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Nonlinear control of mine ventilation networks☆

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Abstract

Ventilation networks in coal mines serve the critical task of maintaining a low concentration of explosive or noxious gases (e.g., methane). Due to the objective of controlling fluid flows, mine ventilation networks are high-order nonlinear systems. Previous efforts on this topic were based on multivariable linear models. The designs presented here are for a nonlinear model. Two control algorithms are developed. One employs actuation in all the branches of the network and achieves a global regulation result. The other employs actuation only in branches not belonging to the tree of the graph of the network and achieves regulation in a (non-infinitesimal) region around the operating point. The approach proposed for mine ventilation networks is also applicable to other types of fluid networks like gas and water distribution networks, irrigation networks, and possibly to building ventilation.

Keywords: Nonlinear control; Mine ventilation networks; Flow control; HVAC

1. Introduction

Coal as a source of fossil fuel energy should remain in abundance for a considerable time after petroleum reserves are exhausted. One of the principal difficulties in underground coal mines is the presence of poisonous and explosive gases like methane. Accidents claiming the lives of coal miners have been numerous through the history and continue to this day.

Modern coal mines contain elaborate ventilation facilities that allow to regulate the concentration of methane. In such ventilation systems the objective is usually not to directly control the concentrations but to control the air flow rates through individual branches of the ventilation network. The actuation available ranges from a few fans/compressors strategically located through the network (and often directly connected to the outside environment), to actively controlled "doors" that are in many of the branches of the network. The problem of controlling mine ventilation received considerable attention in the 1970s and the 1980s [1,2,6–11].

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It is clear that a mine ventilation network is a multivariable control problem where acting in one branch can affect the flow in the other branches in an undesirable way. For this reason, mine ventilation needs to be approached in a model-based fashion, as a fluid flow network (in much of the same way one would model an electric circuit) and as a multivariable control problem.

Pioneering work on this topic was performed by Kocić [5] who considered a linearized lumped-parameter dynamic model of a mine ventilation network and developed a methodology for designing linear feedback laws for it. He discovered structural properties that allowed him to decouple the problem into SISO problems and avoid the use of generic, highly complicated MIMO control tools. However, he did not take advantage of the graph theoretic properties of the network, which forced him to both neglect the nonlinearities (essential in this fluid flow problem) and to employ dynamic output-feedback compensators where static output feedback would suffice. We provide these improvements in this article.

The control model of a mine ventilation network consists of Kirchhoff's current and voltage laws (algebraic equations) and fluid dynamical equations of individual branches (differential equations). The branches are modeled using lumped parameter approximations of incompressible Navier–Stokes equations that take a form whose electric equivalent is an RL characteristic with a nonlinear resistance. To be precise, the pressure drop over a branch is approximated to be proportional to the square of the air flow rate (nonlinear resistive term) and to the air flow acceleration (linear inductive term).

A model written using Kirchhoff's algebraic equations and the branch characteristic differential equation constitute a non-minimal representation of the control model. It is clear that, due to the mass conservation at the branching points (nodes) of the network, airflows in many of the branches will be inter-dependent. Hence, the minimal system representation will be of lower order than the number of branches.

This intuition becomes systematic when one employs graph theoretic concepts from circuit theory [3]. Each network can be divided into a set of branches called a *tree* (they connect all the nodes of the graph without creating any loops) and the complement of the tree, called a co-tree, whose branches are referred to as the *links*. The minimal system representation of the dynamics of the network is given by the flow through the links.

While it is to possible to control the airflows only in independent branches—the links—and therefore necessary to put actuators only in those branches, the physical possibility to put actuators also in the tree branches allows to approach the control problem in two distinct ways. The first approach that we pursue actuates all the branches and yields a global stability result for this nonlinear system. The second approach actuates only the independent, link branches and yields a regional (around the operating point in the state space) result.

A peculiarity of the problem is that, while the model is affine in the control inputs, they do not appear in an additive manner. Since the inputs to the system are resistivities of the branches (modulated by the openings of "doors" in the branches), the control inputs are always multiplied by quadratic nonlinearities.

As the reader shall see in Section 4, following a complicated model development in the preceding sections, the last step of the nonlinear control design amounts to multivariable feedback linearization. This might normally raise the issue of modeling uncertainties but in this class of systems they are minor as tunnel lengths and diameters are easy to measure.

The method developed employs full state measurement because coal mine tunnels are always equipped with pressure, flow, and methane concentration sensors.

The paper is organized as follows. In Section 2 we introduce the constitutive equations and develop separately the non-minimal and the minimal representation of the system. In Section 3 we develop feedback laws that employ actuation in all the branches of the network, while in Section 4 we develop feedbacks for inputs only in the independent branches. We close with an example, chosen of minimal order to illustrate the main issues in the problem and the design algorithms.

2. Model of mine ventilation network system

2.1. Pipe flow dynamics and Kirchhoff's laws for mine ventilation networks

In order to develop the model of a mine ventilation network, we first establish the dynamical equation of one branch. For simplicity, we make the following assumptions: (A1) the air is incompressible; (A2) the temperatures in all branches are identical. Under assumptions (A1) and (A2), one branch of the mine ventilation network is described with the following equations [5,12,13]:

$$\frac{dQ_j}{dt} + K_j R_j |Q_j| Q_j = K_j H_j, \tag{1}$$

where Q_j is airflow quantity through a branch $j, R_j = r_j l_j$ are aerodynamic resistances, r_j are specific aerodynamic resistances of the branches, l_j are lengths of the branches, $H_j = p_{l_j} - p_{l_{j0}}$ are pressure drops of the branches, p_{l_j} are absolute pressures at the end of the branches, $p_{l_{j0}}$ are absolute pressures at the beginning of the branches, $K_j = S_j/\rho l_j$ are inertia coefficients, S_j are cross-sections of the branches, ρ is air density, $j = 1, \ldots, n$ and n is the number of network branches.

