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Feedback control of particle dispersion in bluff body wakes

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We address the problem of imposing particle dispersion, or mixing, in the wake of a bluff body by means of feedback control. The only means of actuation is a pair of blowing and suction slots located on the cylinder wall, and the only means of sensing is a collection of pressure sensors, also located on the cylinder wall. Numerical simulations show that a very simple static feedback control law suppresses vortex shedding at Reynolds number $Re = 60$, and more interestingly, also enhances particle dispersion when run in destabilizing mode. At $Re = 45$, for which natural vortex shedding does not occur, the control law run in destabilizing mode effectively initiates vortex shedding, which in turn leads to significantly enhanced particle dispersion. Simulations suggest that the actuation slots should be placed well ahead of the separation point, and that the performance of the controller is quite insensitive to the direction of the jets.

1. Introduction

Mixing processes are encountered frequently in engineering applications, and the quality of the mixture that they produce is usually crucial for the effectiveness of some downstream process. A prime example of this is combustion engines (Annaswamy and Ghoniem 1995), where the quality of the mixture of fuel and air will decide how much power can be extracted from the engine. As a consequence, mixing has been the focus of much research, both experimental and theoretical, but without reaching a unified theory, either for the generation of flows that mix well due to external forcing, or for the quantification of mixing in such flows (see Ottino (1989) for a review). The theoretical approaches to analysis of mixing have focused on applying modern dynamical systems theory (Ottino 1989, Swanson and Ottino 1990, Rom-Kedar *et al.* 1990, Mezić 1994, Haller and Poje 1998) in order to quantify mixing taking place in a given flow. Thus, they lend little help to the problem of *generating* a fluid flow that mixes well.

Recently, control systems theory has been applied to mixing problems. In D'Allessandro *et al.* (1999), the mixing protocol that maximizes entropy among all the possible periodic sequences composed of two shear flows orthogonal to each other was derived. In Noack *et al.* (2000), the optimal vortex trajectory in the flow induced by a single vortex in a corner subject to a controlled external strain field was found using tools from dynamical systems theory. The resulting trajectory was stabilized using control theory. In Aamo *et al.* (2001), it was proposed to use feedback control in order to enhance existing instability mechanisms in a model flow. Simple, destabilizing feedback control laws were designed, that significantly enhanced mixing in simulations of the Navier–Stokes equation using small control effort. Partially motivated by Mezić (2001), these control laws were shown to have certain optimality properties in Aamo *et al.* (2002). A framework for destabilization of linear systems and mixing enhancement was presented in Bamieh *et al.* (2001).

It was demonstrated in Aamo *et al.* (2001, 2002, 2003), that static output feedback from pressure measurements on the wall to wall transpiration of fluid induces strong mixing in both 2D channel flows and a 3D pipe flows. The controllers were of the form

$$u_n(s) = k(p(s) - p(s^*)) \tag{1}$$

where u_n is the wall-normal velocity component, or amount of wall transpiration, p is pressure, s denotes a location on the wall, s^* denotes the location on the wall resulting from mirroring s about the centre line of the flow domain, and k is a constant feedback gain. In other words, the amount of wall transpiration applied at any given point s on the wall is simply proportional to the pressure difference between two suitable points on the wall. Apart from being very simple, one of the main features of the control law is its symmetry in the sense that $u_n(s) = -u_n(s^*)$, which due to incompressibility maintains the mass balance. Motivated by the results in Aamo *et al.* (2001, 2002, 2003), we present in this paper a simulation study that investigates the feasibility of enhancing particle dispersion in the wake of a circular cylinder (2D) using simple feedback control laws similar to those used in Aamo *et al.* (2001, 2002, 2003). The remainder of this paper is organized as follows: the numerical model setup is presented in §2; Section 3 deals

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with the stabilization problem in bluff body flows; Section 4 presents simulation results for particle dispersion, and conclusions are offered in § 5.

2. Model setup

All simulations have been performed using FLUENT† on a grid of approximately 4000 nodes, shown in figure 1. The grid extends $5D$ upstream of the cylinder center, $20D$ downstream, and $\pm 5D$ in the vertical direction, where D is the diameter of the cylinder. The minimum grid cell area is $0.0016D^2$. The density of nodes in our grid is comparable to that used by other authors who study the controlled cylinder flow, for instance Min and Choi (1999). While the total number of nodes in their grid is larger, so is their computational domain. Near the cylinder, the density of nodes is about the same, even though they look at flows at $Re = 160$, which is more than twice the Reynolds number we are studying. At the cylinder wall, no-slip boundary conditions are applied except in the locations of the actuators which employ suction and blowing. The details of the actuation mechanism is given in § 3.3.

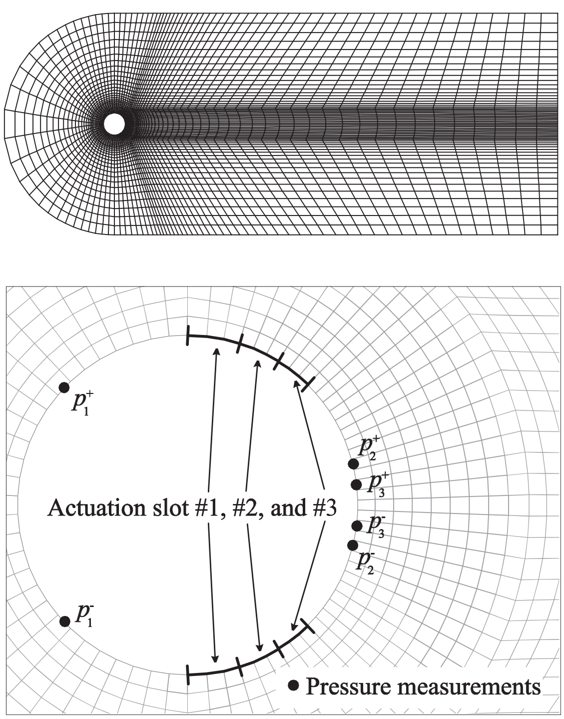

Figure 1. Grid showing slots for blowing and suction, and locations at which pressure measurements are taken.

† FLUENT is a commercial computational fluid dynamics (CFD) package available from Fluent Inc.

Uniform free stream conditions ($U = U_\infty$, $V = 0$, where U is the flow velocity in the x direction, and V is the flow velocity in the y direction) are applied at the inlet and lateral boundaries, while the flow exit is treated as a zero-normal-gradient outlet boundary. The time step is $\Delta t = 0.05U_\infty/D$. For details of the Navier–Stokes solver and the particle tracker in FLUENT, we refer the reader to the FLUENT documentation. Validation simulations have been performed and documented by Fluent Inc. on the exact same grid that we are using in this work, for Reynolds number $Re = 20$, 40, and 100. The simulation results are reported to be in satisfactory agreement with results from Coutanceau and Dafaye (1991), and Braza *et al.* (1986), in terms of the length of the recirculation region for $Re = 20$ and 40, and in terms of the shedding frequency for $Re = 100$. The flows we study in this paper have Reynolds number $Re = 45$ or 60. For further details from the validation results, we refer the reader to the FLUENT documentation. Figure 1 also shows the configuration of sensors and actuators used. Actuation is applied in the form of suction or blowing of fluid through the indicated slots, whereas sensing is in the form of pressure measurements on the cylinder wall. The feedback control loop, consisting of sensing, computational logic, and actuation, was implemented using the *User Defined Functions* capability in FLUENT. The simulations are grouped in terms of the Reynolds number, defined as

$$Re = \frac{\rho U_\infty D}{\mu} \tag{2}$$

where U_∞ is the free stream velocity, D is the cylinder diameter, and ρ and μ are density and viscosity of the fluid, respectively. When the Reynolds number is small, the flow is *subcritical* and tends to a steady state which is symmetric about the streamwise axis, but when $Re > 47$, the flow is *supercritical* and vortex shedding occurs. In this case, vortices are shed alternately from the top and bottom parts of the cylinder, bringing the flow into a periodic state. In this paper, we will study flows at Reynolds number 60 (supercritical) and 45 (subcritical). Simulation results are reported in terms of the lift coefficient, magnitude of the control, snap shots of particle distributions, and snap shots of vorticity fields. The lift coefficient is defined as the force acting on the cylinder in the vertical direction, multiplied by a normalization factor, and the vorticity is given as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{3}$$

where u and v are the streamwise and vertical components of velocity, respectively. Unless otherwise stated, actuation is via Slot #2 (see Figure 1).

3. Stabilization

The stabilization problem for the flow past a circular cylinder has been subject to much attention. Approaches range from numerical simulations of *ad-hoc* feedback control configurations (see, e.g., Park *et al.* 1994), to application of modern optimal control theory on various sensor/actuator configurations (see, e.g., Shritharan 1992, Min and Choi 1999, Protas and Styczek 2002). The objective of this section is twofold: (1) to reproduce earlier results which used feedback from a single velocity measurement downstream of the cylinder in order to stabilize the supercritical flow, and (2) to show that stabilization can be achieved when the velocity measurement is replaced by pressure measurements on the cylinder surface. The former case primarily serves as a reference to compare the latter case with, but it also provides validation of the model. The control configuration of the latter case is motivated by the control laws designed using Lyapunov stability analysis in Aamo *et al.* (2001, 2002, 2003), which successfully enhance mixing in 2D channel flow and 3D pipe flow when run in destabilizing mode. The latter control configuration is also justified from a practical point of view, since taking measurements on the cylinder surface rather than in the interior of the flow seems more feasible.

3.1. *Initial condition*

The initial condition for the simulations is obtained by starting from a perturbed velocity field and running the simulation for 500 time units. The result is a periodic steady state, as indicated by the lift coefficient plotted in figure 2(a). The vorticity field at the end of this initial run is shown in figure 2(b), where the von Kármán vortex street is clearly visible. For a thorough review of the dynamics of the cylinder wake, see Williamson (1996).

