta_u$, respectively where $\hat{k}$, represents the estimate of the parameter and $\bar{k}$, $\underline{k}$ are the maximum and minimum bounds of the parameter, respectively.

The inverse of the transformation (42) is given by

$$u(x,t) = \frac{1}{\hat{b}}\Big(w(x,t) - \hat{\theta}^T\xi^X(x,t) - \sum_{i=1}^{n}\hat{\beta}_i^T \xi^{a_i}(x,t) - \hat{\beta}_u^T\xi^U(x,t) + (K-\hat{\gamma}^T)\int_0^x e^{(A_0+B(K-\hat{\gamma}^T))(x-y)} \times Bw(y,t)dy + (K-\hat{\gamma}^T)e^{(A_0+B(K-\hat{\gamma}^T))x}X(t)\Big). \quad (50)$$

Using (18), (24), (26) and (28), Eq. (50) is written as

$$u(x,t) = \zeta_U(x,t) - \frac{1}{\hat{b}}\hat{\beta}_u^T Z(t-x) + \frac{e^{Dx}}{\hat{b}}\hat{\beta}_u^T\delta(t), \quad (51)$$

where

$$\zeta_U(x,t) = \frac{1}{\hat{b}}\Big(w(x,t) - \Big(\hat{\theta}^T - \frac{1}{b}\hat{\beta}_u^T\Big)\xi^X(x,t) - \sum_{i=1}^{n}\Big(\hat{\beta}_i^T - \frac{a_i}{b}\hat{\beta}_u^T\Big)\xi^{a_i}(x,t) + (K-\hat{\gamma}^T) \times e^{(A_0+B(K-\hat{\gamma}^T))x}X(t) + (K-\hat{\gamma}^T) \times \int_0^x e^{(A_0+B(K-\hat{\gamma}^T))(x-y)}Bw(y,t)dy\Big). \quad (52)$$

Theorem 1. *Let Assumptions 1–4 hold and consider the closed-loop system consisting of (21)–(29), the control law (30) and the update laws given by (35)–(39). There exists $\kappa^* > 0$ such that for any $\kappa \in (0, \kappa^*[$, there exist positive constants R and ρ independent of the initial conditions such that for all initial conditions satisfying $(X^0, \Xi_X^0, \Xi_i^0, \Xi_U^0, \delta^0, u^0, \xi^{X^0}, \xi^{a_i 0}, \xi^{U^0}, \hat{b}^0, \hat{\gamma}^0, \hat{\theta}^0, \hat{\beta}_i^0, \hat{\beta}_u^T) \in \mathbb{R}^n \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times L_2(0,D) \times L_2(0,D) \times L_2(0,D) \times L_2(0,D) \times \Pi_b \times \Pi_\gamma \times \Pi_\theta \times \Pi_{\beta_i} \times \Pi_{\beta_u}$, the following holds:*

$$\Upsilon(t) \le R\left(e^{\rho\Upsilon(0)} - 1\right), \quad \forall t \ge 0 \quad (53)$$

where

$$\Upsilon(t) = |X(t)|^2 + |\Xi_X(t)|^2 + \sum_{i=1}^{n}|\Xi_i(t)| + \int_0^D \Big(u_{\beta_u}(x,t)^2 + \xi^{X^T}(x,t)\xi^X(x,t) + \sum_{i=1}^{n}\xi^{a_i T}(x,t)\xi^{a_i}(x,t)\Big)dx + \tilde{b}^2(t) + \tilde{\gamma}^T(t)\tilde{\gamma}(t) + \tilde{\theta}^T(t)\tilde{\theta}(t) + \sum_{i=1}^{n}\tilde{\beta}_i(t)^T\tilde{\beta}_i(t) + \tilde{\beta}_u(t)^T\tilde{\beta}_u(t). \quad (54)$$

Furthermore,

$$\lim_{t\to\infty} X(t) = 0, \qquad \lim_{t\to\infty}\hat{b}(t)U(t) + \hat{\beta}_u^T(t)\Xi_U(t) = 0. \quad (55)$$