Like an electrical network, a mine ventilation network must satisfy Kirchhoff's current law, i.e., the airflow out of any node is equal to the flow into that node. Mathematically, Kirchhoff's current law for mine ventilation networks can be expressed as

$$\sum_{j=1}^{n} E_{Qij} Q_j = 0, \quad i = 2, \ldots, n_c - 1 \tag{2}$$

or

$$E_Q Q = 0, \tag{3}$$

where n_c is the number of nodes in the network, Q is a vector of airflow quantities, E_Q is a full rank matrix of order $(n_c - 2) \times n$ and $E_Q = [E_{Qij}]$, the values of E_{Qij} are defined as follows: $E_{Qij} = 1$ if branch j is connected to node i and the air flow goes away from node i, $E_{Qij} = -1$ if branch j is connected to node i and the air flow goes into node i, $E_{Qij} = 0$ if branch j is not connected to node i.

Let us assume that the mine ventilation network employs one main fan that is connected with the ambient outside of the mine. Also let node 1 be connected to the fan branch. Then the airflow in the fan branch can be expressed as

$$\sum_{j=1}^{n} e_{Qmj} Q_j = Q_m, \tag{4}$$

or

$$e_{Q_m} Q = Q_m, \tag{5}$$

where Q_m is airflow quantity through fan (<u>m</u>ain) branch, $e_{Qm} = [e_{Qm1}, \ldots, e_{Qmn}]$ is $1 \times n$ matrix, includes the values of e_{Qmj}, $j = 1, \ldots, n$ are defined as follows: $e_{Qmj} = 1$ if branch j is connected to node 1 and the air flow goes away from node 1, $e_{Qmj} = -1$ if branch j is connected to node 1 and the air flow goes into node 1, $e_{Qmj} = 0$ if branch j is not connected to node 1.

Similarly, a mine ventilation network also satisfies Kirchhoff's voltage law, i.e., the sum of the pressure drops around any loop in the network must be equal to zero, or mathematically,

$$\sum_{j=1}^{n} E_{Hij} H_j = 0, \quad i = 1, \ldots, l - k, \tag{6}$$

or

$$E_H H = 0, \tag{7}$$

where H_j is the pressure drop of the branch j, l is a number of the links in the network, $l = n - n_c + 1$; H is a vector of pressure drops, E_H is $(l-k) \times n$ fundamental mesh matrix, in which each mesh is formed by a link and a unique chain in the tree connecting two endpoints of the link, k is a number of meshes, containing fan branch, it is equal to the number of links, connected to the fan branch at its end. $E_H = [E_{Hij}]$, the elements of E_{Hij} are defined as follows: $E_{Hij} = 1$ if branch j is contained in mesh i and has the same direction, $E_{Hij} = -1$ if branch j is contained in mesh i and has the opposite direction, $E_{Hij} = 0$ if branch j is not contained in mesh i.

Considering meshes, containing the fan branch, express the pressure drop in the fan branch as

$$\sum_{j=1}^{n} e_{Hmij} H_j = -H_m, \quad i = 1, \ldots, k, \tag{8}$$

or

$$e_{H_m} H = -H_m, \tag{9}$$

where H_m is the pressure drop of the fan branch, e_{Hm} is $k \times n$ matrix, includes the values of e_{Hmij}, $j = 1, \ldots, n$ which defined as follows: $e_{Hmij} = 1$ if branch j is contained in mesh i and has the same direction, $e_{Hmij} = -1$ if branch j is contained in mesh i and has the opposite direction, $e_{Hmij} = 0$ if branch j is not contained in mesh i.

The dynamics of the fan branch can be expressed as

$$H_m = d - R_m Q_m, \tag{10}$$

where d denotes the equivalent pressure drop generated by fan, and R_m is the resistance coefficient in the fan branch.

2.2. Non-minimal model of the network

In order to establish the state equation, one has to find independent variables as states of the system. By virtue of the concepts of a tree and a link, they can easily be found. So the first step is to describe the tree of the mine ventilation network such that the fan branch is contained in it, and take the airflow quantities of link branches as state variables. For convenience of analysis, we label the air flow quantities of link branches from 1 to $N - n_c + 1$, where $N = n + 1$. Define

$$Q = \begin{bmatrix} Q_c \\ Q_a \end{bmatrix} = \begin{bmatrix} Q_1 \\ \vdots \\ Q_{N-n_c+1} \\ Q_{N-n_c+2} \\ \vdots \\ Q_n \end{bmatrix}, \quad H = \begin{bmatrix} H_c \\ H_a \end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_{N-n_c+1} \\ H_{N-n_c+2} \\ \vdots \\ H_n \end{bmatrix}, \tag{11}$$

so that Q_c and H_c matrices describe airflow quantity and pressure drop, respectively, in the links, and Q_a and H_a matrices describe them in the tree branches, excluding the fan branch.

With the notation

$$Q_D^2 = \mathrm{diag}(Q_1|Q_1|,\dots,Q_n|Q_n|), \quad K = \mathrm{diag}(K_1,\dots,K_n) = \begin{bmatrix} K_c & 0 \\ 0 & K_a \end{bmatrix}, \tag{12}$$

(1) can be rewritten as

$$\dot{Q} = -KQ_D^2R + KH. \tag{13}$$

Proposition 2.1. *There exist matrices A, B, C, Y_{RQ}, Y_Q and Y_d of appropriate dimensions so that the full order model of mine ventilation network can be expressed as*

$$\dot{Q} = AQ_D^2R + BQ + Cd, \tag{14}$$

$$H = Y_{RQ}Q_D^2R + Y_QQ + Y_dd, \tag{15}$$

where Q is the state, R and d are the inputs, and H is the output of the system.