3.2. *Stabilization by velocity feedback*

The stabilization problem involves suppressing vortex shedding in order to obtain zero lift and a steady flow field which is symmetric about the streamwise axis. Suppression of vortex shedding by feedback control has been subject to much attention. It is well known that vortex shedding at $Re = 60$ can be effectively suppressed using feedback from a single velocity measurement downstream of the cylinder to a pair of actuators located as shown in figure 1 (Park *et al.* 1994). The time evolution of the lift coefficient

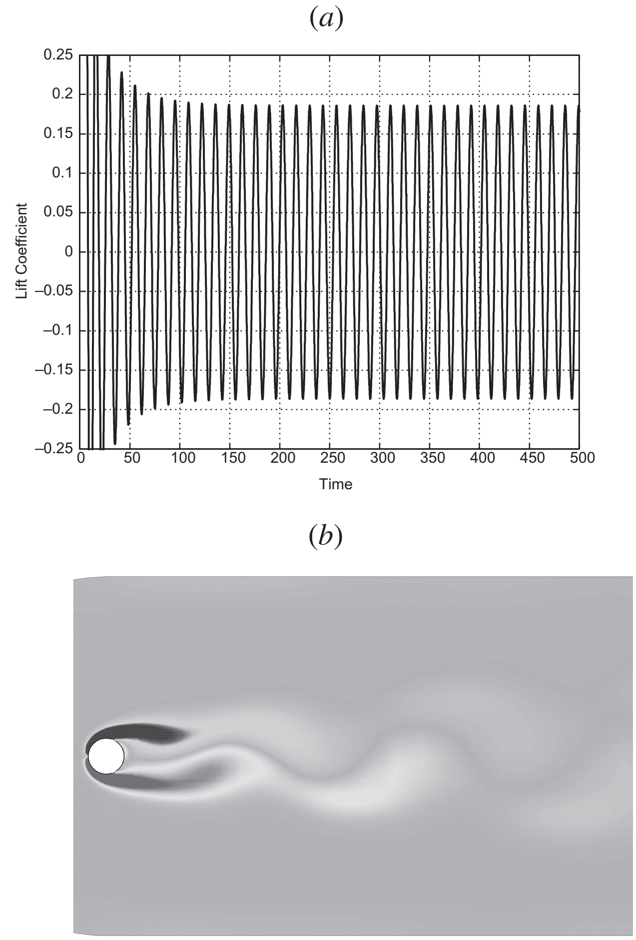

Figure 2. Lift coefficient (a) for initial simulation, and vorticity field (b) at $t = 500$, for $Re = 60$.

for this case is shown in figure 3(a) and the control effort is shown in figure 3(b). They indicate that the vortex shedding is weakened, which is confirmed by the nearly symmetric (about the streamwise axis) vorticity map at $t = 750$ shown in figure 4, and the streamlines at $t = 1000$ shown in figure 5. The controlled velocity field continues to oscillate a little, though. In terms of the lift coefficient, it continues to vary between $\approx \pm 0.02$ beyond the time interval shown in figure 3, which is about 10% of its amplitude in the uncontrolled case.

3.3. *Stabilization by pressure feedback*

In the previous section, vortex shedding was suppressed using feedback from one single velocity measurement. The measurement was taken downstream of the cylinder, in the interior of the fluid,

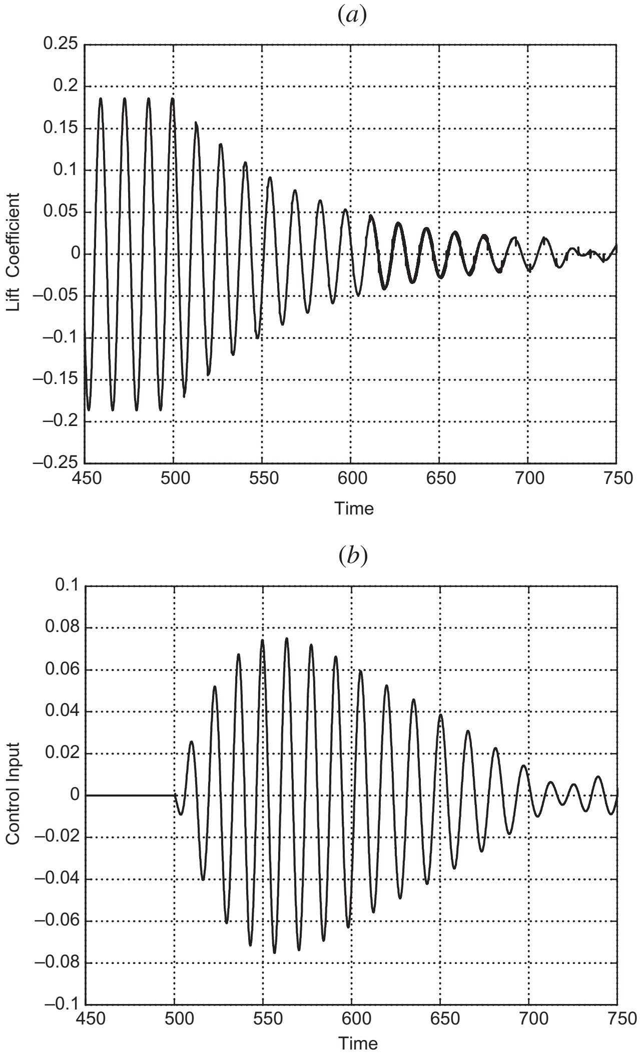

Figure 3. Lift coefficient (*a*) and control input (*b*) for stabilizing control at $Re = 60$. Feedback from velocity measurement.

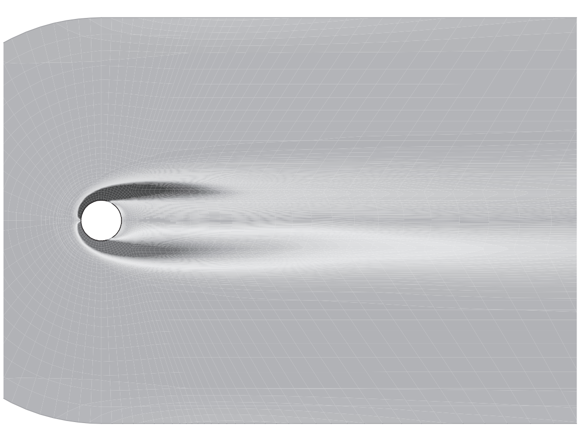

Figure 4. Vorticity field at $t = 750$ for stabilizing control at $Re = 60$. Feedback from velocity measurement.

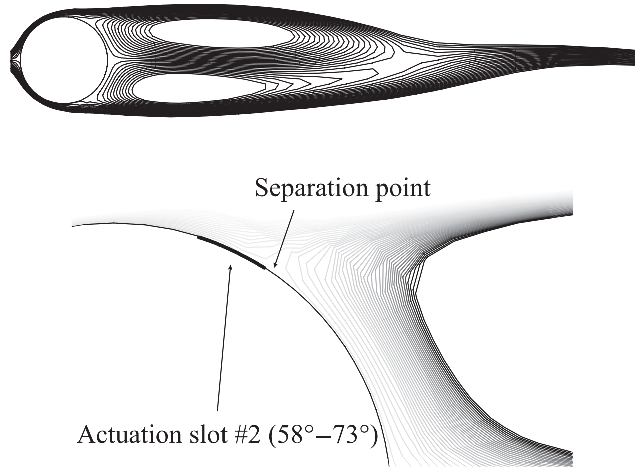

Figure 5. Streamlines at $t = 1000$ for stabilizing control at $Re = 60$. Feedback from velocity measurement.

and is thus hard to obtain in practice. It is much more convenient to be able to rely on measurements taken on the surface of the cylinder. An attempt at using pressure feedback to suppress vortex shedding at $Re = 60$ using a pair of blowing/suction slots was made in Gunzburger and Lee (1996). That attempt failed, but the problem was resolved by adding a third actuator. Here, we instead add pressure measurements, which are easily obtained in practice. Using a combination of pressure measurements, as shown in figure 1, we successfully suppress vortex shedding, as indicated by the lift coefficient, control effort and vorticity map in figures 6 and 7. The feedback law has the form

$$\tau = k(a_1 \Delta p_1 + a_2 \Delta p_2 + a_3 \Delta p_3) \qquad (4)$$

where k, a_1, a_2 and a_3 are constants, and $\Delta p_i = p_i^+ - p_i^-$, for $i = 1, 2, 3$. τ assigns the maximum value of the velocity profile for the flow through the actuation slot, as indicated in figure 8. From these simulation results, we conclude that the configuration of sensors and actuators that we have chosen have comparable control authority to configurations studied by others, but with the benefit of having all sensors and actuators located on the boundary of the cylinder. With this sensor/actuator configuration, we now move on to the main topic of this paper, which is particle dispersion. Motivated by Aamo *et al.* (2001), we simply switch the sign of the feedback gain k in (4), turning (4) into a destabilizing control rather than a stabilizing one, and carry out a simulation study which shows that the resulting feedback control increases particle dispersion in the wake of the cylinder.

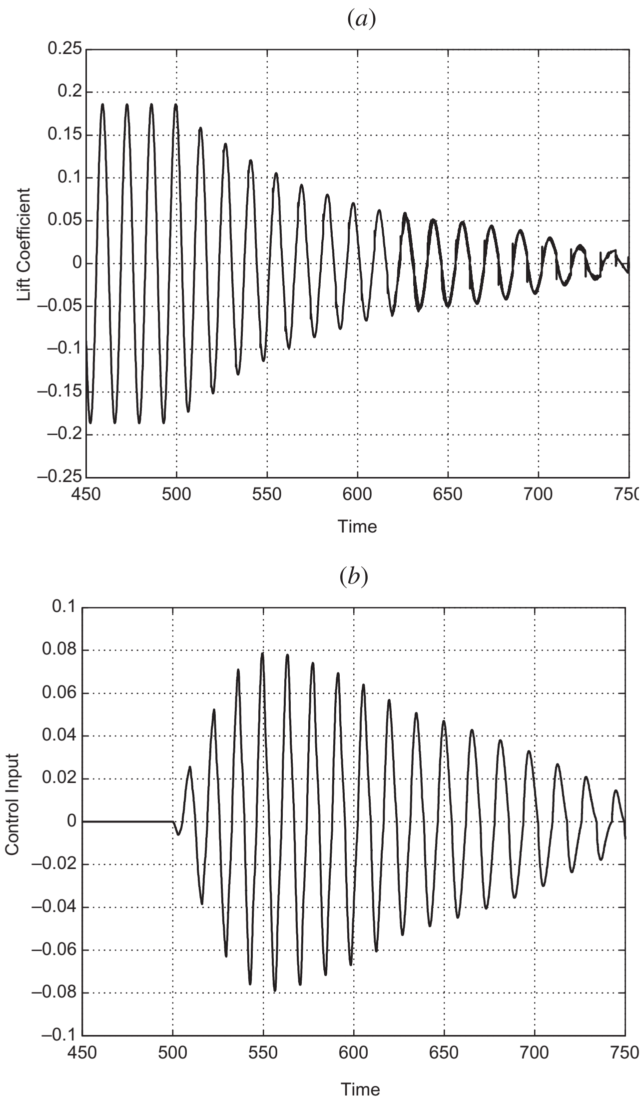

Figure 6. Lift coefficient (*a*) and control input (*b*) for stabilizing control at $Re = 60$. Feedback from pressure measurement.