5. Stability proof

The transformed system is given by

$$\dot{X}(t) = (A_0 + BK)X + B\Big(\tilde{\gamma}^T X(t) + \tilde{\theta}^T\xi^X(0) + \sum_{i=1}^{n}\tilde{\beta}_i^T\xi^{a_i}(0) + \tilde{\beta}_u^T\xi^U(0) + \frac{1}{\hat{b}}w(0) + \tilde{b}u(0) + \theta_\delta^T\delta(t)\Big), \quad (56)$$

$$w_t(x,t) = w_x(x,t) + \dot{\hat{b}}p_b(x,t) + \dot{\hat{\gamma}}p_\gamma(x,t) + \dot{\hat{\theta}}^T p_\theta(x,t) + \sum_{i=1}^{n}\dot{\hat{\beta}}_i^T p_{\beta_i}(x,t) + \dot{\hat{\beta}}_u^T p_{\beta_u}(x,t) - \tilde{b}q_b(x,t) - \tilde{\gamma}^T q_\gamma(x,t) - \tilde{\theta}^T q_\theta(x,t) - \sum_{i=1}^{n}\tilde{\beta}_i^T q_{\beta_i}(x,t) - \tilde{\beta}_u^T q_{\beta_u}(x,t) - Ke^{(A_0+B\hat{\gamma})x}B\theta_\delta^T\delta(t), \quad (57)$$

$$w(D,t) = 0, \quad (58)$$

where

$$p_b = u(x,t) - (K-\hat{\gamma}^T)\int_0^x e^{(A_0+B\hat{\gamma})(x-y)}Bu(y,t)dy, \quad (59)$$

$$p_\gamma = -X(t)(K-\hat{\gamma}^T)xe^{(A_0+B\hat{\gamma})}B - B(K-\hat{\gamma}^T)\int_0^x e^{(A_0+B\hat{\gamma})(x-y)}B\zeta(y,t)dy, \quad (60)$$

$$p_\theta = \xi^X(x,t) - \int_0^x \xi^X(y,t)B^T e^{(A_0+B\hat{\gamma})(x-y)}(K-\hat{\gamma}^T)^T dy, \quad (61)$$

$$p_{\beta_i} = \xi^{a_i}(x,t) - \int_0^x \xi^{a_i}(y,t)B^T e^{(A_0+B\hat{\gamma})(x-y)}(K-\hat{\gamma}^T)^T dy, \quad (62)$$

$$p_{\beta_u} = \xi^U(x,t) - \int_0^x \xi^U(y,t) B^T e^{(A_0+B\hat{\gamma})(x-y)} (K-\hat{\gamma}^T)^T dy, \quad (63)$$

$$q_b = u(0)(K-\hat{\gamma}^T) e^{(A_0+B\hat{\gamma})x} B, \quad (64)$$

$$q_\gamma = X(t)(K-\hat{\gamma}^T) e^{(A_0+B\hat{\gamma})x} B, \quad (65)$$

$$q_\theta = \xi^X(0)(K-\hat{\gamma}^T) e^{(A_0+B\hat{\gamma})x} B, \quad (66)$$

$$q_{\beta_i} = \xi^{a_i}(0)(K-\hat{\gamma}^T) e^{(A_0+B\hat{\gamma})x} B, \quad (67)$$

$$q_{\beta_u} = \xi^U(0)(K-\hat{\gamma}^T) e^{(A_0+B\hat{\gamma})x} B. \quad (68)$$

Before the main proof of theorem, we state the following lemma.