Proof. The matrices E_H, E_Q, e_{H_m} and e_{Q_m} can be represented in the form:

$$E_H = [E_{Hc}E_{Ha}], \quad E_Q = [E_{Qc}E_{Qa}], \tag{16}$$

$$e_{H_m} = [e_{H_{mc}}e_{H_{ma}}], \quad e_{Q_m} = [e_{Q_{mc}}e_{Q_{ma}}], \tag{17}$$

where

$$\begin{bmatrix} E_{H_c} \\ e_{H_{mc}} \end{bmatrix} = I_{l\times l}, \tag{18}$$

$$E_{Q_a} = I_{(N-l-1)\times(N-l-1)}, \quad e_{Q_{ma}} = 0. \tag{19}$$

Let us now express the tree airflow quantities through link airflows. From (3), (11) and (16), we have

$$[E_{Qc}E_{Qa}]\begin{bmatrix} Q_c \\ Q_a \end{bmatrix} = 0. \tag{20}$$

With (19)

$$Q_a = -E_{Q_a}^{-1}E_{Q_c}Q_c = -E_{Q_c}Q_c. \tag{21}$$

Now express the link pressure drops through the fan branch pressure drop and tree pressure drops. From (7), (9) and (17), we can get

$$\begin{bmatrix} E_H \\ e_{H_m} \end{bmatrix} H = \begin{bmatrix} E_{Hc} \\ e_{H_{mc}} \end{bmatrix} H_c + \begin{bmatrix} E_{Ha} \\ e_{H_{ma}} \end{bmatrix} H_a = \begin{bmatrix} 0 \\ 1 \end{bmatrix} H_m. \tag{22}$$

From this equation, using (18) one can find H_c as

$$H_c = -\begin{bmatrix} E_{Ha} \\ e_{H_{ma}} \end{bmatrix} H_a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} H_m. \tag{23}$$

Using (10), rewrite (23) as

$$H_c = S_{Ha}H_a + R_m S_Q Q + S_d d, \tag{24}$$

where

$$S_{Ha} = -\begin{bmatrix} E_{Ha} \\ e_{H_{ma}} \end{bmatrix}, \tag{25}$$

$$S_Q = \begin{bmatrix} 0 \\ e_{Q_m} \end{bmatrix} = [S_{Q_c} \quad S_{Q_a}], \tag{26}$$

$$S_d = -\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{27}$$

With (13), (24), differentiating (3), we have

$$H_a = \zeta_{RQ} Q_D^2 R + \zeta_Q Q + \zeta_d d, \tag{28}$$

where

$$\zeta_{RQ} = (E_{Qc}K_c S_{Ha} + K_a)^{-1} E_Q K = [\zeta_{RQc} \quad \zeta_{RQa}], \tag{29}$$

$$\zeta_Q = -(E_{Qc}K_c S_{Ha} + K_a)^{-1} E_{Qc} K_c R_m S_Q = [\zeta_{Qc} \quad \zeta_{Qa}], \tag{30}$$

$$\zeta_d = -(E_{Qc}K_c S_{Ha} + K_a)^{-1} E_{Qc} K_c S_d. \tag{31}$$

One should mention, that the inverse of $E_{Qc}K_cS_{Ha} + K_a$, from Eqs. (29)–(31), exists, which will be shown in Lemma 2.1. Substituting (28) into (24), it can be expressed as

$$H_c = S_{Ha}\zeta_{RQ} Q_D^2 R + (S_{Ha}\zeta_Q + R_m S_Q)Q + (S_d + S_{Ha}\zeta_d)d. \tag{32}$$

With (11), (28) and (32), we have

$$\begin{aligned} H &= \begin{bmatrix} S_{Ha}\zeta_{RQ} \\ \zeta_{RQ} \end{bmatrix} Q_D^2 R + \begin{bmatrix} S_{Ha}\zeta_Q + R_m S_Q \\ \zeta_Q \end{bmatrix} Q + \begin{bmatrix} S_d + S_{Ha}\zeta_d \\ \zeta_d \end{bmatrix} d \\ &= Y_{RQ} Q_D^2 R + Y_Q Q + Y_d d, \end{aligned} \tag{33}$$

where

$$Y_{RQ} = \begin{bmatrix} S_{Ha}\zeta_{RQ} \\ \zeta_{RQ} \end{bmatrix}, \quad Y_Q = \begin{bmatrix} S_{Ha}\zeta_Q + R_m S_Q \\ \zeta_Q \end{bmatrix}, \quad Y_d = \begin{bmatrix} S_d + S_{Ha}\zeta_d \\ \zeta_d \end{bmatrix}.$$

Substituting (33) into (13), rewrite it as

$$\begin{aligned} \dot{Q} &= -K(I - Y_{RQ})Q_D^2 R + KY_Q Q + KY_d d \\ &= AQ_D^2 R + BQ + Cd, \end{aligned} \tag{34}$$

where

$$A = -K(I - Y_{RQ}), \quad B = KY_Q, \quad C = KY_d. \qquad \square$$

Lemma 2.1. *The inverse of* $E_{Qc}K_cS_{Ha}+K_a$ *exists.*

Proof. We number the branches in the following way: the links are enumerated from 1 to $l=N-n_c+1$, the first branch connects with the fan branch, and the tree branches are enumerated from l to N, where the fan branch is the last one. The loop and the node equations, including the fan branch, can be expressed as

$$\begin{bmatrix} E_{H_c} & E_{H_a} & 0 \\ e_{H_{mc}} & e_{H_{ma}} & 1 \end{bmatrix} \begin{bmatrix} H_c \\ H_a \\ H_m \end{bmatrix} = 0, \tag{35}$$

$$\begin{bmatrix} E_{Q_c} & E_{Q_a} & 0 \\ -e_{Q_{mc}} & -e_{Q_{ma}} & 1 \end{bmatrix} \begin{bmatrix} Q_c \\ Q_a \\ Q_m \end{bmatrix} = 0. \tag{36}$$

