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Nonlinear and Adaptive Control Design—Miroslav Kristiċ, Ioannis Kanellakopoulis, and Petar V. Kokotoviċ (New York: Wiley, 1995). *Reviewed by David Mayne.*

This is an unusual book, highly original and educative. The *adaptive backstepping* procedure, developed by the authors as recently as 1990, broke many barriers, permitting adaptive controllers to be designed for a much wider class of nonlinear systems than was previously possible. Since then, the three authors have made considerable progress in developing the basic idea to the point where it provides well-understood procedures for the design of controllers for a wide range of uncertain nonlinear systems. In the first place, therefore, the book is a report on new research in an important area—adaptive nonlinear control; it "opens a view to the largely unexplored landscape of nonlinear systems with uncertainties." But to view it merely as a specialized research monograph would be wrong. It is written in a style which, in contrast to many research texts, is, with occasional lapses, highly accessible. The approach is pedagogical; the reader is gently taught the important advances made by the authors, and prior specialized knowledge of nonlinear or adaptive control is not required. Reading the text is a pleasure rather than a demanding exercise. And in following the text, the reader acquires a familiarity with many important concepts.

The book, because of its originality, cannot be easily placed in the context of existing literature on adaptive control. Milestones in this literature (at least for this reviewer) include the introduction of the self-tuning idea [1], the first proof of stability for systems of arbitrary relative degree [2], the dramatic introduction of a new approach (error normalization) [3], [4] and its extension to continuous time [5], [6], the development of robust adaptive controllers following the revealing simulations in [7], the still-to-be explored possibilities offered by hybrid adaptive control, the extension of adaptive control to nonlinear systems [8], [9], and backstepping. Error normalization marked a change of direction which has permeated most of the literature on adaptive control, including that on nonlinear adaptive control, where linearization has allowed the import of techniques developed initially for linear adaptive control [8] and has permitted controller and estimator design to be decoupled. There are now many excellent texts in adaptive control, a recent and excellent example being [10]. The methodology in *Nonlinear and Adaptive Control Design* differs markedly from most of the literature (which, in the main, employs error normalization), being closer in spirit to the earlier work in [2] where the discerning reader may observe the use of a form of backstepping to obtain derivative-free adaptive control of linear systems with high relative degree. The elaboration of backstepping as a powerful and consistent design methodology for both nonlinear and adaptive control emerges for the first time in *Nonlinear and Adaptive Control Design.*

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Grossly simplifying, the major theme can be simply stated: first, control of scalar nonlinear systems (or those satisfying a matching condition) is simple; second, control of complex systems (of a specified structure) can be decomposed into a sequence of simple control problems by a process called *backstepping*. To support the first contention, the following points are made in the excellent introduction (Chapter 1) and in the introductory material of later chapters. For the scalar system

$$\dot{x} = u + \phi(x)$$

$V(x) = (1/2)x^2$ is a *control Lyapunov function* ($\inf_u \dot{V}(x) = \inf_u x(u+\phi(x)) < 0$) so that a stabilizing state feedback controller (e.g., $u = \alpha(x) \triangleq -\phi(x) - cx$, yielding $\dot{V}(x) = -cx^2$) is easily determined. This controller is also *linearizing* since the controlled system satisfies $\dot{x} = -cx$, but one of the points forcibly made by the book is that linearizing control, which has received much attention, is often wasteful. Thus, if $\phi(x) = -x^3$, stabilization easily can be seen to be achieved by the effortless controller $u = 0$ which yields $\dot{V}(x) = -x^4$; linearization requires unnecessary control effort. Uncertainty is also easily dealt with in scalar systems. For the system

