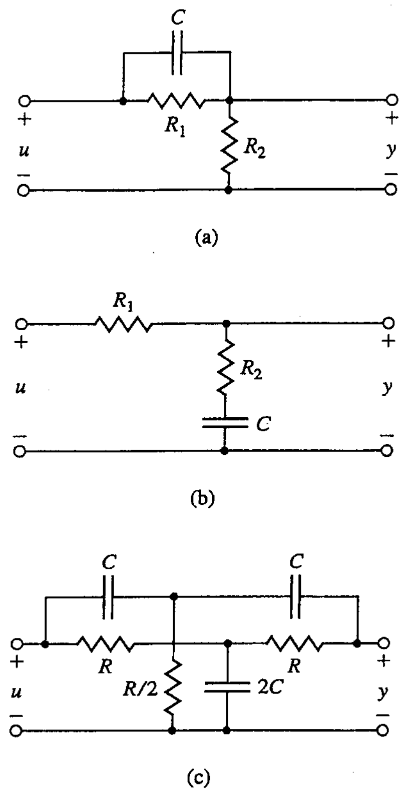
you would compute **F** and **G** for the standard state-variable description of the equations of motion.

2.9 Use node analysis to write the dynamic equations for the circuits shown in Fig. 2.40 and listed below:

a) lead network
b) lag network
c) notch network

FIGURE 2.40
Lead (a), lag (b) and notch (c) circuits



2.10 Using the step command in MATLAB, compare the response of each of the circuits in Problem 2.9 to a unit-step input. Assume $R = R_1 = R_2 = 1\,\Omega$ and $C = 1$ F.

2.11 Use node analysis to write the dynamic equations for the op-amp circuits in Fig. 2.41 and listed below. Assume ideal operational amplifiers in every case.

a) first-order op-amp lead network
b) second-order op-amp circuit
c) Sallen-Key circuit

2.12 Write the state equations for the electrical network shown in Fig. 2.42. Choose as state variables the capacitor voltage v_C and inductor current i_L.

Problems for Section 2.6

2.29. Figure 2.55 shows a simple pendulum system in which a cord is wrapped around a fixed cylinder. The motion of the system that results is described by the differential equation

$$(l + R\theta)\ddot{\theta} + g\sin\theta + R\dot{\theta}^2 = 0,$$

where

$$l = \text{length of the cord in the vertical (down) position},$$

$$R = \text{radius of the cylinder}.$$

(a) Write the state-variable equations for this system.

(b) Linearize the equation around the point $\theta = 0$, and show that for small values of θ the system equation reduces to an equation for a simple pendulum, that is,

$$\ddot{\theta} + (g/l)\theta = 0.$$

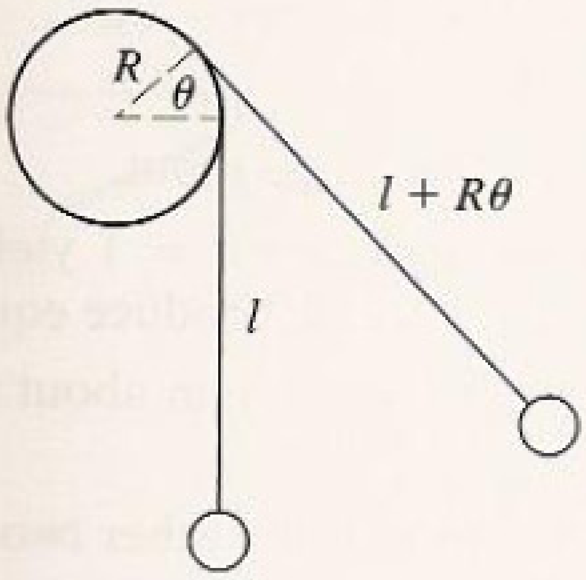


Figure 2.55

2.30. A schematic for the satellite and scientific probe for the Gravity Probe-B (GP-B) experiment is sketched in Fig. 2.56. Assume that the mass of the spacecraft plus helium tank, m_1, is 2000 kg and that the mass of the probe, m_2, is 1000 kg. A rotor will float inside of the probe and will be forced to follow the probe with a capacitive forcing mechanism; however, this will have no effect on m_2. The spring constant of the coupling, k, is 3.2×10^6. The viscous damping, b, is 4.6×10^3.

(a) Write the equations of motion for the system consisting of masses m_1 and m_2 using the inertial position variables, y_1 and y_2.

(b) The actual disturbance, u, is a micrometerorite and the resulting motion is very small. Therefore, rewrite your equations with the scaled variables $z_1 = \frac{y_1}{10^6}$, $z_2 = \frac{y_2}{10^6}$, and $v = 1000u$.

(c) Put the equations in state-variable form using the state $\mathbf{x} = [z_1 \quad \dot{z}_1 \quad z_2 \quad \dot{z}_2]^T$, the output $y = z_2$, and the input an impulse, $u = 10^{-3}\delta(t)$ N·sec on mass m_1.