Lemma 3. *There exists a constant M_c such that the following bounds hold*

$$\dot{\hat{b}}(t) \int_0^D (1+x) w(x,t) p_b(x,t) dx \le \kappa_b c_w M_c \Big(|X(t)|^2 + |\xi^X(0)|^2 + \sum_{i=1}^n |\xi^{a_i}(0)|^2 + \|w(t)\|^2 + |\delta(t)|^2 \Big), \quad (69)$$

$$\dot{\hat{\gamma}}(t) \int_0^D (1+x) w(x,t) p_b(x,t) dx \le \kappa_\gamma c_w M_c \Big(|X(t)|^2 + \|w(t)\|^2 + \|\xi^X(t)\|^2 + \sum_{i=1}^n \|\xi^{a_i}(t)\|^2 \Big), \quad (70)$$

$$\begin{aligned} &\Big(\dot{\hat{\theta}}^T(t) \int_0^D (1+x) w(x,t) p_\theta(x,t) dx + \sum_{i=1}^n \dot{\hat{\beta}}_i^T(t) \\ &\times \int_0^D (1+x) w(x,t) p_{\beta_i}(x,t) dx + \dot{\hat{\beta}}_u^T(t) \\ &\times \int_0^D (1+x) w(x,t) p_{\beta_u}(x,t) dx \Big) \\ &\le 3 \max\{\kappa_\theta, \kappa_{\beta_i}, \kappa_{\beta_u}\} c_w M_c \\ &\times \Big(|X(t)|^2 + |\xi^X(0)|^2 + \sum_{i=1}^n |\xi^{a_i}(0)|^2 + \|w(t)\|^2 \\ &+ \|\xi^X(t)\|^2 + \sum_{i=1}^n \|\xi^{a_i}(t)\|^2 + |\delta(t)|^2 \Big), \end{aligned} \quad (71)$$

where

$$\begin{aligned} M_c = \frac{3M_D}{M_{N_L}} \max\Big\{ &\frac{1}{b_{\min}} \Big(\bar{\theta} + \frac{1}{b_{\min}}\bar{\beta}\Big)^T \Big(\bar{\theta} + \frac{1}{b_{\min}}\bar{\beta}\Big), \\ &\frac{1}{b_{\min}} \Big(\bar{\beta} + \frac{\bar{a}}{b_{\min}}\bar{\beta}\Big)^T \Big(\bar{\theta} + \frac{1}{b_{\min}}\bar{\beta}\Big), \lambda_{\max}\{PBB^TP\}, \\ &(KM_\gamma BD)^2 + (\bar{\gamma}^T M_\gamma BD)^2, (|KM_\gamma BD|) + (|\bar{\gamma} M_\gamma BD|) \bar{Z}^T \bar{Z} \Big\}, \end{aligned} \quad (72)$$

with

$$M_\gamma = \max_{\gamma \in \Pi_\gamma} e^{(A+B\gamma)D}, \quad (73)$$

$$\begin{aligned} M_D = \max_{\gamma \in \Pi_\gamma} \Big\{ &1 + D, D\left(1 + D(1 + 2(KM_\gamma B)^2)\right), \\ &((K-\gamma^T) M_\gamma BD) \bar{Z}^T \bar{Z}, \lambda_{\max}\{PBB^TP\} \Big\}, \end{aligned} \quad (74)$$

$$M_{N_L} = \min\{1, \lambda_{\min}\{P\}, \lambda_{\min}\{P_G\}, \lambda_{\min}\{P_G\}, c_w, c_0, c_\gamma\}, \quad (75)$$

and using Assumptions 1 and 2, the fact that $Z(t) = MW(t)$ and the parametric solution of M, the bound for $Z(t)$ is given by

$$|Z(t)|^2 < \bar{Z}^T \bar{Z} = 2 g_{\max}^2 \max_{\omega \in [\omega_{\min}, \omega_{\max}]} \lambda_{\max}\{M^T M\}. \quad (76)$$

Proof. It is possible to show the bounds by using the Cauchy–Schwarz and Young's inequalities, the change of coordinates for the double integrals, and (18), (51). ∎