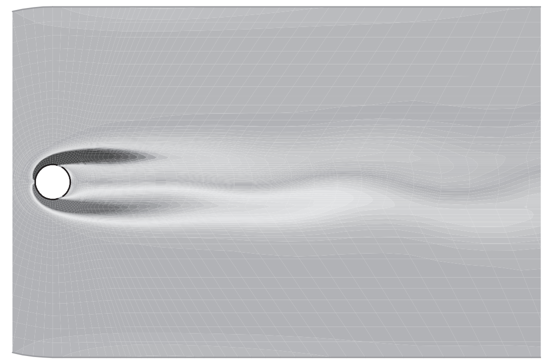

Figure 7. Vorticity field at $t = 750$ for stabilizing control at $Re = 60$. Feedback from pressure measurement.

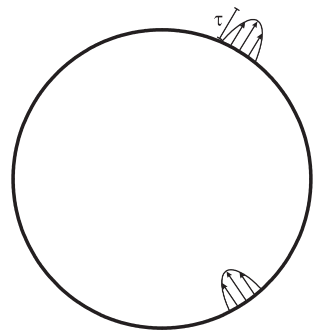

Figure 8. Cylinder showing velocity profiles for the flow through the actuation slots.

4. Particle dispersion

4.1. *Measuring particle dispersion*

We will measure particle dispersion in the same way one would in a lab experiment, by injecting a tracer dye. In the simulations, the tracer dye will be represented by massless particles injected into the flow at six locations upstream of the cylinder. As the tracer particles are advected with the flow, they reveal the Lagrangian motion of fluid particles, providing a picture of the structure of the wake of the cylinder. Thus, we are able to measure the vertical width over which particles are dispersed, and this will be our measure of how well particle dispersion is enhanced by control.

4.2. *Supercritical flow at* Re = *60*

4.2.1. *Initial condition*. The initial condition for the supercritical flow simulations is the same as in § 3.1, that is, the periodic steady state of natural vortex shedding.

4.2.2. *Controlled flow*. As shown by the way the initial state is generated in § 3.1, the natural state for the supercritical flow is not a steady flow that inherits the symmetry of the geometry of the problem, but rather a periodic orbit. In other words, vortex shedding occurs naturally in the uncontrolled, supercritical flow, leading to considerable unsteadiness in the wake behind the cylinder. The objective of applying feedback control in this case is to increase this unsteadiness even further. While it is clear that the effect of control will not extend downstream to infinity (where the velocity is uniform), the effect of the control is significant in the vicinity of the cylinder. Figures 9–12 show lift coefficient, control input, vorticity maps and particle dispersion plots for the uncontrolled flow and the controlled flow with three different feedback gains. The lift force increases with increased control effort, and

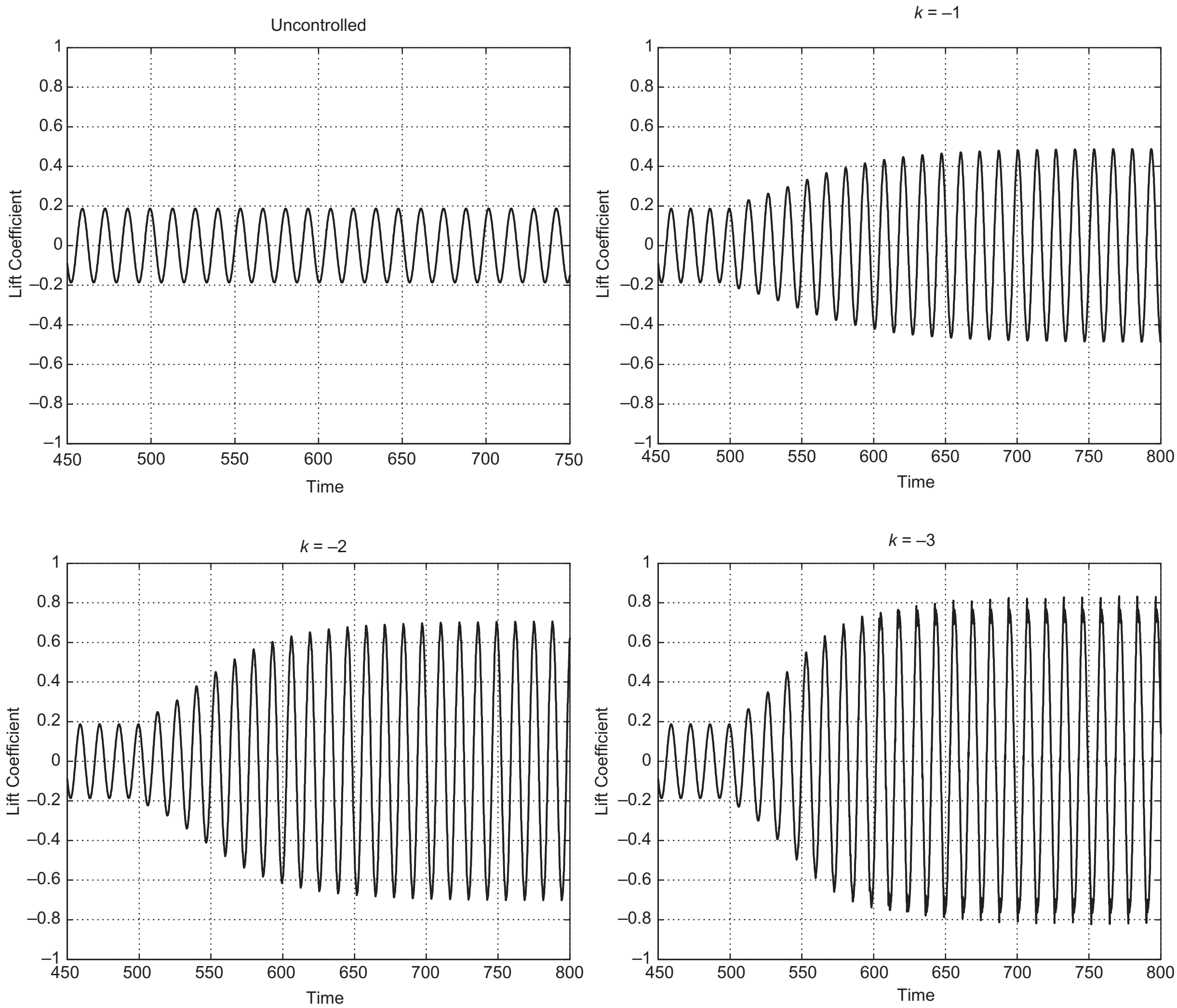

Figure 9. Lift coefficient for $Re = 60$.

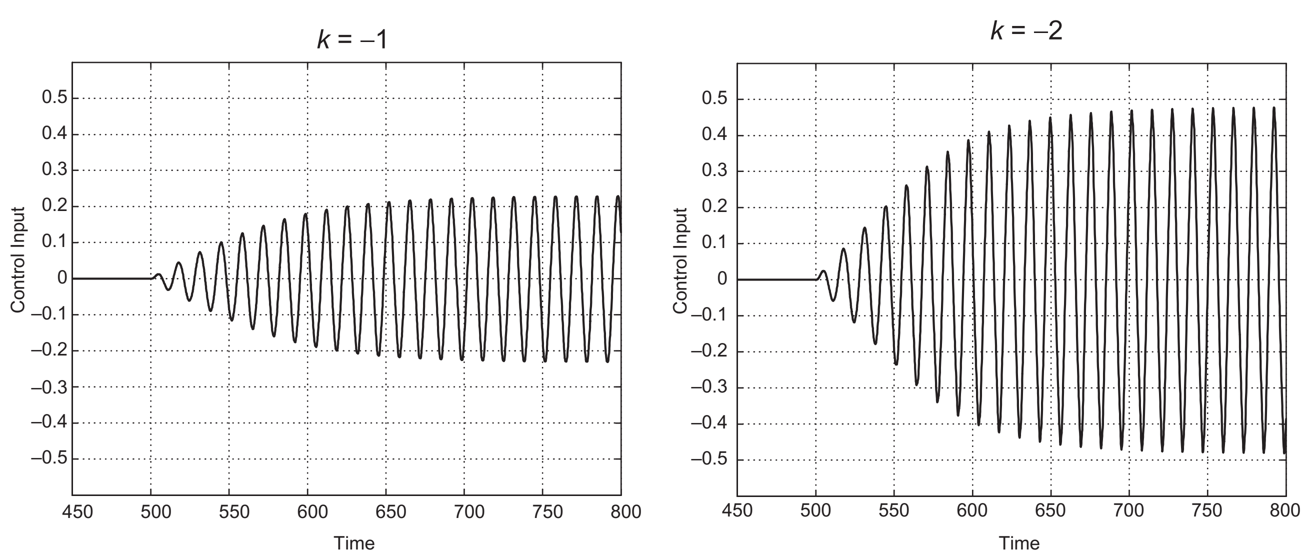

Figure 10. Control input for $Re = 60$.

