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Brief paper

An adaptive algorithm for control of combustion instability☆

Andrzej Banaszuk[a,*], Kartik B. Ariyur[b], Miroslav Krstić[b], Clas A. Jacobson[a]

[a]United Technologies Research Center, MS15, 411 Silver Lane, East Hartford, CT 06108, USA
[b]Department of MAE, UCSD, La Jolla, CA 92093-0411, USA

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Abstract

We propose an adaptive algorithm for control of combustion instability suitable for reduction of acoustic pressure oscillations in gas turbine engines, and main burners and augmentors of jet engines over a large range of operating conditions, and supply an experimental demonstration of oscillation attenuation, the first for a large industrial-scale gas turbine combustor. The algorithm consists of an Extended Kalman Filter based frequency tracking observer to determine the in-phase component, the quadrature component, and the magnitude of the acoustic mode of interest, and a phase shifting controller actuating fuel-flow, with the controller phase tuned using extremum-seeking. The paper also identifies a closed-loop model with phase-shifting control of combustion instability from experimental data; supplies stability analysis of the adaptive scheme based upon the identified model, and stable extremum-seeking designs used in experiments.

1. Introduction

Lower emissions requirements have motivated development of lean premixed combustors for industrial gas turbines. Their susceptibility to thermoacoustic pressure oscillations, and subsequent decreased durability have motivated a large body of research on combustion instability control. Prior experimental results and model-based analysis show that pressure measurement and a simple phase-shifting controller with an appropriately chosen phase-shift to actuate either fuel-injection or a loudspeaker is sufficient for suppression of oscillations, given enough control authority (Lang, Poinsot, & Candel, 1987; Hathout, Annaswamy, Fleifil, & Ghoniem, 1998; Cohen, Rey, Jacobson, & Anderson, 1998; Banaszuk, Jacobson, Khibnik, & Mehta, 1999a; Banaszuk, Jacobson, Khibnik, & Mehta, 1999b). The difficulty in determining the optimal phase shift that minimizes pressure oscillations, either by analysis or by experiment, especially in large industrial-scale combustors that operate over a wide range of conditions, has led researchers to call for the use of adaptive schemes (Seume et al., 1997).

This paper proposes an adaptive scheme to find the optimal phase shift online (from pressure measurement to fuel-injection), that is based on extremum seeking and motivated by physical modeling; identifies a closed loop model with phase-shifting control of combustion instability from experimental data; supplies stability analysis of the adaptive scheme based upon the identified model; develops stable extremum-seeking designs and provides the first successful result on oscillation minimization in an industrial-scale 4 MW gas turbine combustor.[1] The algorithm achieved the objective of monotonically reducing oscillation amplitudes below uncontrolled levels from all initial conditions. An Extended Kalman Filter (La Scala, 1994) based frequency tracking observer (Banaszuk, Zhang, & Jacobson, 2000b) was used

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* Corresponding author. Tel.: +1-860-610-7381; fax: +1-860-610-7909.
E-mail address: banasza@utrc.utc.com (A. Banaszuk).

[1] Conducted on a single nozzle rig at United Technologies Research Center (UTRC) in August 1998 before the experiments described in Johnson, Neumeier, Lubarsky, Lee, Neumaier, and Zinn (2000) and Murugappan, Gutmark, and Acharya, 2000.

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to reliably detect the in-phase and quadrature components of the dominant bulk mode of pressure oscillation over a wide range of operating conditions (bulk-mode frequency varying from 150 to 250 Hz). This prevented other frequencies and noise from entering the phase-shifting feedback. The control phase was updated using classical perturbation-based extremum seeking (Krstic, 2000) while the control gain was fixed.

The paper is organized as follows: Section 2 details derivation of an averaged model of pressure magnitude dynamics as a function of phase-shifting control; Section 3 presents experimental identification of the averaged model; Section 4 presents control-phase tuning by extremum seeking along with stability analysis, and Section 5 presents adaptive oscillation attenuation on the 4 MW single nozzle rig.

2. Averaged pressure magnitude model of a controlled combustion process

In this section we derive a simplified model of combustion instability for analysis of the control algorithm. A simplified version of the physics-based model of coupled acoustics and heat release from Jacobson, Khibnik, Banaszuk, Cohen, and Proscia (2000) is

$$\dot{v}_n(t) = -\frac{V_n^{ss}}{C_d^2 L_n} v_n(t) - \frac{1}{\rho_3 L_n} p_c(t), \tag{1}$$

$$\begin{aligned}\dot{p}_c(t) =& \frac{\rho_3 A_n}{C_c} v_n(t) - \frac{\kappa C_d A_e}{\sqrt{T_4} C_c} p_c(t) \\ &+ h(v_n(t-\tau), w_f(t-\tau)),\end{aligned} \tag{2}$$

where $v_n(t)$ is the perturbation velocity at nozzle exit and V_n^{ss} the steady-state velocity, $p_c(t)$ is the perturbation pressure in the combustor, and $w_f(t)$ fuel mass flow rate (the control variable). ρ_3 is density at the nozzle exit, L_n denotes effective nozzle length, C_d is the discharge coefficient, C_c is the capacitance of combustor volume, A_n is the physical area of nozzle cross section, A_e is the physical area of exit cross section, κ is the constant in choked flow equation, T_4 is the exit temperature in combustor. τ is the heat release delay due to transport of the mixture of fuel and air to the flame front, mixing of cold mixture with the hot products in the combustor recirculation zone, and chemical reaction. The heat release function $h(\cdot,\cdot)$ represents the mass flow addition due to heat release. We assume that $h(0,0)=0$. We also assume that the combustor is operating at a lean condition in which the heat release rate is an increasing function of fuel to air mass flow ratio, i.e., $h(v,w)$ is an increasing function of w if v is held constant and a decreasing function of v if w is held constant.