Proof of Theorem. Consider the following Lyapunov function

$$\begin{aligned} V(t) = \ln(N(t)) &+ \frac{c_w}{\kappa_b} \tilde{b}^2 + \frac{c_w}{\kappa_\gamma} \tilde{\gamma}^T \tilde{\gamma} + \frac{c_w}{\kappa_\theta} \tilde{\theta}^T \tilde{\theta} \\ &+ c_w \sum_{i=1}^n \frac{1}{\kappa_{\beta_i}} \tilde{\beta}_i^T \tilde{\beta}_i + \frac{c_w}{\kappa_{\beta_u}} \tilde{\beta}_u^T \tilde{\beta}_u + \epsilon_\delta \delta^T P_G \delta \end{aligned} \quad (77)$$

where $\epsilon_\delta > \frac{1}{\lambda_{\min}\{Q_G\}} \Big(4 \max\{\kappa_b, \kappa_\gamma, \kappa_\theta, \kappa_{\beta_i}, \kappa_{\beta_u}\} c_w M_c + \lambda_{\max}\{PB\theta_\delta^T \theta_\delta B^T P\} + (KM_\gamma BD)^2 \lambda_{\max}\{\theta_\delta \theta_\delta^T\} \Big)$. Substituting (35)–(39), using Lemma 3 and Young's inequality for cross terms, the time derivative of (77) is given by

$$\begin{aligned} \dot{V} \le -\frac{(M_d - 5 c_w \kappa^* M_c)}{N_L(t)} \Big(&|X(t)|^2 + |\varXi_0(t)|^2 + \sum_{i=1}^n |\varXi_i(t)|^2 \\ &+ |\xi^X(0,t)|^2 + \sum_{i=1}^n |\xi^{a_i}(0,t)|^2 + |w(0,t)|^2 + \|\xi^X(t)\|^2 \\ &+ \sum_{i=1}^n \|\xi^{a_i}(t)\|^2 + \|w(t)\|^2 \Big) - M_\delta |\delta(t)| \end{aligned} \quad (78)$$

where

$$M_\delta = \epsilon_\delta \lambda_{\min}\{Q_G\} - 4 c_w \kappa^* M_c - \lambda_{\max}\{PB\theta_\delta^T \theta_\delta B^T P\}, \quad (79)$$

$$\begin{aligned} M_d = \min\Big\{ &\lambda_{\min}\{Q\} - 2 - \lambda_{\max}\{P_G G N N^T G^T P_G\} \\ &- n \lambda_{\max}\{P_G l l^T P_G\} - 2 c_0 \lambda_{\max}\{N^T N\}, \lambda_{\min}\{Q_G\} \\ &- 1 - 2c_0, \lambda_{\min}\{Q_G\} - 1 - 2c_\gamma, c_0, c_\gamma, \\ &c_w - \lambda_{\max}\{PBB^TP\} \Big\}. \end{aligned} \quad (80)$$

By choosing

$$\kappa^* = \frac{M_d}{5 c_w M_c} \quad (81)$$

and $(\kappa_b, \kappa_\gamma, \kappa_\beta, \kappa_\theta, \kappa_{\beta_u}) \in [0; \kappa^*[^5$, we obtain $\dot{V}(t) \le 0$ and

$$V(t) \le V(0). \quad (82)$$

From the transformation (42) and its inverse (50), by using (18) and (34), the inequalities between the states are written as

$$\|w(t)\|^2 \le r_1 \|u_{\beta_u}(t)\|^2 + r_2 \|\xi^X(t)\|^2 + r_3 \sum_{i=1}^n \|\xi^{a_i}\|^2 + r_4 |X(t)|^2, \quad (83)$$

$$\|u_{\beta_u}(t)\|^2 \le s_1 \|w(t)\|^2 + s_2 \|\xi^X(t)\|^2 + s_3 \sum_{i=1}^n \|\xi^{a_i}\|^2 + s_4 |X(t)|^2, \quad (84)$$

where r_1, r_2, s_1, s_2 are sufficiently large positive constants. From (77) and (84), it follows that

$$\begin{aligned} &\tilde{b}^2 + \tilde{\gamma}^T \tilde{\gamma} + \tilde{\theta}^T \tilde{\theta} + \sum_{i=1}^n \tilde{\beta}_i^T \tilde{\beta}_i + \tilde{\beta}_u^T \tilde{\beta}_u + \delta^T(t)\delta(t) \\ &\le \left(\frac{(n+4)\kappa^*}{c_w} + \frac{\epsilon_\delta}{\lambda_{\min}\{P_G\}} \right) V(t), \end{aligned} \quad (85)$$