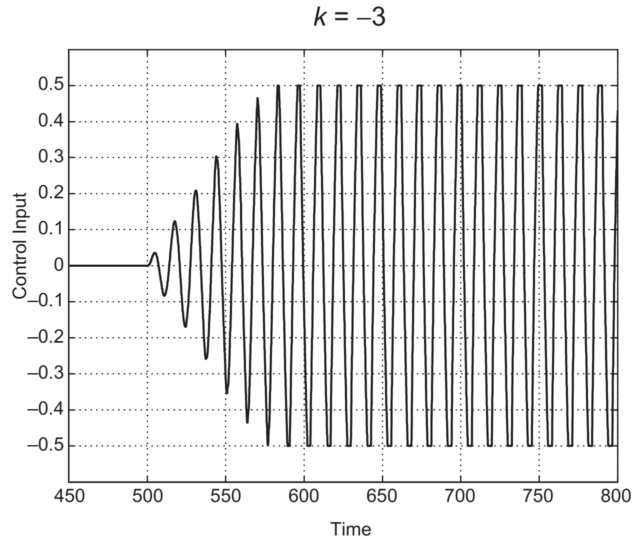

Figure 10. *Continued.*

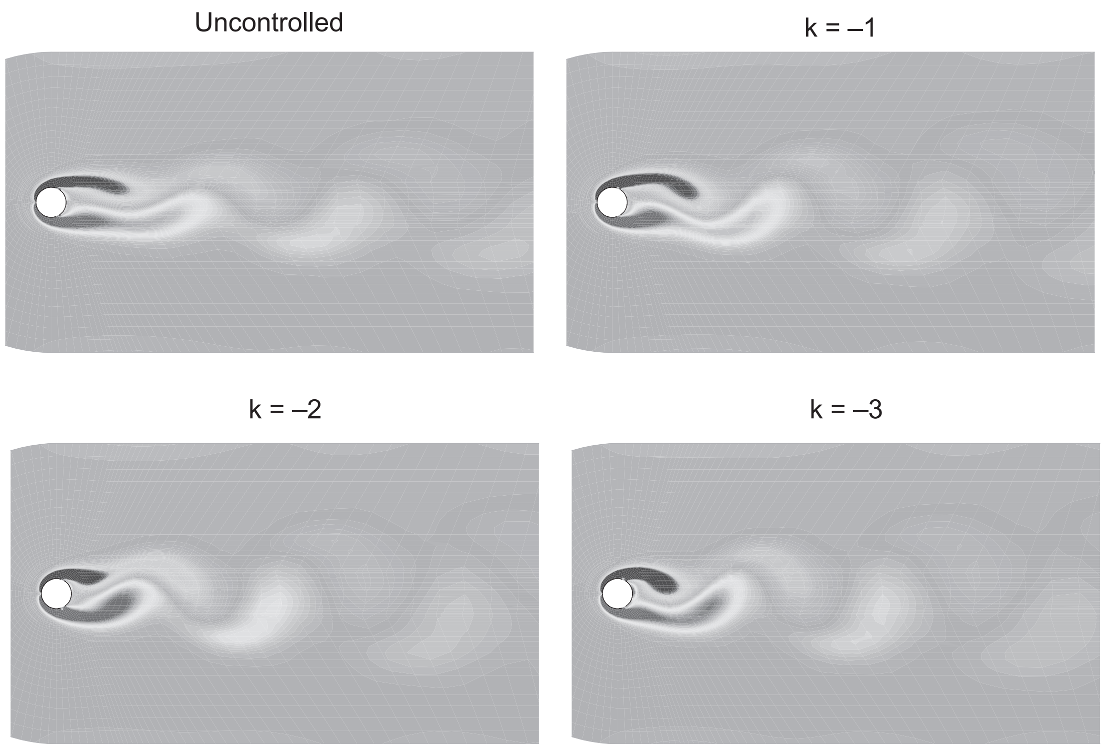

Figure 11. Vorticity field for $Re = 60$ at $t = 800$.

the vortical structures are intensified. From the particle dispersion plots in figure 12, we can quantify the effect of our control. At $x = 10$, that is, 10 radii downstream, the width of the cylinder wake is increased by 30% in the controlled case with $k = -3$ relative to the uncontrolled case. At $x = 20$ (20 radii downstream), the width of the cylinder wake is increased by 24%. Qualitatively, the results show that the width of the wake increases with increased feedback gain ($|k|$); so the wake can be made wider at the expense of increased control effort.

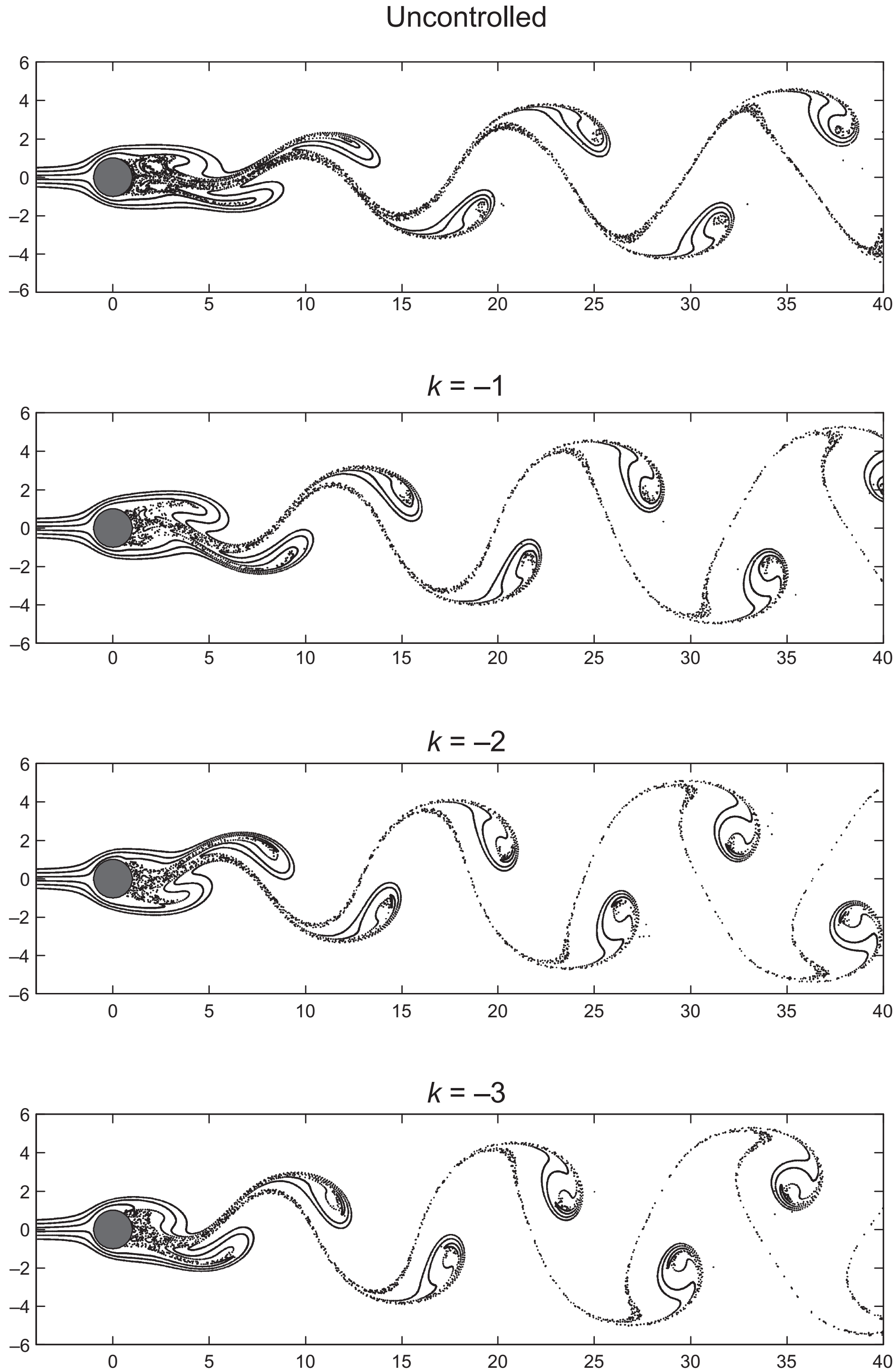

Figure 12. Particle distribution for $Re = 60$ at $t = 800$.

4.3. *Subcritical flow at* Re = *45*

4.3.1. *Initial condition*. As in the previous case, the initial condition is obtained by running the simulation for 500 time units starting from a perturbed velocity field. In this case, which is subcritical, the disturbances are dampened out, as suggested by the lift coefficient plot of figure 13(*a*), and confirmed by the high degree of symmetry in the vorticity map shown in figure 13(*b*).

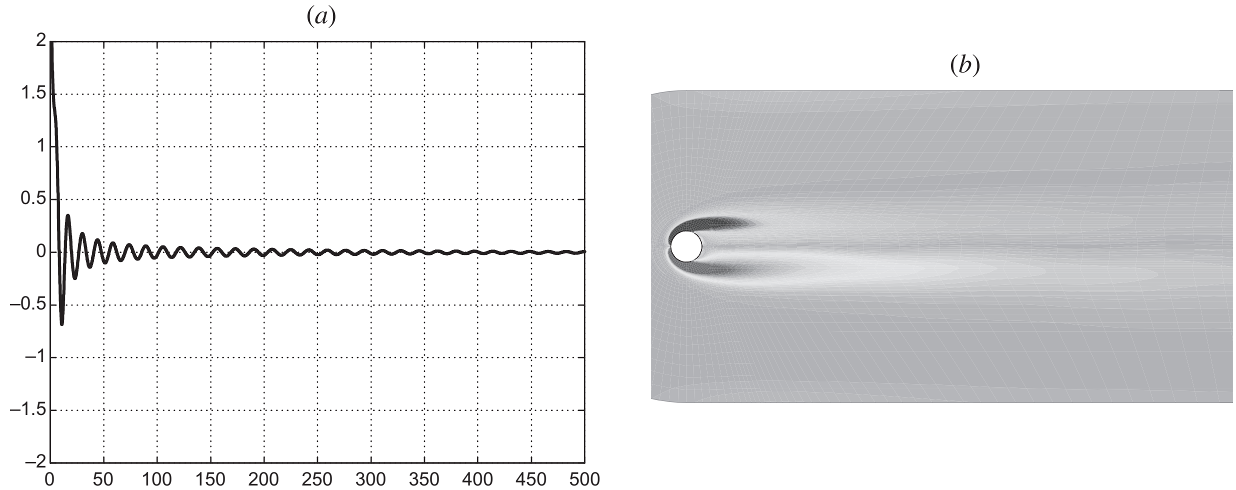

Figure 13. Lift coefficient (*a*) for initial simulation, and vorticity field (*b*) at $t = 500$, for $Re = 45$.