It can be shown [3, p. 493], that

$$\begin{bmatrix} E_{H_a} & 0 \\ e_{H_{ma}} & 1 \end{bmatrix} = -[E_{Q_c}^{\mathrm{T}} \quad -e_{Q_{mc}}^{\mathrm{T}}],$$

or

$$\begin{bmatrix} E_{H_a} \\ e_{H_{ma}} \end{bmatrix} = -E_{Q_c}^{\mathrm{T}}. \tag{37}$$

From (19) and (37), E_Q is of full rank. So $E_QK^{1/2}$ can be factorized by singular value decomposition [4] as

$$E_QK^{1/2} = U\Sigma V, \tag{38}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \cdots & \sigma_{n_c-2} & 0 \end{bmatrix}, \quad \sigma_i \neq 0, \quad i=1,\ldots,n_c-2. \tag{39}$$

With (38) and (39), we can write

$$E_QKE_Q^{\mathrm{T}} = U\Sigma VV^{\mathrm{T}}\Sigma^{\mathrm{T}}U^{\mathrm{T}} = U\Sigma\Sigma^{\mathrm{T}}U^{\mathrm{T}}.$$

So we have

$$\det(E_QKE_Q^{\mathrm{T}}) \neq 0. \tag{40}$$

Substituting (17) and (19) into (40), we get

$$\det(E_{Q_c}K_cE_{Q_c}^{\mathrm{T}} + K_a) \neq 0. \tag{41}$$

With (25), (19), (37) and (41), we have the following:

$$\det(E_{Q_c}K_cS_{Ha}^{\mathrm{T}}+K_a)=\det\left(-E_{Q_c}K_c\begin{bmatrix}e_{H_{ma}}\\ E_{H_a}\end{bmatrix}+K_a\right)$$
$$=\det\left(E_{Q_c}K_cE_{Q_c}^{\mathrm{T}}+K_a\right)\neq 0. \tag{42}$$

So the inverse of $E_{Qc}K_cS_{Ha}+K_a$ exists. □

2.3. Minimal model of the network

In the previous subsection we have established the full model of mine ventilation network, which is of order n. The states of the system are not independent, so one needs to find the minimal representation. In this subsection, we establish a minimal model of the mine ventilation network.

Define

$$Q_{cD}^2=\mathrm{diag}(Q_1|Q_1|,\ldots,Q_{N-n_c+1}|Q_{N-n_c+1}|), \tag{43}$$

$$Q_{aD}^2=\mathrm{diag}(Q_{N-n_c+2}(Q_c)|Q_{N-n_c+2}(Q_c)|,\ldots,\ Q_n(Q_c)|Q_n(Q_c)|), \tag{44}$$

$$R=[R_c^{\mathrm{T}}\quad R_a^{\mathrm{T}}]^{\mathrm{T}}, \tag{45}$$

where the dependence on Q_c in (44) should be understood in the sense of (21).

Proposition 2.2. *There exist matrices A_c, A_{ca}, B_c and C_c of appropriate dimensions so that the minimal model of mine ventilation network system can be expressed as*

$$\dot{Q}_c=A_cQ_{cD}^2R_c+A_{ca}Q_{aD}^2R_a+B_cQ_c+C_cd, \tag{46}$$

$$H_a=\zeta_{RQ_c}Q_{cD}^2R_c+\zeta_{RQ_a}Q_{aD}^2(Q_c)R_a+\zeta_{Q_c}Q_c+\zeta_dd, \tag{47}$$

where Q_c is a state, R_c, R_a and d are the control inputs, and H_a is the system output.

Proof. First we should mention, that $S_{Qa}=0$, which follows from (19) and (26). Also, from (30), $\zeta_{Qa}=0$. Substituting (44) into (32), we get

$$H_c=S_{Ha}\zeta_{RQ_c}Q_{cD}^2R_c+S_{Ha}\zeta_{RQ_a}Q_{aD}^2(Q_c)R_a+(S_{Ha}\zeta_{Q_c}+R_mS_{Q_c})Q_c+(S_d+S_{Ha}\zeta_d)d. \tag{48}$$

From (13), we have

$$\dot{Q}_c=-K_cQ_{cD}^2R_c+K_cH_c. \tag{49}$$

Substituting now (48) into (49),

$$\dot{Q}_c=(-K_c+K_cS_{Ha}\zeta_{RQ_c})Q_{cD}^2R_c+K_cS_{Ha}\zeta_{RQ_a}Q_{aD}^2(Q_c)R_a$$
$$+K_c(S_{Ha}\zeta_{Q_c}+R_mS_{Q_c})Q_c+K_c(S_d+S_{Ha}\zeta_d)d$$
$$=A_cQ_{cD}^2R_c+A_{ca}Q_{aD}^2R_a+B_cQ_c+C_cd, \tag{50}$$

where

$$A_c=-K_c+K_cS_{Ha}\zeta_{RQ_c}, \tag{51}$$

$$A_{ca}=K_cS_{Ha}\zeta_{RQ_a}, \tag{52}$$

$$B_c = K_c(S_{Ha}\zeta_{Q_c} + R_m S_{Q_c}), \tag{53}$$

$$C_c = K_c(S_d + S_{Ha}\zeta_d). \tag{54}$$

From (21), (28) and (30), we have

$$H_a = \zeta_{RQ_c} Q_{cD}^2 R_c + \zeta_{RQ_a} Q_{aD}^2 (Q_c) R_a + \zeta_{Q_c} Q_c + \zeta_d d. \qquad \square \tag{55}$$