$$\dot{x} = u + \phi(x)\Delta(t)$$

where $\Delta(t)$ is unknown but uniformly bounded (knowledge of the bound is *not* required), a stabilizing controller $u = \alpha(x)$ for the unperturbed system ($\dot{x} = u$ in this example) is first determined, and to it is added *nonlinear damping* of the form $-s(x)x$, yielding a controller $u = \alpha(x) - s(x)x$; the nonlinear damping term is chosen to dominate the uncertainty for "large" x. For example, if $\phi(x) = x^2$ and $\Delta(t) = e^{-kt}$, the controller $u = \alpha(x) = -x$ stabilizes the unperturbed system but allows the state of the perturbed system to diverge to infinity in *finite* time if $x(0) > 1+k$. With this controller, $\dot{V} = -x^2 + \Delta(t)x^3$, the second term clearly dominating for large x, suggesting the possibility of finite escape. However, the addition of nonlinear damping of the form $s(x) = \phi^2(x)$ yields a controller $u = -x - x\phi^2(x)$ which ensures

$$\dot{V}(x) = -x^2 + x\phi(x)\Delta(t) - x^2\phi^2(x) \le -x^2 + \Delta^2(t)/4$$

the last inequality resulting from completing the square $(x^2\phi^2(x) - x\phi(x)\Delta(t) = [x\phi(x) - \Delta(t)/2]^2 - \Delta^2(t)/4)$. Clearly $\dot{V} < 0$ for $x \notin L \triangleq \{x | V(x) \le \|\Delta(\cdot)\|_\infty^2/8\}$ so that all trajectories of the perturbed system are bounded and converge to the compact level set L.

Designing scalar adaptive systems is also relatively simple. The system

$$\dot{x} = u + \theta\phi(x)$$

where θ is unknown (but constant, satisfying $\dot{\theta} = 0$) can be treated as an uncertain system if a bound for θ is known. But better performance can often be obtained by estimating θ using

$$\dot{\hat{\theta}} = \tau(\hat{\theta}, x).$$

Defining $\tilde{\theta}$ to be equal to $\theta - \hat{\theta}$, the composite system satisfies

$$\dot{x} = u + \hat{\theta}\phi(x) + \tilde{\theta}\phi(x)$$
$$\dot{\hat{\theta}} = \tau(\hat{\theta}, x)$$

where the uncertainty is now $\tilde{\theta}\phi(x)$; with a "good" estimator, $\tilde{\theta}\phi(x)$ is "smaller" than $\theta\phi(x)$. This two-dimensional system has two "controls" (u and τ). A suitable Lyapunov function is $V(x, \tilde{\theta}) = x^2/2 + \tilde{\theta}^2/2$, yielding (since $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$)

$$\dot{V} = x\dot{x} + \tilde{\theta}\dot{\tilde{\theta}} = x[u + \hat{\theta}\phi(x)] + \tilde{\theta}[-\tau(\hat{\theta}, x) + x\phi(x)].$$

The *certainty equivalence* control law $u = -cx - \hat{\theta}\phi(x)$ coupled with the estimator dynamics $\tau(\hat{\theta}, x) = x\phi(x)$ gives $\dot{V} = -cx^2 \le 0$, yielding (via the LaSalle stability theorem) global asymptotic stability. This choice of estimator dynamics eliminates terms involving $\tilde{\theta}$ in the equation for $\dot{V}$; the estimator dynamics are, therefore, determined by the choice of Lyapunov function. In contrast, in the estimator approach to adaptive control, estimator dynamics are independently obtained by setting up a "filtering" problem

$$\dot{\theta} = 0$$
$$y = \psi\theta$$

from which an estimator for θ may be obtained; ψ is known as the regressor. For our example, if the certainty equivalence controller $u = -x - \phi(x)\hat{\theta}$ is employed, then

$$\dot{x} = -x + \phi(x)\tilde{\theta}$$

so that $x = (\phi(x)\tilde{\theta})_f$ where $(\cdot)_f$ denotes $[1/(s+1)](\cdot)$. An appropriate observation equation is