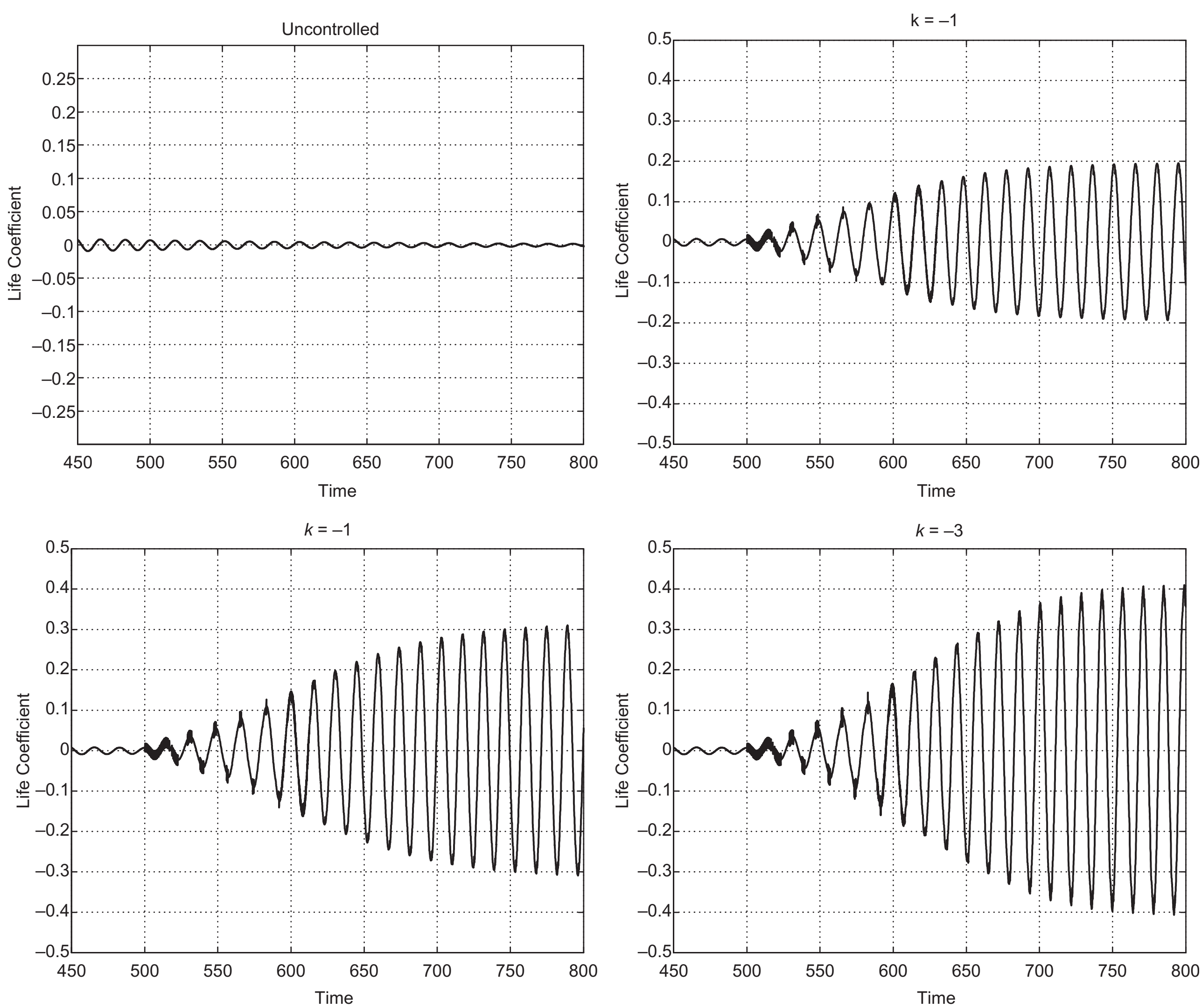

Figure 14. Lift coefficient for $Re = 45$.

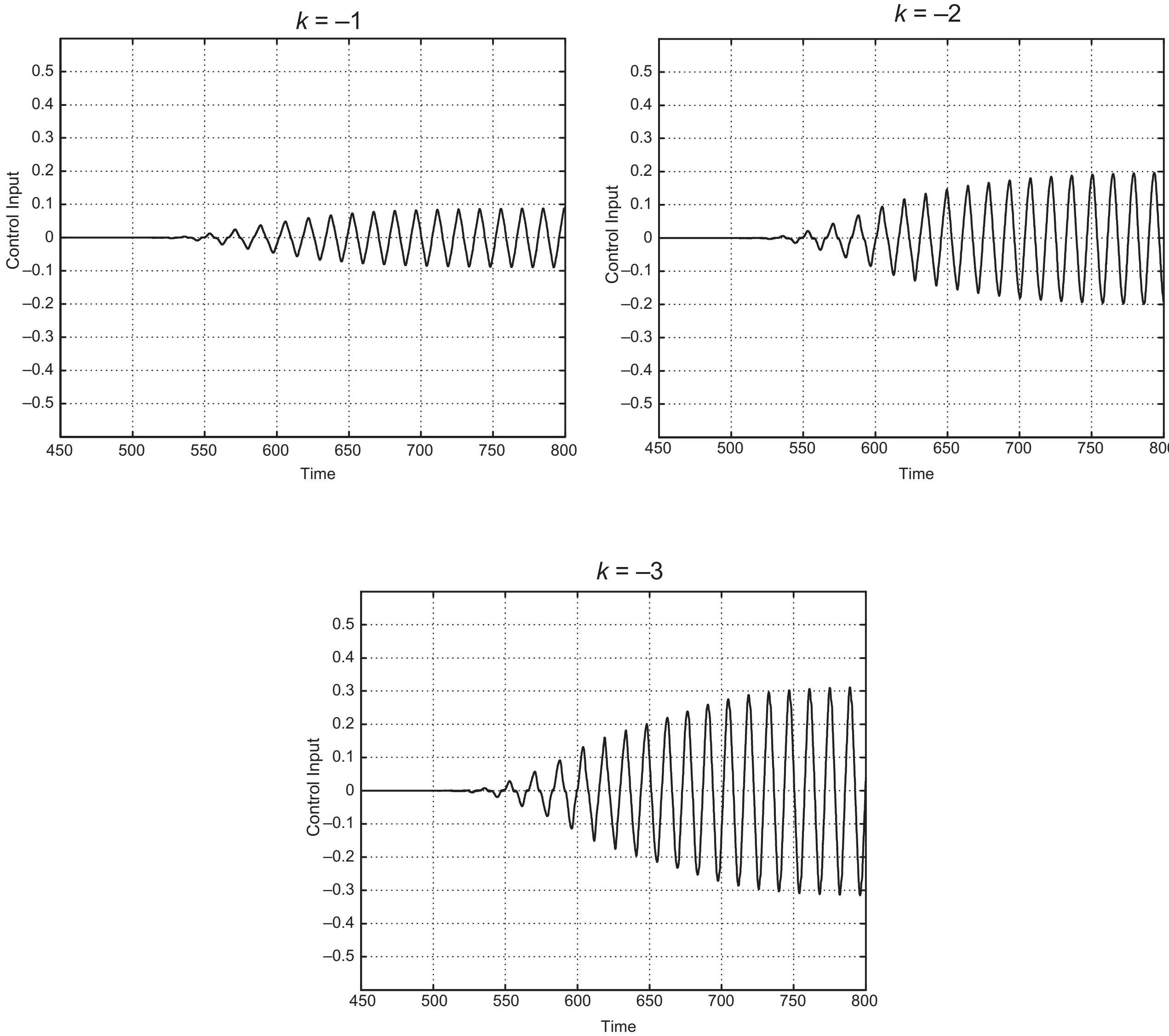

Figure 15. Control input for $Re = 45$.

The initial condition for the runs with feedback control is thus the steady state velocity field, only slightly perturbed.

4.3.2. *Controlled flow*. Figures 14–17 show lift coefficient, control input, vorticity maps and particle dispersion plots for the uncontrolled flow and the controlled flow with three different feedback gains. Similarly to the $Re = 60$ case, the lift force increases with increased control effort, and the vortical structures are intensified. From the particle dispersion plots in figure 17, it is clear that the width of the vortex street increases with increased control effort. The most interesting point in this case, which is subcritical so that vortex shedding does not occur naturally, is that vortex shedding can be initated by our simple feedback control.

4.3.3. *Effects of varying slot location*. As mentioned before, for this subcritical flow, the velocity field tends to a symmetric steady state. An interesting characteristic of the steady state at this Reynolds number is that two circulation cells develop behind the cylinder, as shown in figure 18, which shows streamlines at the end of the uncontrolled simulation. Therefore, fluid near the horizontal centreline behind the cylinder (and not too far downstream) does not flow in the direction of the main flow, but instead

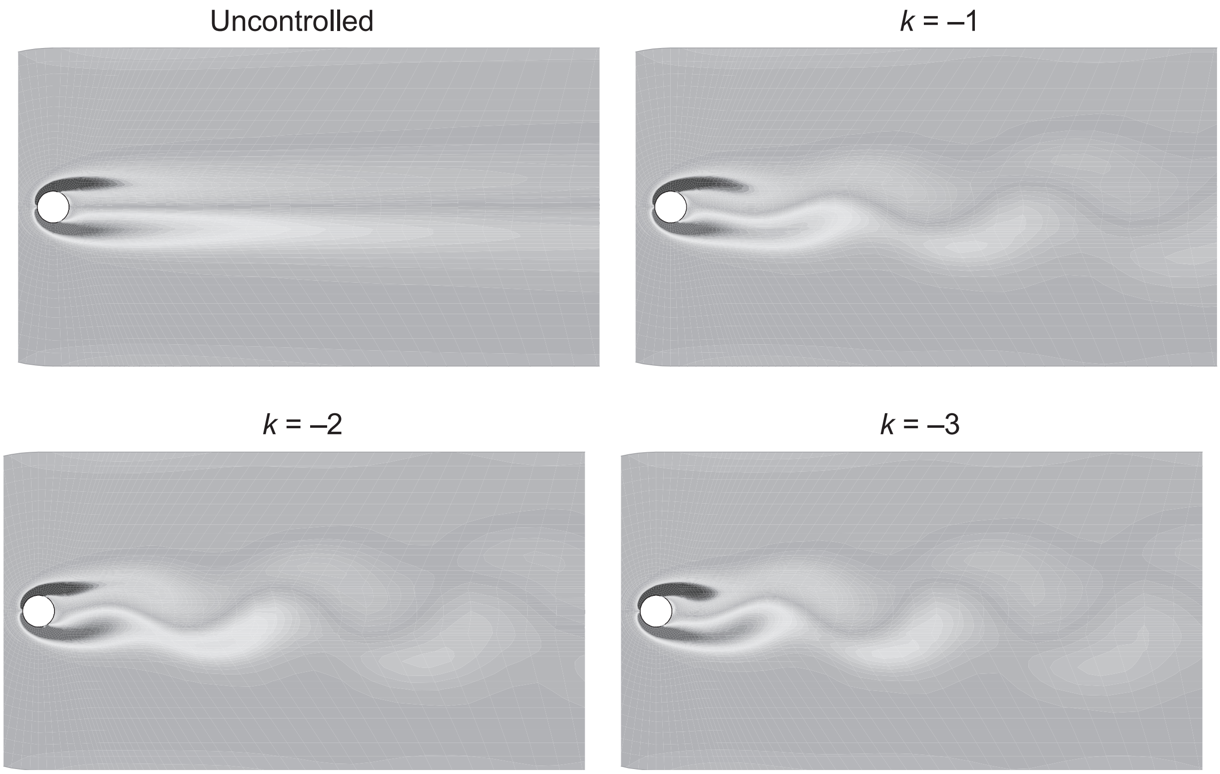

Figure 16. Vorticity field for $Re = 45$ at $t = 800$.

flows *backwards*, towards the cylinder. This leads to the existence of a separation point on the cylinder wall, as sketched in figure 19. It is of interest to investigate how the performance of the control varies with the location of the actuation relative to the separation point. In figure 18, a zoom on the cylinder wall of the streamlines at the end of the uncontrolled simulation shows the location of the separation point in this case, along with the position of slots #2 and #3. Slot #2, which has been used so far, is located just ahead of the separation point, slot #3 is collocated with the separation point, and slot #1 (not shown in figure 18) is located ahead of slot #2 (recall figure 1). In this section we look at the effect of actuating via slots #1 and #3, instead of slot #2. Figures 20–22 compare the results in the three cases, in terms of the lift coefficient, control input, and particle distribution. They show clearly that the location of the jets should be ahead of the separation point.