$$|X(t)|^2 \le \frac{1}{\lambda_{\min}\{P\}}\left(e^{V(t)}-1\right) \tag{86}$$

$$|\Xi_0(t)|^2 \le \frac{1}{\lambda_{\min}\{P_G\}}\left(e^{V(t)}-1\right), \tag{87}$$

$$|\Xi_i(t)|^2 \le \frac{1}{\lambda_{\min}\{P_G\}}\left(e^{V(t)}-1\right), \quad i=1,\ldots,n \tag{88}$$

$$\|\xi^X(t)\|^2 \le \frac{1}{c_0}\left(e^{V(t)}-1\right), \tag{89}$$

$$\|\xi^{a_i}(t)\|^2 \le \frac{1}{c_\gamma}\left(e^{V(t)}-1\right), \quad i=1,\ldots,n \tag{90}$$

$$\|u_{\beta_u}(t)\|^2 \le \left(\frac{s_1}{c_w}+\frac{s_2}{c_0}+\frac{ns_3}{c_\gamma}+\frac{s_4}{\lambda_{\min}\{P\}}\right)\left(e^{V(t)}-1\right). \tag{91}$$

From the definition $\Upsilon(t)$, it follows that

$$\Upsilon(t) \le R\left(e^{V(t)}-1\right), \tag{92}$$

where $R = \left(\frac{s_1}{c_w}+\frac{s_2+1}{c_0}+\frac{n(s_3+1)}{c_\gamma}+\frac{s_4+1}{\lambda_{\min}\{P\}}+\frac{n+1}{\lambda_{\min}\{P_G\}}\right)$ and using the fact $\ln(1+c_c) \le c_c$ for $c_c > -1$ and (83), we obtain

$$V(0) \le \rho\Upsilon(0) \tag{93}$$

with $\rho = \Big(c_w\big(r_1+r_2+nr_3+r_4+\frac{1}{\kappa_b}+\frac{1}{\kappa_a}+\frac{1}{\kappa_\theta}+\frac{n}{\kappa_{\beta_i}}+\frac{1}{\kappa_{\beta_u}}\big)+c_0+nc_\gamma+\lambda_{\max}\{P\}+(n+1)\lambda_{\max}\{P_G\}\Big)$. Therefore we obtain the stability result in theorem. We now prove the convergence of the state $X(t)$. From (82), we obtain the uniform boundedness of $|X(t)|$, $\|\xi^X(t)\|$, $\|\xi^{a_i}(t)\|$, $\|w(t)\|$, $|\hat b(t)|$, $|\hat\gamma(t)|$, $|\hat\theta(t)|$, $|\hat\beta_i(t)|$, $|\hat\beta_u(t)|$, $|\Xi_0(t)|$, $|\Xi_i(t)|$ and $|\delta(t)|$. From (6), (16), (18), it follows that $|\Xi_X(t)|$ and $|\Xi_U(t)|$ are bounded. From (84), it is obtained that $\|u_{\beta_u}(t)\|$ is also bounded in time. From (30), we get uniformly boundedness of $U(t)$. Thus we get that $u(0,t)$ is uniformly bounded for $t \ge D$. Finally, with (56), we obtain that $dX(t)^2/dt$ is uniformly bounded for $t \ge D$. Since $|X(t)|$ is square integrable, from (82), we conclude from Barbalat's lemma (Liu & Krstic, 2001) that $X \to 0$ as $t \to \infty$. Moreover, from (82) it follows that $\|w(t)\|$, $\|\xi^X(t)\|$, $\|\xi^{a_i}(t)\|$ are square integrable. From (84), we obtain the square integrability of $\|u_{\beta_u}(t)\|$. In addition to this, from (16) and (82), it follows that $|\Xi_X(t)|$ and $|\Xi_i|$ are also square integrable. Therefore, using (30), the square integrability of $\hat b(t)U(t)+\hat\beta_u^T(t)\Xi_u(t)$ is obtained. Furthermore,