The pressure drop in the fan branch can be described as

$$H_m = d - R_m e_{Q_{mc}} Q_c. \tag{56}$$

3. Design with controls in all branches

In this section, we use R_c, R_a and d as controls. The inputs R_a and d are referred to as auxiliary inputs [5] (thus a subscript "a"). As we shall see in the next section, they are not necessary, i.e., the system can be successfully controlled with R_c alone, but the auxiliary inputs applied for more effective control. Let us choose control laws as

$$R_c = (K_c Q_{cD}^2)^{-1}(K_c H_{cr} + \lambda Q_{ce}), \tag{57}$$

$$R_a = (K_a Q_{aD}^2)^{-1}(K_a H_{ar} + \lambda Q_{ae}), \tag{58}$$

$$d = H_{mr} + R_m Q_m, \tag{59}$$

where H_{cr}, H_{mr} and H_{ar} are the reference (equilibrium) values of H_c, H_m and H_a, respectively, $Q_{ce} = Q_c - Q_{cr}$, $Q_{ae} = Q_a - Q_{ar}$, in which Q_{cr} and Q_{ar} are the reference (equilibrium) values of Q_c and Q_a, respectively, and λ is a constant, that will be defined later. Clearly, H_r and Q_r need to satisfy Kirchhoff's laws for the mine ventilation network.

With the control laws given by (57)–(58), we have the following result.

Theorem 3.1. *For the system described by* (14) *and* (15), *under the control laws* (57)–(58), *the following results hold*:

(i) $H(t) \equiv H_r = [H_{cr}^T, H_{ar}^T]^T$;
(ii) $Q = Q_r = [Q_{cr}^T, Q_{ar}^T]^T$ *is exponentially stable*;
(iii) *suppose that* $Q_i(0) \geqslant 0$, $Q_{ir} > 0$ *and* $\lambda < \min_i K_i R_{ir} Q_{ir}$, *then* $R_i(t) > 0$, $\forall t \geqslant 0$, *where* $i = 1, \ldots, n$.

Proof. (i) Differentiating (3), we have

$$E_Q \dot{Q} = 0. \tag{60}$$

Substituting (16), (13), (43) and (44) into (60), we get

$$E_{Q_c}(-K_c Q_{cD}^2 R_c + K_c H_c) - K_a Q_{aD}^2 R_a + K_a H_a = 0. \tag{61}$$

Substituting (23) into Eq. (61), we rewrite it as

$$E_{Q_c}(-K_c Q_{cD}^2 R_c + K_c S_{Ha} H_a + K_c S_d H_m) - K_a Q_{aD}^2 R_a + K_a H_a = 0. \tag{62}$$

Rearranging this, we have

$$E_{Q_c}K_cQ_{cD}^2R_c + K_aQ_{aD}^2R_a = E_{Q_c}K_cS_{Ha}H_a + E_{Q_c}K_cS_dH_m + K_aH_a. \tag{63}$$

Finally, substituting (57) and (58) into (63), we get the following result:

$$\lambda E_{Q_c}Q_{ce} + \lambda Q_{ae} + E_{Q_c}K_cH_{cr} + K_aH_{ar} = E_{Q_c}K_cS_{Ha}H_a + E_{Q_c}K_cS_dH_m + K_aH_a. \tag{64}$$

From Kirchhoff's law for airflow quantities (3) we can see that $E_{Q_c}Q_{ce} + Q_{ae} = 0$, so substituting this into (64) and taking into account (23), we rewrite (64) in the form

$$E_{Q_c}K_cS_{Ha}H_{ar} + K_aH_{ar} = E_{Q_c}K_cS_{Ha}H_a + K_aH_a. \tag{65}$$

Subtracting $E_{Q_c}K_cS_{Ha}H_{ar} + K_aH_{ar}$ from both sides of (65), we get

$$(E_{Q_c}K_cS_{Ha} + K_a)(H_a - H_{ar}) = 0, \tag{66}$$

From Lemma 2.1, the inverse of $(E_{Q_c}K_cS_{Ha} + K_a)$ exists, so

$$H_a = H_{ar}. \tag{67}$$

Comparing (10) and (59), it is easy to see, that

$$H_m = H_{mr}. \tag{68}$$

Substituting (67) and (68) into (23), we have

$$H_c = H_{cr}. \tag{69}$$

With (67), (68) and (69), we get $H = H_r$.

(ii) After substitution (57) and (58) into (13), the closed loop system becomes

$$\dot{Q} = -\lambda Q_e. \tag{70}$$

Since $\dot{Q} = \dot{Q}_e$, then $\dot{Q}_e = -\lambda Q_e$, which implies (ii).

(iii) From (70), the solutions are

$$Q_{ei} = Q_{ei}(0)e^{-\lambda t}, \quad i = 1, \ldots, n. \tag{71}$$

Substituting (67)–(69) and (71) into (57),

$$\begin{aligned} R_i(t) &= (K_iQ_i^2)^{-1}(K_iH_{ri} + \lambda Q_{ei}) \\ &= (K_iQ_i^2)^{-1}[K_iH_{ri} + \lambda e^{-\lambda t}(Q_i(0) - Q_{ri})], \quad i = 1, \ldots, n. \end{aligned} \tag{72}$$

From (72), if

$$Q_i(0) \geqslant 0, \quad Q_{ir} > 0, \tag{73}$$

$$\lambda < \min_i K_iR_{ri}Q_{ri}, \tag{74}$$

then

$$R_i(t) > 0, \quad \forall t \geqslant 0, \quad i = 1, \ldots, n. \qquad \Box$$

Remark 3.1. In a practical mine ventilation implementation, the minimal branch resistance $R_i(t)$, corresponding to the actuator "door" fully open, will be not zero, but some positive value that is due to the resistance of the tunnel walls.