(d) Using the numerical values, enter the equations of motion into MATLAB in the form

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}u, \tag{2.131}$$

$$y = \mathbf{H}\mathbf{x} + Ju \tag{2.132}$$

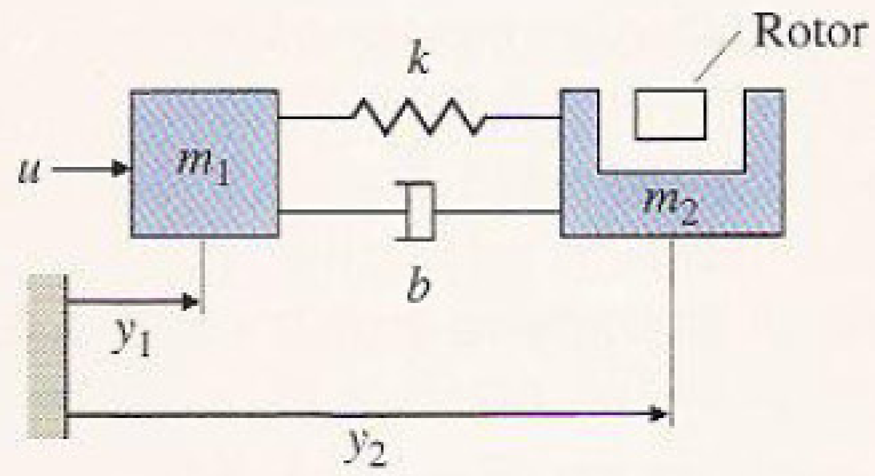


Figure 2.56
Schematic diagram of the GP-B satellite and probe

and define the MATLAB system: sysGPB = ss(F,G,H,J). Plot the response of y caused by the impulse with the MATLAB command impulse(sysGPB). This is the signal the rotor must follow.

2.31. The circuit shown in Fig. 2.57 has a nonlinear conductance G such that $i_G = g(v_G) = v_G(v_G - 1)(v_G - 4)$. The state differential equations are

$$\frac{di}{dt} = -i + v,$$

$$\frac{dv}{dt} = -i + g(u - v),$$

where i and v are the states and u is the input.

(a) One equilibrium state occurs when $u = 1$ yielding $i_1 = v_1 = 0$. Find the other two pairs of v and i that will produce equilibrium.

(b) Find the linearized model of the system about the equilibrium point $u = 1$, $i = v_1 = 0$.

(c) Find the linearized models about the other two equilibrium points.

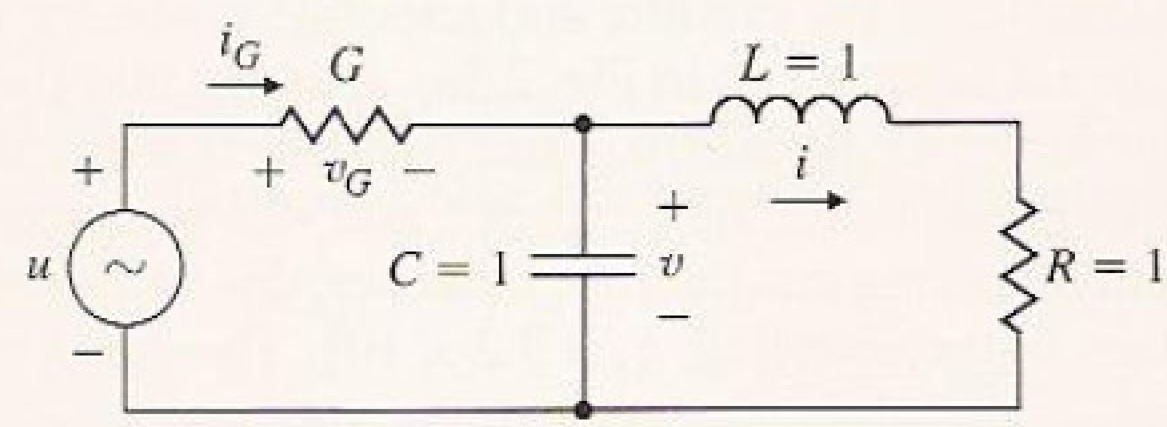


Figure 2.57
Nonlinear circuit for Problem 2.31

2.32. Consider the circuit shown in Fig. 2.58; u_1 and u_2 are voltage and current sources, respectively, and R_1 and R_2 are nonlinear resistors with the following characteristics:

Resistor 1 : $i_1 = G(v_1) = v_1^3$,
Resistor 2 : $v_2 = r(i_2)$,

where the function r is defined in Fig. 2.59.

(a) Show that the circuit equations can be written as

$$\dot{x}_1 = G(u_1 - x_1) + u_2 - x_3$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_1 - x_2 - r(x_3).$$