$$\begin{aligned}
\frac{\mathrm{d}\left(\hat b(t)U(t)+\hat\beta_u^T\Xi_u(t)\right)^2}{\mathrm{d}t} &= \left(\hat b(t)U(t)+\hat\beta_u^T\Xi_u(t)\right)\\
&\times\Big(-\dot{\hat\theta}(t)^T\Xi_X(t)-\hat\theta^T(t)\dot\Xi_X(t)-\sum_{i=1}^n(\dot{\hat\beta}_i(t)^T\Xi_i(t)\\
&+\hat\beta_i^T(t)\dot\Xi_i(t))-\dot{\hat\gamma}^T(t)p_\gamma(D,t)+Ke^{(A_0+B\hat\gamma(t))x}\dot X(t)\\
&+K\dot{\hat\theta}(t)^T\int_0^D\xi^X(y,t)e^{(A_0+B\hat\gamma(t))(D-y)}B\mathrm{d}y\\
&+\sum_{i=1}^n K\dot{\hat\beta}_i(t)^T\int_0^D\xi^{a_i}(y,t)e^{(A_0+B\hat\gamma(t))(D-y)}B\mathrm{d}y\\
&+K\dot{\hat\beta}_u(t)^T\int_0^D\xi^U(y,t)e^{(A_0+B\hat\gamma(t))(D-y)}B\mathrm{d}y\\
&+\dot{\hat b}(t)\int_0^D u(y,t)e^{(A_0+B\hat\gamma(t))(D-y)}B\mathrm{d}y+KB\zeta(D,t)\\
&-Ke^{(A_0+B\hat\gamma^T(t))D}B\zeta(0,t)\\
&+K\int_0^D(A_0+B\hat\gamma^T(t))e^{(A_0+B\hat\gamma^T(t))(D-y)}\zeta(y,t)\mathrm{d}y\Big).
\end{aligned} \tag{94}$$

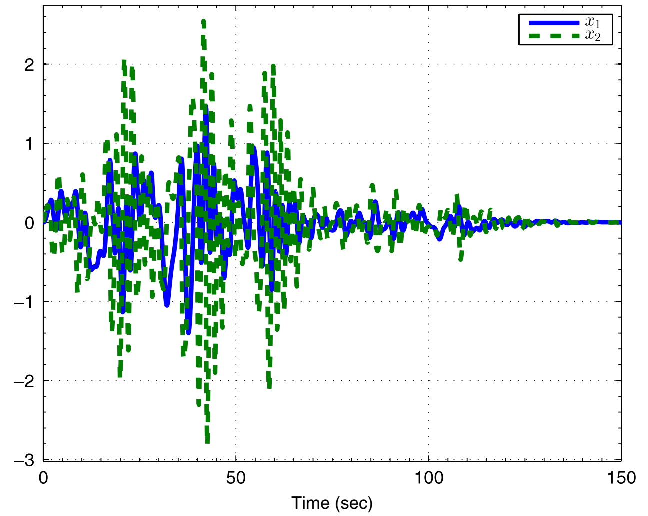

Fig. 1. System's response for the simulation example.

The signals $\dot{\hat b}(t)$, $\dot{\hat\gamma}(t)$, $\dot{\hat\theta}(t)$, $\dot{\hat\beta}_i(t)$, $\dot{\hat\beta}_u(t)$ are uniformly bounded over $t \ge 0$ according to (35)–(39). From (18), it follows that $\|\xi^U(t)\|$ is bounded. Consequently, from (50), the boundedness of $\|u(t)\|$ is obtained. Moreover, by using the uniform boundedness of $X(t)$, $\dot X(t)$, $\|\zeta(t)\|$, $U(t)$, $\Xi_U(t)$ and the parameter estimations, we obtain the uniform boundedness of $\mathrm{d}\big(\hat b(t)U(t)+\hat\beta_u^T\Xi_u(t)\big)^2/\mathrm{d}t$. Then, from Barbalat's lemma (Liu & Krstic, 2001) that $\big(\hat b(t)U(t)+\hat\beta_u^T(t)\Xi_u(t)\big) \to 0$ as $t \to \infty$.