4.3.4. *Effects of varying jet direction.* In this section we investigate how the performance of the control system varies with the angle of the actuation jets (location is fixed at slot #2). Three cases are simulated, with the jets blowing in the vertical direction, wall-normal direction, and approximately aligned with the separatrix (see figure 23). Figures 24–26 show the results of these simulations in terms of the lift coefficient, control input and particle distribution. The results indicate that the control system is not very sensitive to the direction of the jets. A slight decrease in performance is detected when the jets are aligned with the separatrix, which is the direction for which the jet has the least vertical impact of the three directions simulated.

5. Concluding Remarks

We have addressed the problem of imposing particle dispersion, or mixing, in the wake of a bluff body by means of feedback control. The only means of actuation is a pair of blowing and suction slots located on the cylinder wall, and the only means of sensing is a collection of pressure sensors, also located on the cylinder wall. It was shown, in numerical simulations, that a very simple static feedback control law suppresses vortex shedding at Reynolds number $Re = 60$, and more interestingly, also enhances particle dispersion when run in destabilizing mode. At $Re = 45$, for which natural vortex shedding does not occur, the control law effectively initiates vortex shedding, which in turn leads to significantly enhanced particle dispersion. Simulations suggest that the actuation slots should be placed well ahead of the separation point, and that the performance

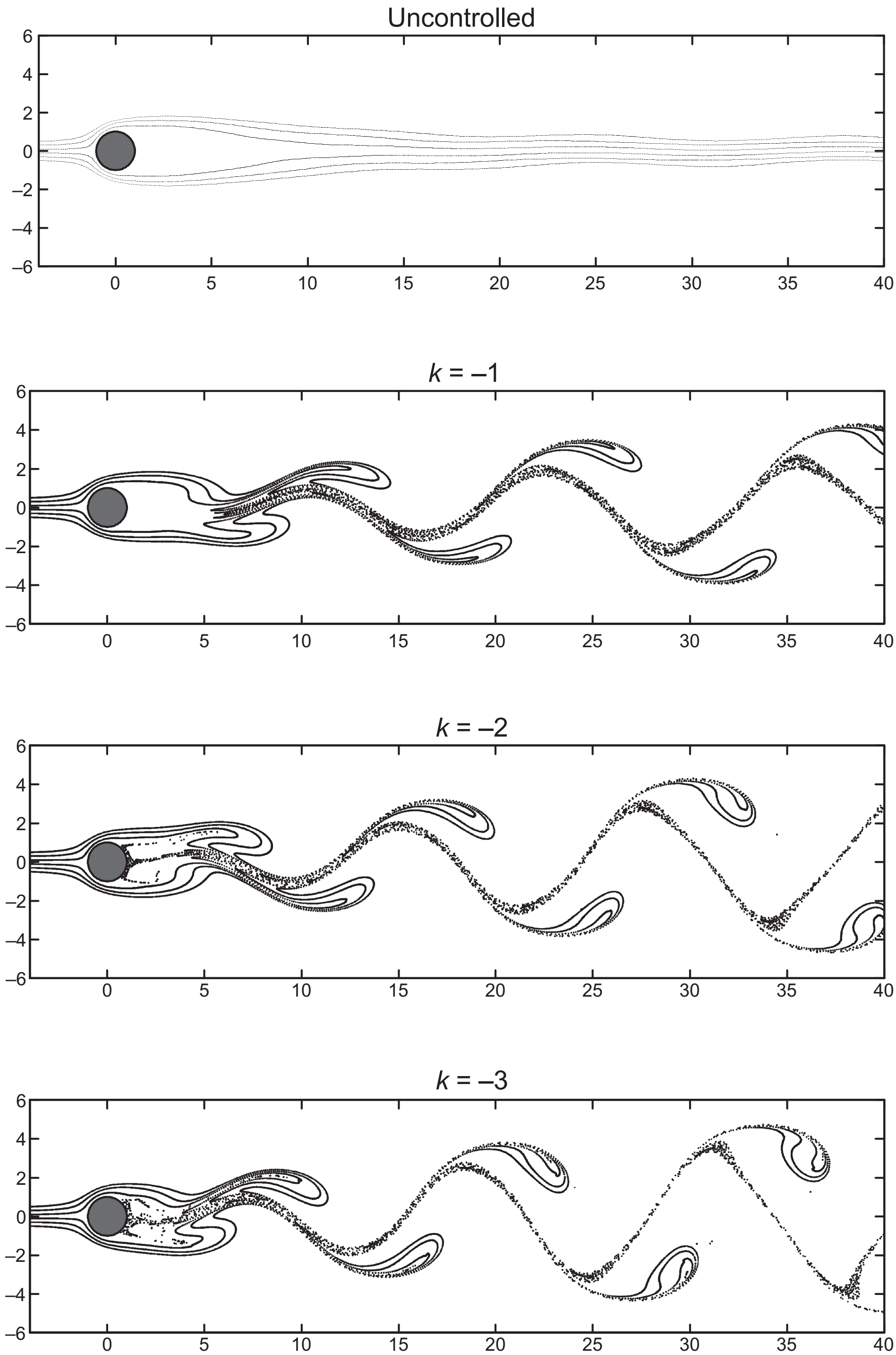

Figure 17. Particle distribution for $Re = 45$ at $t = 800$.

of the controller is quite insensitive to the direction of the jets.

A key feature of the proposed feedback control law is that it automatically finds the appropriate forcing frequency without knowledge of the physical parameters of the flow, such as velocity of incoming flow, fluid viscosity, fluid density or cylinder diameter. Thus, the feedback loop automatically exploits inherent instability mechanisms in the flow to increase mixing. In contrast, open-loop techniques need these parameters for optimizing the forcing frequency. That is, open-loop techniques generally depend on Reynolds number.

While the paper successfully achieves its objective, which is to demonstrate that mixing in bluff body

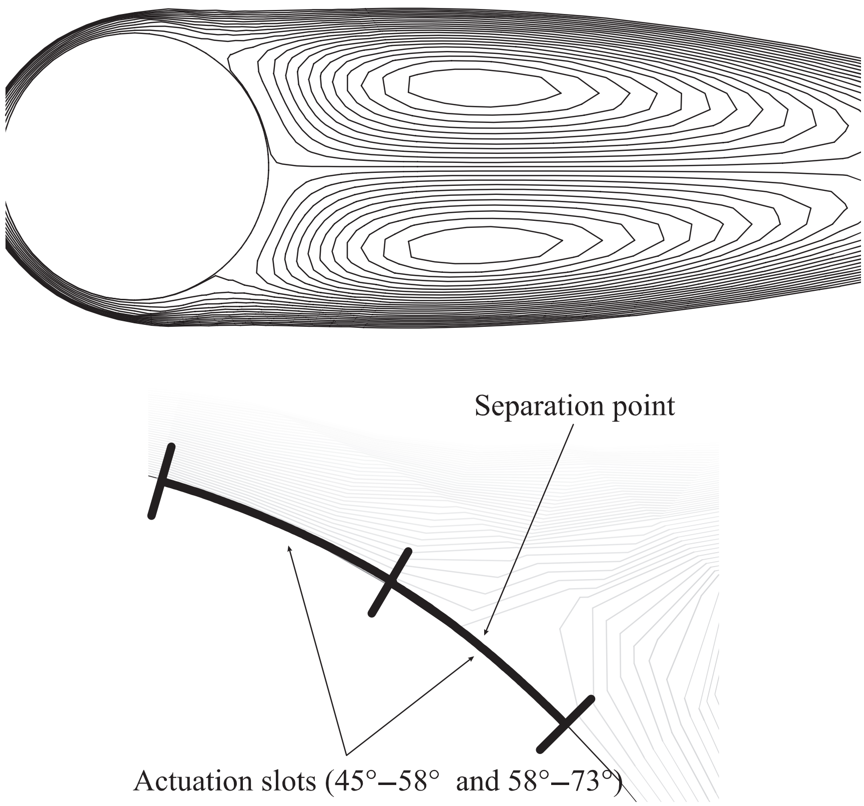

Figure 18. Streamlines at $t = 800$ for the uncontrolled case at $Re = 45$. The lower graph shows the location of the separation point relative to actuation slots #2 and #3.

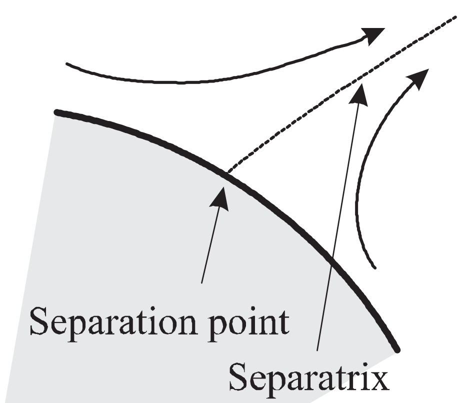

Figure 19. Circulation cells behind the cylinder leads to the existence of separation points and separatrices.

wakes may be enhanced by very simple feedback control, it also raises a number of questions that may serve as motivation for further work. Among the most important are the following:

- As explained above, the proposed feedback control exploits inherent instability mechanisms of the flow, and we therefore believe that mixing is achieved with less effort by closed-loop forcing rather than open-loop forcing. Confirming this by quantifying the energy needed for control and comparing with results achieved with open-loop forcing is an interesting direction for further work.
- The chosen sensing/actuation configuration was shown to be very effective compared to other configurations in channel flow geometries (Aamo 2001, 2002, 2003). The reason seems to be that boundary actuation penetrates into the flow faster by influencing the advective terms in the Navier–Stokes equation (wall normal actuation), rather than the diffusive terms (wall tangential actuation) (although dependent on Reynolds number). Similar comparisons could be carried out for the bluff body flow by comparing actuation by jets versus actuation by cylinder rotation.
- Although quantification of mixing is the object of some controversy, several methods exist which exceed the simple measures that we have employed massively in terms of sophistication. Analysis and comparison using these methods could be carried out to verify the conclusions made in this paper.