4. Design with controls in co-tree only

In this section we achieve the control objective with R_c alone. Choose the control law as

$$R_c = (K_c Q_{cD}^2)^{-1}(K_c H_c + \lambda Q_{ce}), \tag{75}$$

$$R_a = (K_a Q_{arD}^2)^{-1} K_a H_{ar}, \tag{76}$$

$$d = H_{mr} + R_m Q_{mr}. \tag{77}$$

Note that R_a and d are constant. With (11) and (12), the expression for airflow quantities (13) can be rewritten as

$$\dot{Q}_c = -K_c Q_{cD}^2 R_c + K_c H_c, \tag{78}$$

$$\dot{Q}_a = -K_a Q_{aD}^2 R_a + K_a H_a. \tag{79}$$

Substituting (75) into (78),

$$\dot{Q}_c = -\lambda Q_{ce}. \tag{80}$$

This equation clearly indicates exponential stability. However, this stability can be ensured only if the control law (75) is guaranteed to be implementable. A control law that employs negative values of resistance would not be implementable in a mine ventilation network. Thus we need to study feasibility of the feedback (75). While the pressure drop H_c in (75) is preferable for implementation because pressure is easier to measure than the flow rate, for a feasibility study we have to express H_c as a function of the state Q_c. This will allow us to find the function $R_c(Q_c)$.

With (80), differentiating (21), we get airflow quantities for the tree branches

$$\dot{Q}_a = -E_{Q_c}\dot{Q}_c = \lambda E_{Q_c} Q_{ce}. \tag{81}$$

Now let us find the pressure drops. By (79) and (81), H_a can be written as

$$H_a = K_a^{-1}\dot{Q}_a + Q_{aD}^2 R_a = \lambda K_a^{-1} E_{Q_c} Q_{ce} + Q_{aD}^2 R_a. \tag{82}$$

Using (82), we rewrite (23) as

$$\begin{aligned} H_c &= S_{Ha} H_a + S_d H_m \\ &= S_{Ha}(\lambda K_a^{-1} E_{Q_c} Q_{ce} + Q_{aD}^2 R_{ar}) + S_d d_r - R_m S_d e_{Q_{mc}} Q_c. \end{aligned} \tag{83}$$

After substitution (83) into (75), the control law becomes

$$\begin{aligned} R_c(Q_c) &= (K_c Q_{cD}^2)^{-1}[\lambda Q_{ce} + K_c S_{Ha} Q_{aD}^2(Q_c) R_{ar} + \lambda K_c S_{Ha} K_a^{-1} E_{Q_c} Q_{ce} + K_c S_d d_r - K_c R_m S_d e_{Q_{mc}} Q_c] \\ &= (K_c Q_{cD}^2)^{-1}[\lambda(I + K_c S_{Ha} K_a^{-1} E_{Q_c}) Q_{ce} + K_c S_{Ha} Q_{aD}^2(Q_c) R_{ar} + K_c S_d d_r - K_c R_m S_d e_{Q_{mc}} Q_c]. \end{aligned} \tag{84}$$

We are now ready to estimate the feasibility region of the feedback system.

Let $\mathscr{F} = \{Q_c \in R^{N-n_c+1} | R_{ci}(Q_c) \geqslant R_{ci}^{\min},\ i = 1, \ldots, N - n_c + 1\}$ be the feasible control set, where $R_{ci}^{\min}$ is the minimum feasible control values. Define also the sets $B_r = \{\|Q_e\| \leqslant r\}$. Using these designations we can now establish the following result for the system, consisting of the model (46), (47) and the control laws (77), (76) and (84).

Theorem 4.1. *Let r^* be the largest r such that $B_r \subset \mathscr{F}$. Then, $Q = Q_r$ is exponentially stable with the region of attraction that includes B_{r^*}.*

Proof. Consider the Lyapunov function

$$V = \frac{1}{2}\|Q_{ce}\|^2 \tag{85}$$

whose level sets are B_r. For all $Q_e \subset B_{r^*}$ we have $R_{ci} \geqslant R_{ci}^{\min}$, so the closed-loop system can be expressed as

$$\dot{Q}_{ce} = -\lambda Q_{ce}, \tag{86}$$

Differentiating (85) along (86), we obtain

$$\dot{V} = Q_{ce}^{\mathrm{T}}\dot{Q}_{ce} = -\lambda\|Q_{ce}\|^2 = -2\lambda V. \tag{87}$$

From (87), we conclude that $Q = Q_r$ is exponentially stable. □

5. Example

Consider a mine ventilation network control system, which consists of 3 nodes, 3 branches and 1 main fan branch as in Fig. 1. Choosing branches 3 and m as the tree of the network, the loop equations and node equation can be expressed as

$$H_1 - H_3 = 0, \quad H_2 + H_3 = -H_m, \quad Q_1 - Q_2 + Q_3 = 0, \quad Q_2 = Q_m,$$

where

$$E_H = [1 \quad 0 \quad -1], \quad E_Q = [1 \quad -1 \quad 1], \quad e_{H_m} = [0 \quad 1 \quad 1], \quad e_{Q_m} = [0 \quad 1 \quad 0],$$

$$E_{H_c} = [1 \quad 0], \quad E_{H_a} = -1, \quad E_{Q_c} = [1 - 1], \quad E_{Q_a} = 1,$$

$$e_{H_{mc}} = [0 \quad 1], \quad e_{H_{ma}} = 1, e_{Q_{mc}} = [0 \quad 1], \quad e_{Q_{ma}} = 0.$$

Define

$$Q_c = [Q_1 \quad Q_2]^{\mathrm{T}}, \quad Q_a = Q_3, H_c = [H_1 \quad H_2]^{\mathrm{T}}, \quad H_a = H_3.$$

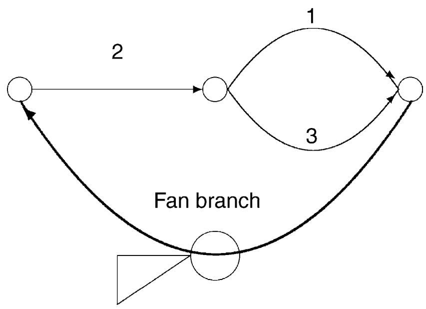

Fig. 1. Mine ventilation network system with 4 branches.