$$y \triangleq x - x_f - u_f = \phi(x)_f\theta$$

which is obtained by passing both sides of the differential equation $\dot{x} = u + \theta\phi(x)$ through the stable filter $1/(s+1)$. A gradient filter takes the form $\dot{\hat{\theta}} = \mu(y - \hat{y})$ where $\hat{y} \triangleq \phi(x)_f\hat{\theta}$; standard filters result in desirable properties of the estimation error (e.g., $\tilde{\theta}$ is uniformly bounded and its derivative is either bounded or square integrable). But these properties cannot be directly employed in stability analysis because of an interesting complication: $\phi(x)_f\tilde{\theta} \ne (\phi(x)\tilde{\theta})_f$ (since $\tilde{\theta}$ is time varying); the difference between these two terms (the "swapping" error) affects the response of the controlled system $(x = (\phi(x)\tilde{\theta})_f)$ and must, therefore, be properly accounted. Even if $\tilde{\theta}$ decays to zero exponentially, the closed-loop system $\dot{x} = -x + \phi(x)\tilde{\theta}$ may still explode in finite time, as discussed above in the context of uncertainty. As in the uncertainty problem, stability may be ensured by adding to the certainty equivalence controller appropriate nonlinear damping which overcomes, at least for large x, the effect of the "uncertainty" $\phi(x)\tilde{\theta}$. Output feedback control (for nonlinear control, control of uncertain systems, and adaptive control) raises similar issues, requiring state estimation, possibly in addition to parameter estimation.

The second contention, that an important class of feedback control problems can be decomposed into a sequence of scalar problems (to which the design procedures mentioned above are applicable), is the major theme of the book, fully addressed in the subsequent nine chapters. The procedure employed is *backstepping*. Backstepping essentially applies scalar design procedures iteratively to each (scalar) equation in a vector differential equation, yielding in the process a Lyapunov function for the transformed system and a stabilizing controller. Indeed, *backstepping* may be profitably regarded as a method for sequentially constructing a state transformation such that the Lyapunov function for the transformed system has a simple, prescribed form such as $\sum z_i^2$ where z is the transformed state. The problems addressed include nonlinear control, nonlinear control of uncertain systems, and nonlinear adaptive control; both state and output feedback are addressed.

The basic methodology employed is best approached through its application to nonlinear control, addressed in Chapter 2. The following simple system:

$$\dot{x}_1 = x_2 + \phi_1(x_1)$$
$$\dot{x}_2 = x_3 + \phi_2(x_1, x_2)$$
$$\dot{x}_3 = u + \phi_3(x_1, x_2, x_3)$$

which has many of the features of the more general systems considered in the book, serves as an appropriate vehicle for understanding the procedure. The transformed state is $z = (z_1, z_2, z_3)$, where $z_1 = x_1$, $z_2 = x_2 - \alpha_1(x_1)$ and $z_3 = x_3 - \alpha_2(x_1, x_2)$. The state transformations are determined sequentially as follows. With $z_1 \triangleq x_1$, we choose $V_1 = z_1^2/2$ so that

$$\dot{V}_1 = z_1[x_2 + \phi(x_1)].$$

The choice $x_2 = \alpha_1(x_1) + z_2$, $\alpha_1(x_1) \triangleq -\phi(x_1) - z_1$ (equivalently, $z_2 = x_2 - \alpha_1(x_1)$) yields

$$\dot{V}_1 = -z_1^2 + z_1 z_2$$

which is negative if we neglect momentarily the term $z_1 z_2$ which will be dealt with in the next iteration. We have effectively used x_2 as a surrogate control variable and determined the "control" law $x_2 = \alpha_1$ by solving a scalar control problem which ensures that $\dot{V}_1 < 0$ when $z_2 = 0$.