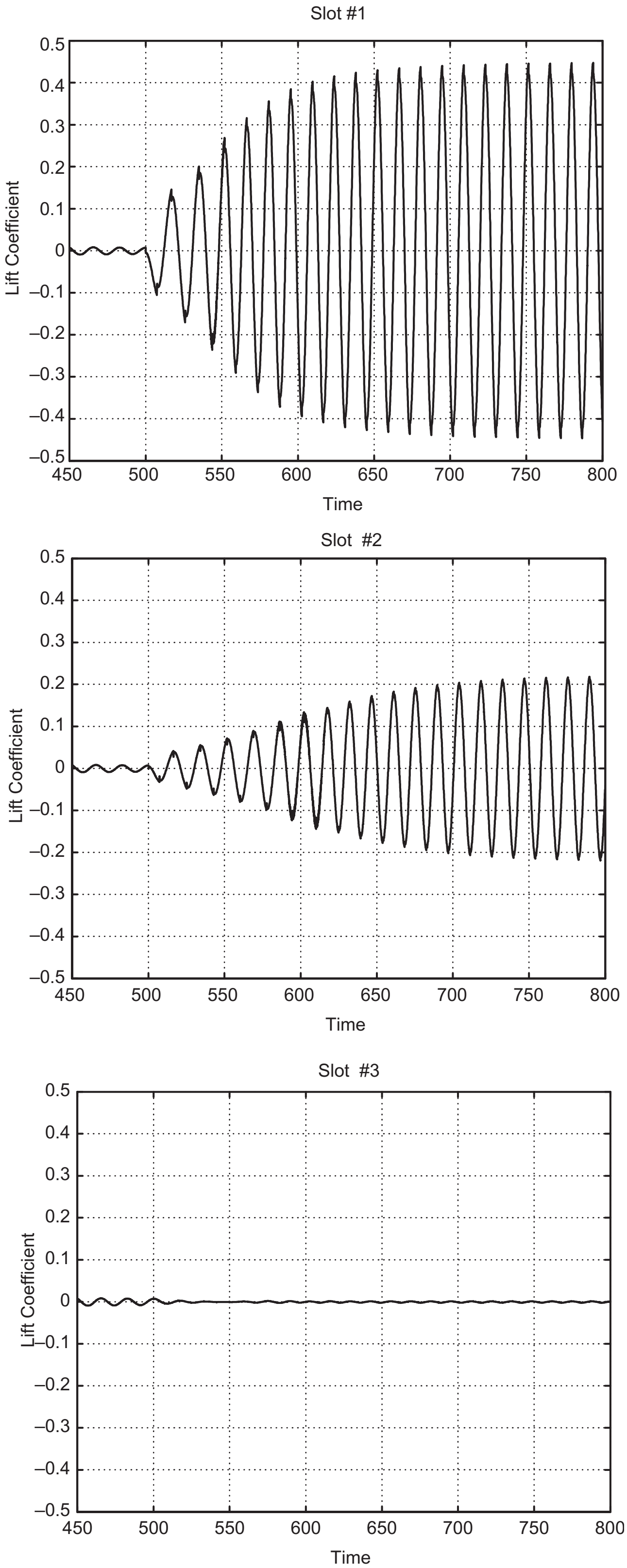

Figure 20. Lift coefficient for $Re = 45$ with control actuation located at three different locations.

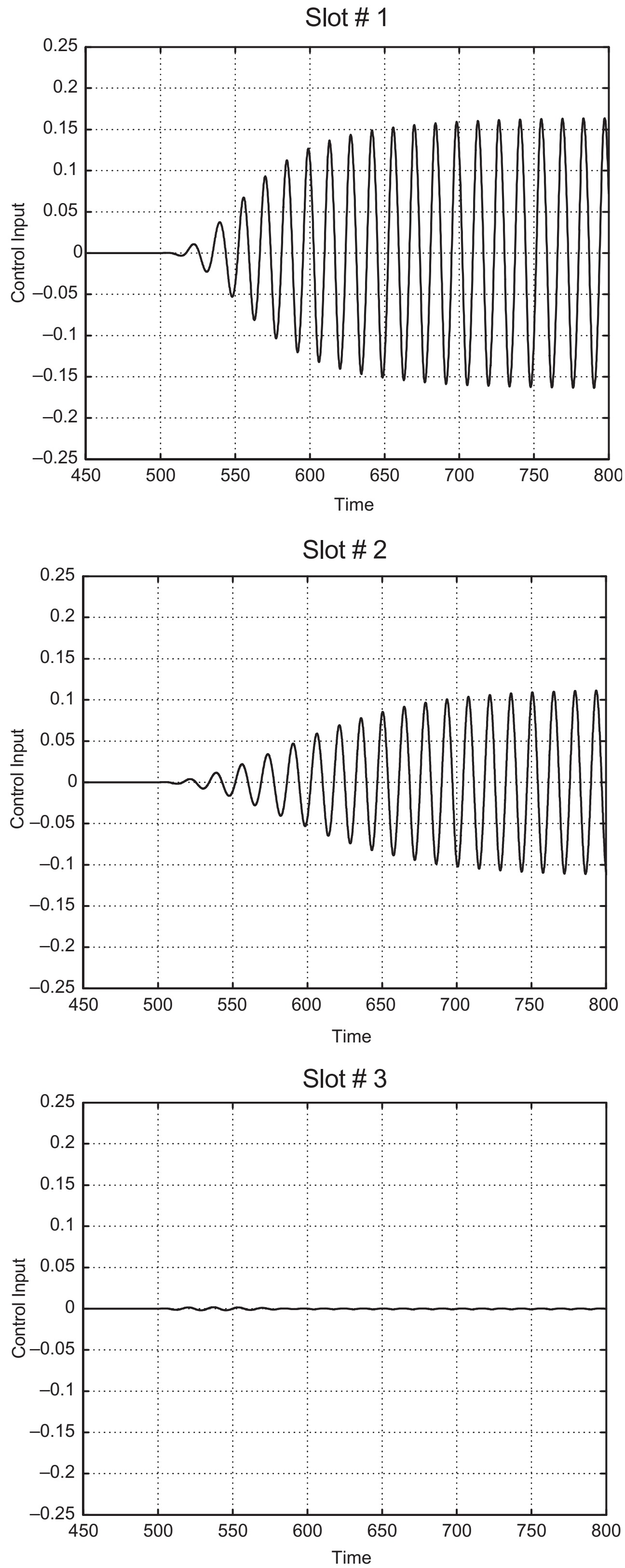

Figure 21. Control input for $Re = 45$ with control actuation located at three different locations.

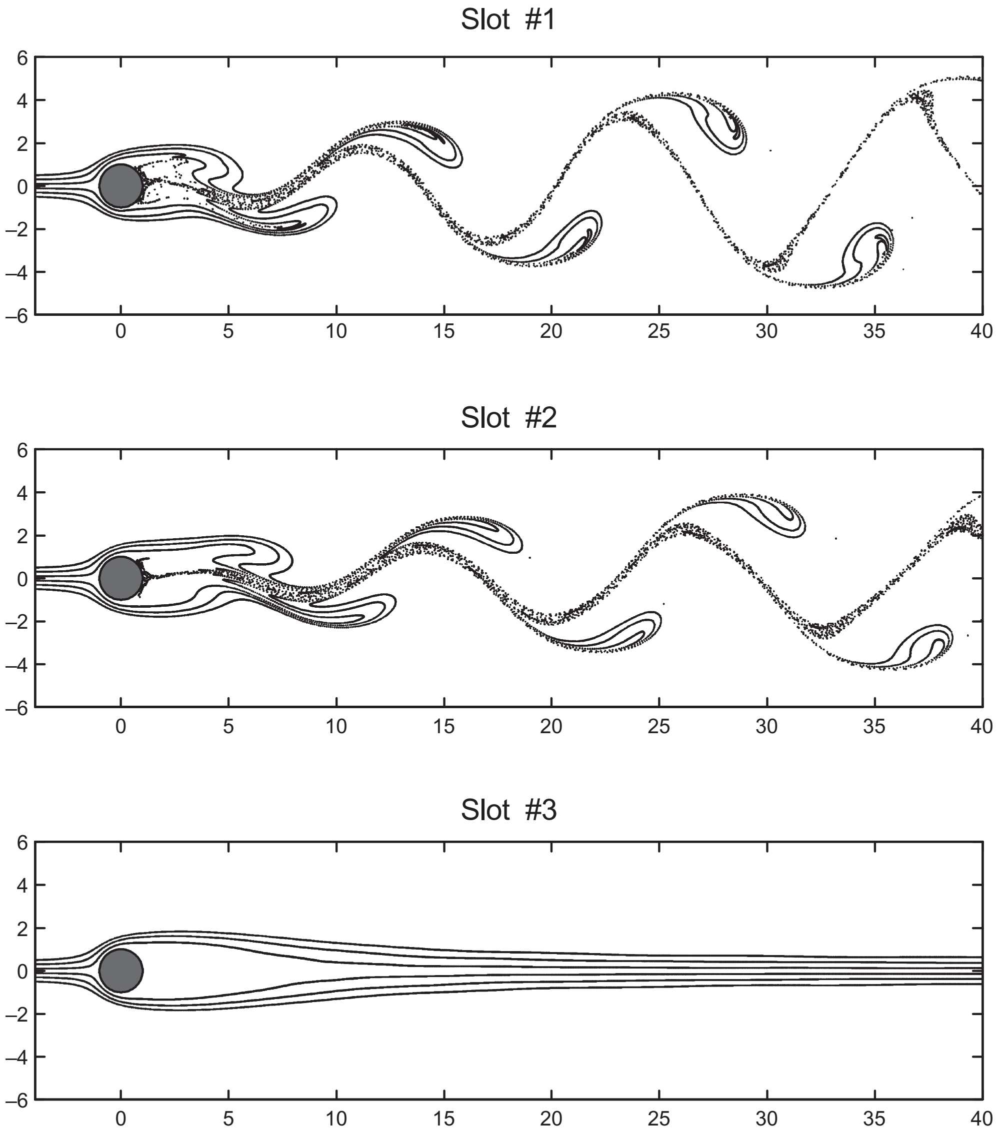

Figure 22. Particle distribution for $Re = 45$ at $t = 800$ with control actuation located at three different locations.