The matrices and vectors in (24), (28) and (33) are

$$S_{Ha} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad S_Q = [S_{Q_c} \quad S_{Q_a}] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad S_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\zeta_{RQ} = -\frac{1}{K_1+K_2+K_3}[K_1 \quad -K_2 \quad -K_3], \quad \zeta_Q = \frac{1}{K_1+K_2+K_3}[-R_{m1}K_1 \quad 0 \quad 0],$$

$$\zeta_d = \frac{1}{K_1+K_2+K_3}K_1,$$

$$Y_{RQ} = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} K_1 & -K_2 & -K_3 \\ -K_1 & K_2 & K_3 \\ -K_1 & K_2 & K_3 \end{bmatrix}, \quad Y_Q = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -R_{m1}(K_2+K_3) & 0 & 0 \\ -R_{m1}K_1 & 0 & 0 \\ -R_{m1}K_1 & 0 & 0 \end{bmatrix},$$

$$Y_d = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} K_2+K_3 \\ K_1 \\ K_1 \end{bmatrix}.$$

The matrices and vectors for the full order system are

$$A = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -K_1(K_2+K_3) & -K_1K_2 & -K_1K_3 \\ -K_1K_2 & -K_2(K_1+K_3) & K_2K_3 \\ -K_1K_3 & K_2K_3 & -K_3(K_1+K_2) \end{bmatrix},$$

$$B = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -R_{m1}K_1(K_2+K_3) & 0 & 0 \\ -R_{m1}K_1K_2 & 0 & 0 \\ -R_{m1}K_1K_3 & 0 & 0 \end{bmatrix},$$

$$C = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} K_1(K_2+K_3) \\ K_1K_2 \\ K_1K_3 \end{bmatrix}.$$

The matrices and vectors for the minimal representation are

$$A_c = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -K_1(K_2+K_3) & -K_1K_2 \\ -K_1K_2 & -K_2(K_1+K_3) \end{bmatrix},$$

$$A_{ca} = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -K_1K_3 \\ K_2K_3 \end{bmatrix},$$

$$B_c = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} -R_{m1}K_1(K_2+K_3) & 0 \\ -R_{m1}K_1K_2 & 0 \end{bmatrix},$$

$$C_c = \frac{1}{K_1+K_2+K_3}\begin{bmatrix} K_1(K_2+K_3) \\ K_1K_2 \end{bmatrix}.$$

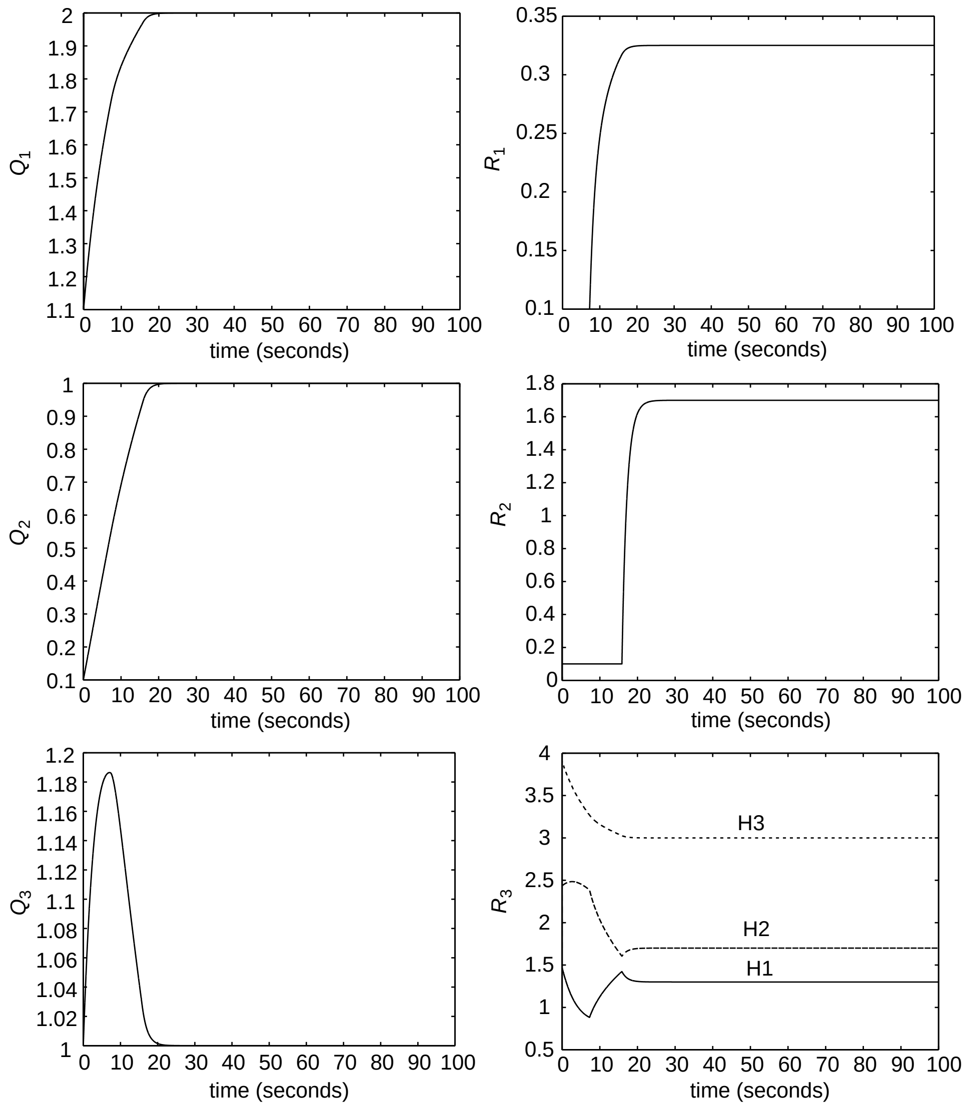

Fig. 2. Responses without auxiliary controllers.