For step 2, we set $V_2 = z_1^2/2 + z_2^2/2$ so that

$$\dot{V}_2 = -z_1^2 + z_1 z_2 + z_2[x_3 + \phi_2(x_1, x_2) - \gamma_1(x_1, x_2)]$$

$(\gamma_1(x_1, x_2) \triangleq \dot{\alpha}_1 = (\partial\alpha_1(x_1)/\partial x_1)[x_2 + \phi_1(x_1)])$ which we see poses a similar problem. We therefore choose x_3 to be the surrogate control (for the second differential equation) and set $x_3 = \alpha_2(x_1, x_2) + z_3$, $\alpha_2(x_1, x_2) \triangleq -\phi_2(x_1, x_2) + \gamma_1(x_1, x_2) - z_1 - z_2$ (this defines the transformed variable $z_3 = x_3 - \alpha_2(x_1, x_2)$). Hence

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3.$$

For the final step we choose $V = V_3 = z_1^2/2 + z_2^2/2 + z_3^2/2$, yielding

$$\dot{V} = -z_1^2 - z_2^2 + z_2 z_3 + z_3[u + \phi_3 - \gamma_2]$$

and choose the control law

$$u = \alpha_2(x_1, x_2, x_3)$$
$$\triangleq -\phi_3(x_1, x_2, x_3) + \gamma_2(x_1, x_2, x_3) - z_2 - z_3$$

where ($\gamma_2 \triangleq \dot{\alpha}_2$) yielding, finally

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2$$

which establishes global, asymptotic stability. The resultant transformed system satisfies $\dot{z} = A(z)z$ where $A(z)$ has a simple, tridiagonal, structure.

The procedure can be easily extended to deal with uncertainty; suppose the system to be controlled is now defined by

$$\begin{aligned}\dot{x}_1 &= x_2 + \phi_1(x_1)\Delta(t)\\ \dot{x}_2 &= x_3 + \phi_2(x_1, x_2)\\ \dot{x}_3 &= u + \phi_3(x_1, x_2, x_3).\end{aligned}$$

The uncertainty Δ is assumed to be uniformly bounded ($\|\Delta(\cdot)_\infty\| \leq \delta$), but the bound δ need not be known. We proceed as before, with $z_1 \triangleq x_1$ and $V_1 = z_1^2/2$ but now add nonlinear damping so that now $\alpha_1(x_1) \triangleq -x_1\phi_1(x_1)^2 - z_1$ (it was previously $-\phi(x_1) - z_1$). Consequently

$$\begin{aligned}\dot{V}_1 &= -z_1^2 + z_1 z_2 - [x_1^2\phi_1(x_1)^2 - \phi_1(x_1)\Delta(t)]\\ &\leq -z_1^2 + z_1 z_2 + \delta^2/4\end{aligned}$$

on completing the square as before. The design now proceeds much as before, with $V = V_3 = z_1^2/2 + z_2^2/2 + z_3^2/2$ satisfying

$$\dot{V} \leq -z_1^2 - z_2^2 - z_3^2 + \delta^2/4$$

so that, once again, all trajectories are bounded and converge to a compact level set of V.

This bald summary does little justice to the flexibility of the backstepping procedure, a flexibility which is highly evident in adaptive control where many variants of backstepping are possible. The discussion of adaptive control commences in Chapter 3, where the simplest variety, adaptive backstepping, is employed. The essence of the method may be illustrated by its application to the system

$$\begin{aligned}\dot{x}_1 &= x_2 + \phi_1(x_1)\theta\\ \dot{x}_2 &= x_3 + \phi_2(x_1, x_2)\theta\\ \dot{x}_3 &= u + \phi_3(x_1, x_2, x_3)\theta\end{aligned}$$

where θ is constant but unknown. Setting $z_1 = x_1$ and choosing $V_1 = z_1^2/2 + \tilde{\theta}_1^2/2$, where $\tilde{\theta}_1 \triangleq \theta - \hat{\theta}$ yields

$$\dot{V}_1 = z_1[x_2 + \phi_1(x_1)\hat{\theta}_1] + \tilde{\theta}_1[z_1\phi_1(x_1) + \dot{\tilde{\theta}}_1].$$