By using (14), (13), (17) and (24)–(27) and the fact that $X \to 0$ as $t \to \infty$ and G is Hurwitz, it is concluded that $\Xi_X(t)$, $\Xi_i(t)$, $\delta(t) \to 0$ as $t \to \infty$. Therefore, considering (21) and the fact that $\big(\hat b(t)U(t)+\hat\beta_u^T\Xi_u(t)\big) \to 0$ as $t \to \infty$, it is obtained that $\frac{1}{\hat b(t-D)}\hat\beta_u(t-D)^T\Xi_U(t-D) \to \frac{1}{b}\nu(t)$ as $t \to \infty$. ■

6. Simulation results

We illustrate the performance of our controller with a second-order system with $\gamma^T = \begin{bmatrix}0.5 & -0.5\end{bmatrix}$, $b = 1$, the unknown disturbance $\nu(t) = 0.3\sin(2t+\pi/5)$, the known delay $D = 1.8$, and initial conditions $x(0) = \begin{bmatrix}0 & 0\end{bmatrix}^T$. It is assumed that $a_{\max} = 1$, $a_{\min} = -1$, $b_{\max} = 2$, $b_{\min} = 0.2$, $|g|_{\max} = 0.6$, $\omega_{\max} = 1$ and $\omega_{\min} = 0.001$. According to the considered bounds on parameters, κ^* is calculated as 0.0013. The control gain K is chosen such that the eigenvalues of A_0+BK are -4, -5 and $c_0 = c_\gamma = 0.01$. We set all the update gains to 0.00125. Finally, the controllable pair (G, l) is chosen as $l = \begin{bmatrix}0, & 1\end{bmatrix}^T$, $G = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} + l\begin{bmatrix}-1.5 & -2.5\end{bmatrix}$. From Figs. 1 and 2, one can observe that $x(t)$ and $\frac{1}{\hat b(t-D)}\hat\beta^T(t-D)\Xi_U(t-D) - \frac{1}{b}\nu(t)$ converge to zero as Theorem 1 predicts.

7. Conclusions

The problem of disturbance cancellation for unknown linear systems with input delay is considered. The problem is converted to an adaptive control problem by representing the disturbance as a constant unknown vector multiplied by a known regressor plus an exponentially decaying disturbance. The delay is represented as

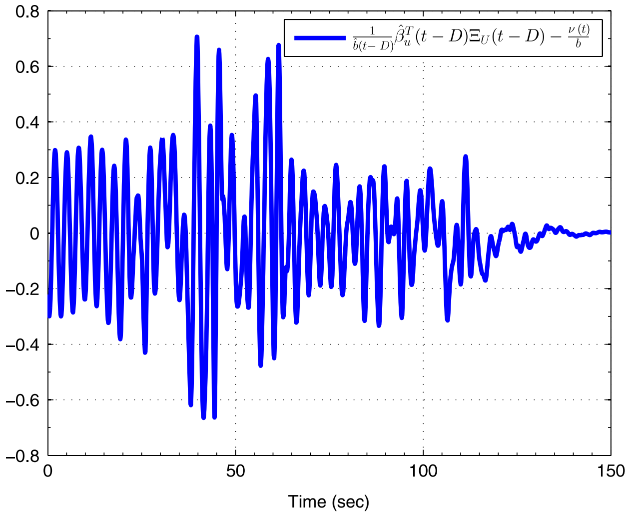

Fig. 2. The disturbance estimation error for the simulation example.

a transport PDE. Using the certainty equivalence principle and the backstepping boundary control procedure for PDEs, an adaptive controller is designed and it is shown that the equilibrium of the closed loop system is stable. Moreover, it is proven that the state $X(t)$ converges to zero as $t \to \infty$ with perfect disturbance estimation. The effectiveness of the controller is demonstrated with a numerical example.

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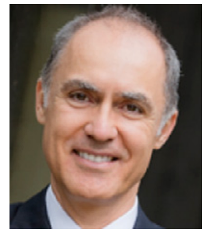

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