The parameters of the system are chosen as $R_m=1$, $K_1=1/10$, $K_2=1/40$ and $K_3=1/10$. First, we investigate the responses of the system without auxiliary controllers. The operating point is $H_{mr}=3.0$, $H_{3r}=1.7$ and $Q_r=[2,1,1]^{\mathrm{T}}$. The control gain is chosen as $\lambda=0.07$. Under initial condition $Q(0)=[1.1,0.1,1.0]^{\mathrm{T}}$, the responses of the system are shown in Fig. 2. The control starts saturated but eventually brings the system to its feasibility region.

Next, we consider the design *with* auxiliary controllers. We start with the same initial condition and set the same reference point. We use a different gain, $\lambda=0.043$. The responses are shown in Fig. 3. Two observations should be made. First, due to a particular choice of initial condition (made to saturate the inputs in Fig. 2), the auxiliary control R_3 is not active. However, the auxiliary control d is active. Second, the activity of d

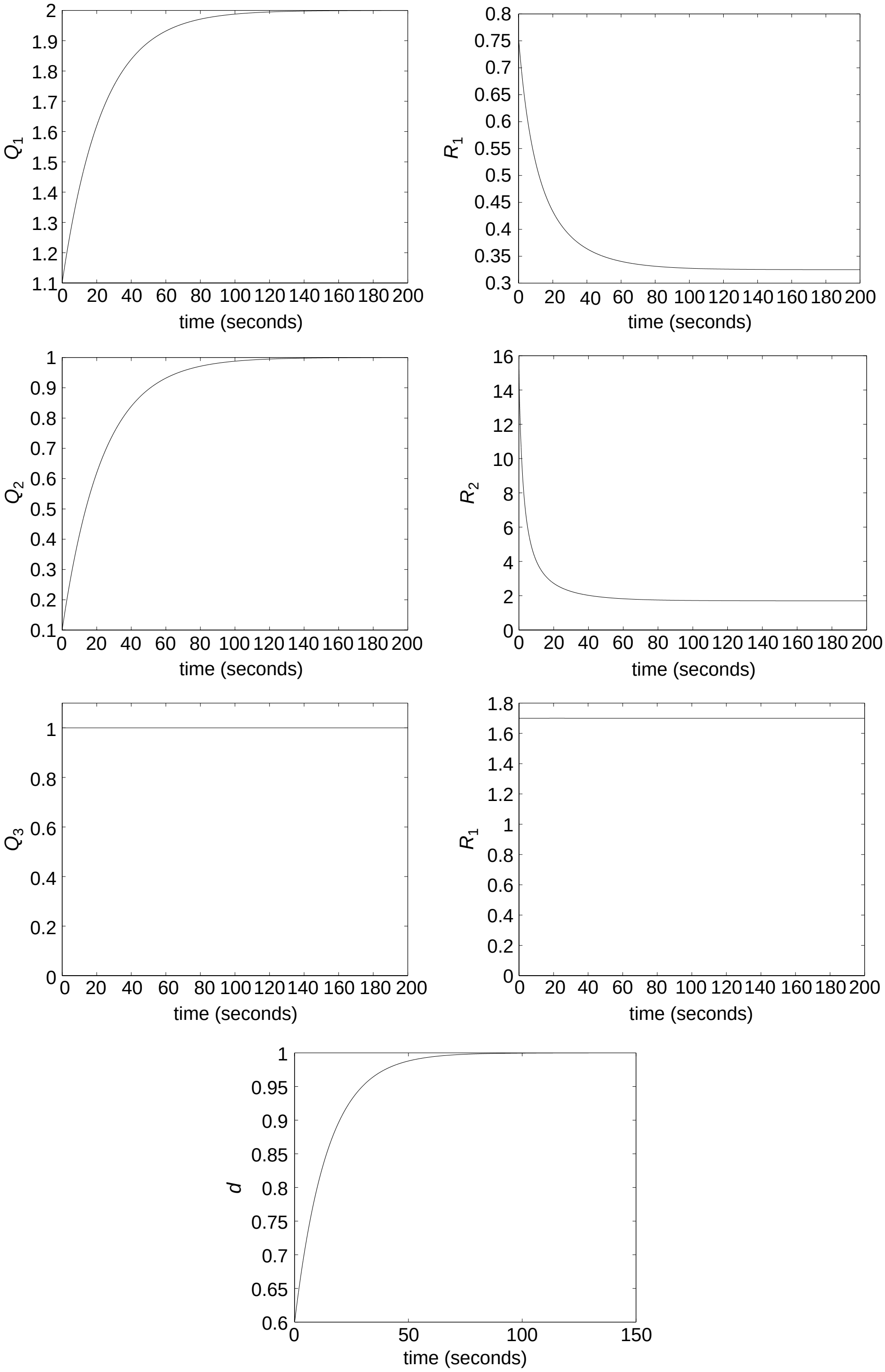

Fig. 3. Responses with auxiliary controllers.

allows R_1 and R_2 not to saturate. Note how d varies in wide range and how R_1 and R_2 have trends that are opposite to those in Fig. 2.

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This paper is dedicated to the memory of Professor Dragomir Kocić.

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