The choices $x_2 = \alpha_1 + z_2$ with $\alpha_1 \triangleq -\phi_1(x_1)\hat{\theta}_1 - z_1$ (certainty equivalence "control") and estimator dynamics $\dot{\hat{\theta}}_1 = -\dot{\tilde{\theta}}_1 = \tau_1$ with $\tau_1 \triangleq x_1\phi(x_1)$ give

$$\dot{V}_1 = -z_1^2 + z_1 z_2$$

as in the nonadaptive case. The choice of estimator removes terms involving $\tilde{\theta}_1$ from $\dot{V}_1$. The procedure can be carried forward to the second and third stages with $V_i = V_{i-1} + z_i^2/2 + \tilde{\theta}_i^2/2$, $i = 2, 3$. Cancellation, for each i of the term involving $\tilde{\theta}_i$ in $\dot{V}_i$ necessitates an estimator, $\dot{\hat{\theta}}_i = -\dot{\tilde{\theta}}_i = \tau_i$, for each component of the state. This feature, referred to as overparameterization, obviously complicates implementation.

Over parameterization is overcome by the introduction of *tuning* functions, the subject of Chapter 4. Consider the adaptive control of

$$\dot{x} = f(x) + F(x)\theta + g(x)u.$$

If, for each θ, there exists a control Lyapunov function $V_a(x, \theta)$ for the modified system

$$\dot{x} = f(x) + F(x)[\theta + \partial V_a/\partial\theta] = g(x)u$$

then an interesting formula due to Sontag may be employed to obtain a stabilizing controller $u = \alpha(x, \theta)$ for the modified system. The controller $u = \alpha(x, \hat{\theta})$, together with estimator $\dot{\hat{\theta}} = \tau(x, \theta) \triangleq [\partial V_a(x, \hat{\theta})F(x)]^T$ when applied to the *original* system, renders $\dot{V}$ negative definite with $V(x, \hat{\theta}) \triangleq V_a(x, \hat{\theta}) + |\tilde{\theta}|^2/2$; α and τ define a stabilizing adaptive controller for the original system. The modified problem yields a modified controller which properly accounts for parameter estimation transients. The functions V_a and α may be obtained by applying backstepping, much as described above, to the original problem, but now *not* cancelling at each stage the terms involving $\dot{\hat{\theta}}$. One estimator is used, and, for each i, a term of the form $\tilde{\theta}[\dot{\hat{\theta}} - \tau_i]$ is retained in the expression for $\dot{V}_i$; τ_i depends on τ_{i-1} and is referred to as a tuning function. At the final stage, τ_n is chosen to eliminate the term involving $\tilde{\theta}$; this yields the (single) estimator ($\dot{\hat{\theta}} = \tau_n$), which, together with an appropriate choice of $u = \alpha_n$, renders $\dot{V}_n$ negative semidefinite, $V_n \triangleq \sum z_i^2/2 + |\tilde{\theta}|^2/2$, enabling stability to be established.

The tuning function approach, although yielding a simpler controller, has to employ an estimator dictated by the choice of Lyapunov function, unlike the situation in linear adaptive control where control and estimator modules may be separately designed. An ingenious method for achieving an analogous modularity is described in Chapter 5. For the system

$$\dot{x} = f(x) + F(x)\theta + g(x)u$$

with a Lyapunov function $V(x, \hat{\theta})$, we have

$$\begin{aligned}\dot{V} = [\partial V/\partial x][f(x) + F(x)\hat{\theta} + g(x)u]\\ + [\partial V/\partial x]F(x)\tilde{\theta} + [\partial V/\partial\hat{\theta}]\dot{\hat{\theta}}.\end{aligned}$$

The approach adopted is to regard $\tilde{\theta}$ and $\dot{\hat{\theta}}$ as uncertainties and to apply the backstepping procedure for uncertain systems, described above. A wide class of estimators can be employed, provided only that $\tilde{\theta}$ and $\dot{\hat{\theta}}$ have certain properties (e.g., $\tilde{\theta}$ is uniformly bounded and $\dot{\hat{\theta}}$ is either uniformly bounded or square integrable). Prior to defining the estimator, nonlinear damping can be incorporated in the backstepping process, as described above, to dominate these uncertainties, at least for "large" x, thus ensuring that all signals are uniformly bounded even if there is no adaptation (since then $\tilde{\theta}$ is constant and $\dot{\hat{\theta}} = 0$). The resultant controller achieves input-to-state stability, as defined by Sontag, with input ($\tilde{\theta}$, $\dot{\hat{\theta}}$). The transformed system satisfies

$$\dot{z} = A(z, \hat{\theta}, t) + W(z, \hat{\theta}, t)\tilde{\theta} + Q(z, \hat{\theta}, t)\dot{\hat{\theta}}$$

where, as before, A has a simple, tridiagonal structure.