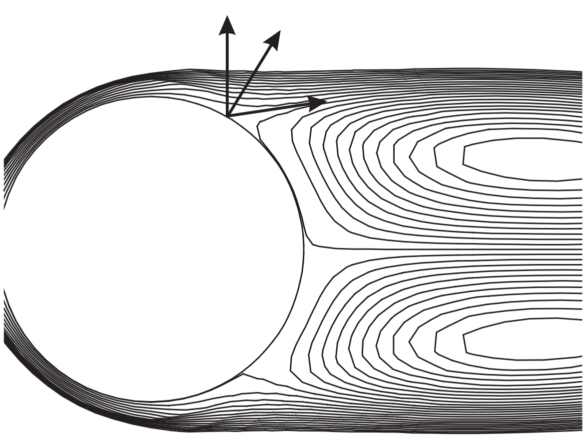

Figure 23. Directions of the jets.

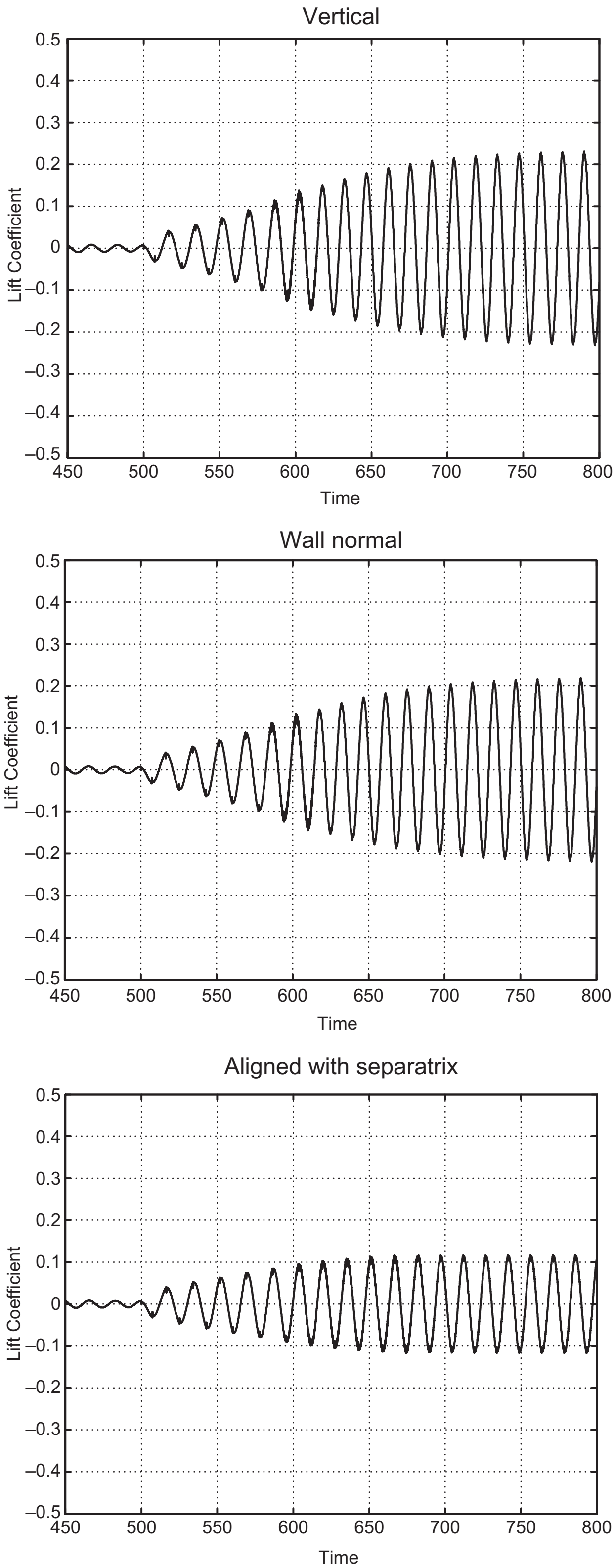

Figure 24. Lift coefficient for $Re = 45$ with control actuation directed in three different directions.

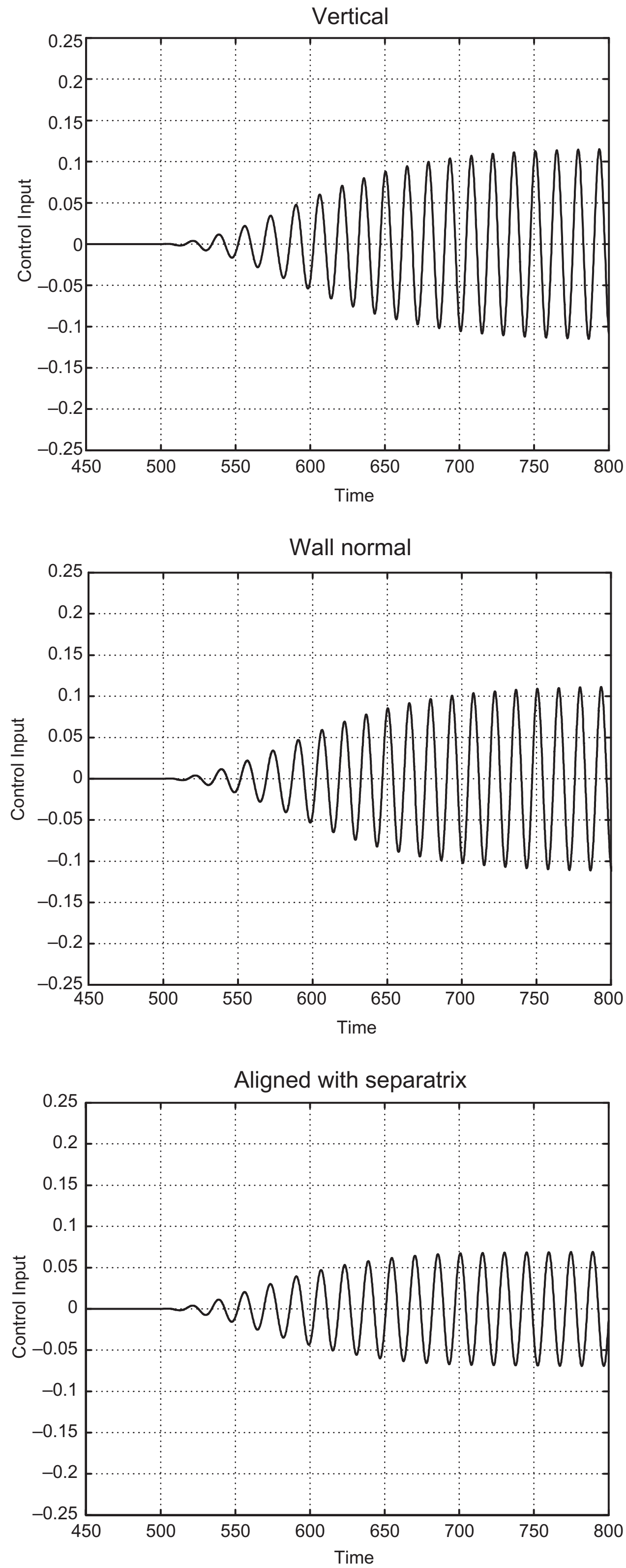

Figure 25. Control input for $Re = 45$ with control actuation directed in three different directions.

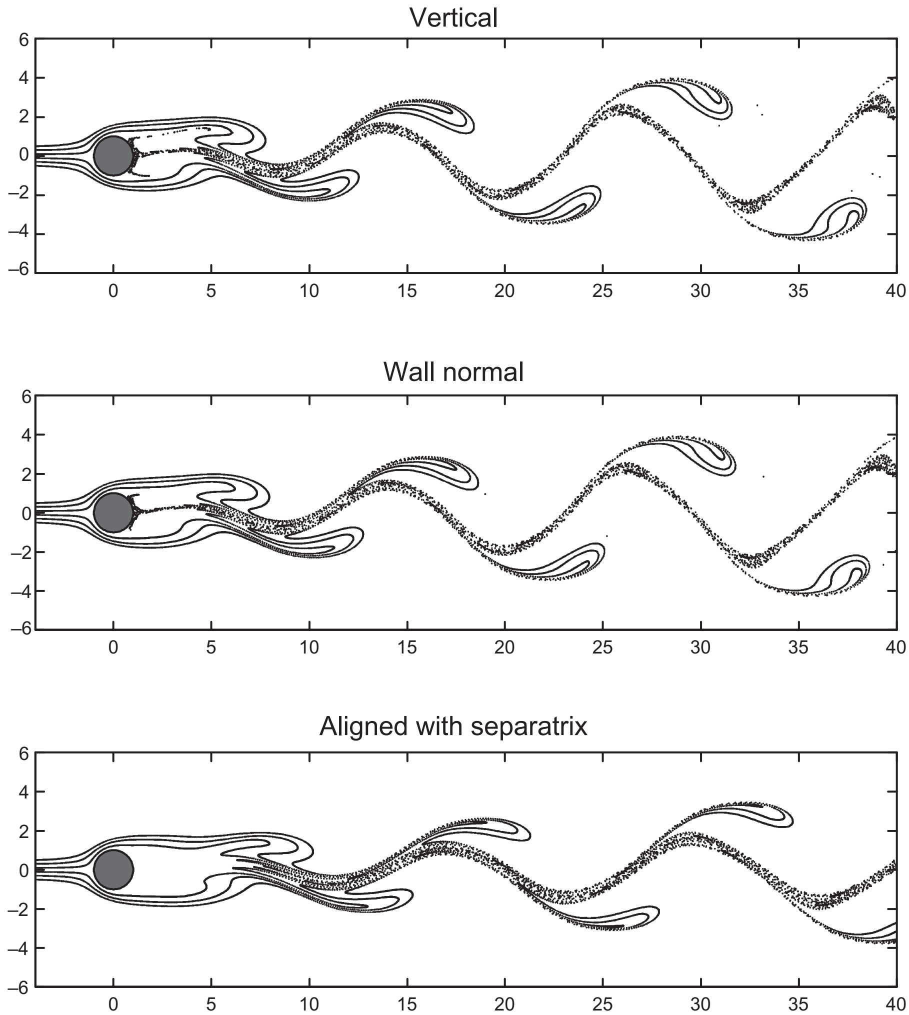

Figure 26. Particle distribution for $Re = 45$ at $t = 800$ with control actuation directed in three different directions.

- In Aamo *et al.* (2002), it was shown mathematically that a control law similar to (1) enjoys certain (inverse) optimality properties in 3D channel flow. It is of interest to extend the analysis to the bluff body flow studied here.

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