To achieve adaptation, an estimator, which ensures that the uncertainty ($\tilde{\theta}$, $\dot{\hat{\theta}}$) has the necessary properties, has to be added. An estimator which incorporates a passive observer of the state z is presented in the remainder of Chapter 5; the operator relating the input $\tilde{\theta}$ to the output $W(z, \hat{\theta}, t)(z - \hat{z})$ is strictly passive. The input to the estimator is the output of the passive observer. A more determined effort to achieve modularity is mounted in Chapter 6, where the use of nonlinear extensions to the "swapping lemma," (due to Morse) permits the use of gradient and least-squares parameter estimators. The authors construct two nonlinear filters with outputs Ω and ψ which provide a "static" observation of the unknown parameter, i.e.,

$$z + \psi = \Omega\tilde{\theta}$$

modulo an exponentially decaying error. This static observation can be used in the usual way to obtain gradient and least-squares parameter estimators with the properties required by the controller. Moreover, using the fact that the swapping error lies in $\mathcal{L}_2$, convergence of $z(t)$ to zero is established.

Attention is turned to output feedback in Chapter 7 but restricted to systems for which exponentially convergent state estimators exist. A simple (nonadaptive) example is

$$\begin{aligned}\dot{x}_1 &= x_2 + \phi_1(y)\\ \dot{x}_2 &= x_3 + \phi_2(y)\\ \dot{x}_3 &= \phi_3(y) + b_0 u\\ y &= x_1\end{aligned}$$

which may be more compactly expressed as $\dot{x} = Ax + bu + \phi(y)$, $y = c^T x$, but more complex systems, possessing zero dynamics, are addressed. Since y is known, the nonlinearity $\phi(y)$ does not hinder the derivation of an exponential observer of the form

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + bu + \phi(y) + k(y - \hat{y})\\ \hat{y} &= c^T \hat{x}.\end{aligned}$$

The state estimation error satisfies $\dot{\tilde{x}} = A_0\tilde{x}$, $A_0 \triangleq A - kc^T$. To obtain a controller, backstepping is now applied, much as in the state feedback case, using, in stage i, the state estimate $\hat{x}_{i+1}$ in place of x_{i+1} as a surrogate control and incorporating nonlinear damping to dominate the exponential state estimation error. Thus, in stage 1, with $z_1 \triangleq x_1 = y$ and $\hat{x}_2 = \alpha_1 + z_2$, we obtain

$$\dot{z}_1 = -z_1 + z_2 + \tilde{x}_2$$

if $\alpha_1 \triangleq -z_1 - \phi_1(y)$. In the second stage, with $\hat{x}_3 \triangleq \alpha_2 + z_3$, we get

$$\dot{z}_2 = \alpha_2 + z_3 + \tilde{x}_3 + \phi_2(y) - \dot{\alpha}_1$$

and α_3 now has to incorporate nonlinear damping to dominate the variable $\tilde{x}_2$ appearing via $\dot{\alpha}_1$. Stability is established using a Lyapunov function of the form $\sum z_j^2/2 + \tilde{x}^T P\tilde{x}/d_j$ where $PA_0 + A_0^T P = -I$.

A theory for adaptive output feedback problems is then developed for systems such as

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta\phi_1(y)\\ \dot{x}_2 &= x_3 + \theta\phi_2(y)\\ \dot{x}_3 &= \theta\phi_3(y) + b_0 u\\ y &= x_1\end{aligned}$$

and extended versions of these equations incorporating stable zero dynamics. A whole range of adaptive controllers is presented, mirroring those developed previously for state feedback systems. The overparameterized controller of Chapter 3 is first extended (in Chapter 7), followed (in Chapter 8) by an extension of the tuning function design of Chapter 4. The modular designs of Chapters 5 and 6 are extended, in Chapter 9, to the output feedback problem.

Chapter 10 presents a self-contained development of the application of these new techniques to linear systems. Readers with a good knowledge of linear adaptive control might like to turn to this chapter rather early to see, in the context of familiar material, the development and application of these new approaches to adaptive control. In the linear case, the system equations have the form

$$\begin{aligned}\dot{x}_1 &= x_2 - a_{n-1}y\\ &\vdots\\ \dot{x}_{\rho-1} &= x_\rho - a_{m+1}y\\ \dot{x}_\rho &= x_{\rho+1} - a_m y + b_m u\\ &\vdots\\ \dot{x}_{n-1} &= x_n - a_1 y + b_1 u\\ \dot{x}_n &= -a_0 y + b_0 u\\ y &= x_1\end{aligned}$$

where $a = (a_0, a_1, \cdots, a_n)$ and $b = (b_0, b_1, \cdots, b_m)$ are the unknown parameters. The zero dynamics are defined by rows $\rho + 1 \cdots n$. Choosing appropriate filters, whose outputs are ψ and Ω, yields the "static" relationship

$$x = \psi + \Omega\theta + \varepsilon \tag{0.1}$$

between the state x and the unknown parameter θ. Omitting the zero dynamics, the system equations are replaced by

$$\begin{aligned}\dot{y} &= b_m v_2 + \psi_2 = \omega^T\theta + \varepsilon_2\\ \dot{v}_2 &= v_3 - k_2 v_1\\ &\vdots\\ \dot{v}_{\rho-1} &= v_\rho - k_{\rho-1}v_1\\ \dot{v}_\rho &= v_{\rho+1} - k_\rho v_1 + u\end{aligned}$$

where $v_1 \cdots v_n$ are components of the first column of Ω. Backstepping can now be applied as before using, in the ith row, v_{i+1} as the surrogate control. The tuning function approach yields a nonlinear adaptive controller whose performance is, in many respects, superior to that of a conventional indirect adaptive controller.

This summary gives some appreciation of the basic methodology but does scant justice to the richness of the text. A useful summary of Lyapunov's stability results is given in Chapter 2, together with a nice introduction to the important work by Artstein and Sontag on control Lyapunov functions. This includes the first presentation in a book of Sontag's construction for obtaining a stabilizing control law from a control Lyapunov function. An extension for adaptive problems is presented in Chapter 4. The useful concepts of input-to-state stability and input-to-state control Lyapunov functions are described in Chapter 5. Passivity is used extensively, particularly in the context of backstepping using equation "blocks" (rather than single equations). The tracking problem is comprehensively treated. The book provides, as does nonlinear control in general, a clearer insight into classical results for linear systems; in particular, the weakness of "certainty equivalence" as a design concept is exposed. Serious examples, effectively illustrating the power of the design procedures, are presented.

I have some grumbles. The book, which introduces some topics so well for the novice, is pretty uncompromising on others. The discussion on zero dynamics could be more helpful. Occasionally the text lapses into research paper style, quoting results with little explanation; better coordination between the various authors would help avoid this. On rare occasions, an undefined term (such as "decrescent" in Lemma 5.6) creeps into the discussion. And perhaps there is too much material, with some developments making earlier approaches redundant. Robustness and constrained control are not treated. But these reservations are all minor; in the context of the exciting glimpse the book gives to the "largely unexplored landscape of nonlinear systems with uncertainties," they fall into insignificance. Providing a novel treatment, based on backstepping, of both nonlinear control *and* adaptive control, it differs substantially from other books, the closest being the recent text [9] which is also devoted to nonlinear and adaptive control and differs in its extensive utilization of differential geometric algorithms, including feedback linearization for both state and output feedback. My overwhelming impression of *Nonlinear and Adaptive Control Design* is best stated in my opening remarks: "this is an unusual book, highly original and educative." I strongly recommend it to anyone interested in nonlinear and adaptive control of uncertain systems.

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Optimal Control Theory for Infinite Dimensional Systems—Xunjing Li and Jiongmin Yong (Boston, MA: Birkhauser, 1995, 488 pages). *Reviewed by Suzanne Lenhart.*

I. Introduction

The Pontryagin maximum principle, the Bellman dynamic programming principle, and the Kalman optimal linear regulator theory are the three cornerstones of finite dimensional control theory [2]. The study of optimal control theory for infinite dimensional systems can be traced back to the early 1960's. The fundamental work of Lions [3] was instrumental in starting this study. The above three cornerstones have been extended to infinite dimensional control systems to some extent [1], [4], [5]. This book gives a summary of the results for infinite dimensional systems related to these cornerstones in optimal control theory.

II. Main Body

This book consists of five parts besides the first two chapters of preliminary analysis results and motivational examples. The first part concerns the existence theory of optimal controls. First- and second-order evolution systems are discussed, which correspond to parabolic and hyperbolic partial differential equations (PDE's), as well as elliptic equations and variational inequalities. Several important measurable selection theorems are presented; these theorems have played essential roles in proving the existence of optimal controls. By posing suitable compactness conditions, the infinite dimensional case is illustrated to be quite similar to the finite dimensional case.

The second part is concerned with the Pontryagin's maximum principle for optimal controls of evolution equations and elliptic PDE's. In all of the problems, difficulty can come from the lack of convexity of the control domain, the presence of the pointwise state constraints, and the nonsmoothness of the state equation (e.g., the variational inequality) and/or the cost functional (e.g., the minimax problem). To treat the possible nonconvexity of the control domain, the spike variation for the control is used; the corresponding "Taylor expansion" for the state is developed. Pointwise state constraints are treated with some penalization together with the Ekeland variational principle. When the state equation or the cost functional is nonsmooth, some specially designed penalty functions and certain stability conditions are introduced to achieve reasonable approximations to the original problem. Boundary control is included in the semilinear elliptic case.

The third part is concerned with the dynamic programming method. The theory of viscosity solutions for infinite dimensional Hamilton–Jacobi–Bellman (HBJ) equations in the Hilbert space case is the central topic here. They adopted the result of Ekeland and Lebourg on the perturbed optimization, instead of the Stegall's result. It is interesting that in proving the result of Ekeland and Lebourg, the Ekeland variational principle has been used. This connection makes the second and the third parts link intrinsically. The optimal switching and impulse control problems are presented. The HBJ equation corresponding to these problems are systems of quasi-variational inequalities. A method of approximation together with the theory of viscosity solutions are adopted for bounded HJB equations.

The fourth part covers the time optimal control problem and the controllability problem. The recent result on the (approximate) controllability of semilinear evolution equations using the method of optimization and penalization is included. Hence, the controllability problem is closely linked to the optimal control problem.

The last part is a self-contained presentation of linear quadratic optimal control problems for finite and infinite time durations. They have covered general evolution equations with bounded controls (e.g., parabolic or hyperbolic equations with distributed controls) and the evolution equations governed by analytic semigroups with unbounded controls (which correspond to parabolic equations with Dirichlet or Neumann boundary controls). For the finite time duration problem, after establishing the existence and uniqueness of the optimal control, they introduce the Fredholm integral equation to synthesize the control problem. Then, the integral form of the Riccati equation is introduced as some additional characterization of the optimal synthesizing operator. On the other hand, for the infinite time duration problem, they take the reverse way by introducing the Riccati equation first and then the Fredholm integral equation. At the same time, they have discussed some frequency domain characterizations of the systems.

III. Conclusions

In the book, some efforts are made to collect or construct many counterexamples in the self-contained fashion. For two examples, a control problem with the value function not having continuous first derivatives (Chapter 6, S1) and the nonconvexity of the reachable

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