One can show that the linearization of the acoustic part of system (1)–(2) has a pair of complex conjugate eigenvalues close to imaginary axis. The acoustic damping is $\alpha_0 = \frac{V_n^{ss}}{2C_d^2 L_n} + \frac{\kappa C_d A_e}{2\sqrt{T_4} C_c}$ and the frequency is $\omega_0 = \frac{A_n}{L_n C_c} - \left(\frac{V_n^{ss}}{2C_d^2 L_n} - \frac{\kappa C_d A_e}{2\sqrt{T_4} C_c}\right)^2$. We are going to replace the velocity in the nozzle v_n with the quadrature component of pressure p_q. This change of coordinates puts the acoustic part of system (1)–(2) in the Jordan canonical form keeping p_c as one of the state variables. The change of coordinates is $p_q = s_1 v_n + s_2 p_c$ and the inverse transformation is $v_n = t_1 p_q + t_2 p_c$, where $t_1 = \frac{\omega_0 C_c}{\rho_3 A_n}$, $t_2 = \left(\frac{V_n^{ss} C_c}{2C_d^2 L_n \rho_3 A_n} - \frac{\kappa C_d A_e}{2\sqrt{T_4}\rho_3 A_n}\right)$, $s_1 = \frac{1}{t_1}$, and $s_2 = \frac{-t_2}{t_1}$.

In new coordinates (1)–(2) takes the form

$$\begin{aligned}\dot{p}_q(t) =& -\alpha_0 p_q(t) - \omega_0 p_c(t) \\ &+ s_2 h(v_n(t-\tau), w_f(t-\tau)),\end{aligned} \tag{3}$$

$$\begin{aligned}\dot{p}_c(t) =& \omega_0 p_q(t) - \alpha_0 p_c(t) \\ &+ h(v_n(t-\tau), w_f(t-\tau)).\end{aligned} \tag{4}$$

We are going to represent system (3)–(4) in polar coordinates $A = \sqrt{p_c^2 + p_q^2}$, $\theta = \arctan\left(\frac{p_q}{p_c}\right)$. (The inverse transformation is $p_c = A \sin\theta$, $p_q = A\cos\theta$.) In these coordinates system (3)–(4) takes the form

$$\begin{aligned}\dot{A}(t) =& -\alpha_0 A(t) + (\sin\theta(t) + s_2 \cos\theta(t)) h \\ &\times (t_1 A(t-\tau)\sin\theta(t-\tau) \\ &+ t_2 A(t-\tau)\cos\theta(t-\tau), w_f(t-\tau)),\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{\theta}(t) =& \omega_0 + (\cos\theta(t) - s_2 \sin\theta(t)) \\ &\times \frac{h(t_1 A(t-\tau)\sin\theta(t-\tau) + t_2 A(t-\tau)\cos\theta(t-\tau), w_f(t-\tau))}{A(t)}.\end{aligned} \tag{6}$$

We assume that the dynamics of (5)–(6) is a harmonic motion with nearly constant frequency

$$\begin{aligned}\omega =& \omega_0 + (\cos\theta(t) - s_2 \sin\theta(t)) \\ &\times \frac{h(t_1 A(t-\tau)\sin\theta(t-\tau) + t_2 A(t-\tau)\cos\theta(t-\tau), w_f(t-\tau))}{A(t)}\end{aligned}$$

and slowly varying magnitude $A(t)$. This justifies application of averaging to obtain an average pressure magnitude dynamics model (Murray et al., 1997). In particular, we assume that $\theta(t) = \omega t$ and thus $\theta(t-\tau) = \omega t - \omega\tau$. Moreover, since the magnitude dynamics time scale is assumed to be much slower than the heat release time delay τ, the approximation $A(t-\tau) = A(t)$ is justified. Using these assumptions and trigonometric identity $b_1 \sin\beta + b_2 \cos\beta = B \sin(\beta - \beta_1)$ for $B = \sqrt{b_1^2 + b_2^2}$ and $\beta_1 = -\arctan\frac{b_2}{b_1}$ we obtain $t_1 A(t-\tau)\sin\theta(t-\tau) + t_2 A(t-\tau)\cos\theta(t-\tau) \approx k_1 A(t)\sin(\theta - \theta_1)$, for $k_1 = \sqrt{t_1^2 + t_2^2}$ and $\theta_1 = -\arctan\frac{t_2}{t_1} - \omega\tau$. Using the same trigonometric identity we obtain $\sin\theta(t) + s_2\cos\theta(t) = k_2 \sin(\theta - \theta_2)$, for $k_2 = \sqrt{1 + s_2^2}$ and $\theta_2 = -\arctan s_2$.

We assume that the fuel valve is driven by control law $w_f(t) = k_c A \sin(\theta - \theta_c)$, where k_c is the control gain, and θ_c is the control phase shift (both relative to measured pressure signal $p_c(t)$). Note that this control law can be implemented as $w_f(t) = k_c(p_c(t)\cos\theta_c - p_q(t)\sin\theta_c)$. To obtain an estimate of the quadrature component $p_q(t)$ from the measured in-phase component $p_c(t)$ one can use a frequency tracking observer Banaszuk et al., 2000b. We neglect the fuel line dynamics and assume that $w_f(t) = k_c A \sin(\theta - \theta_c)$. (Alternatively, if the phase lag due to actuation is known, it could be added to θ_c in the expression for $w_f(t)$.) Thus, $w_f(t-\tau) = k_c A \sin(\theta - \theta_c - \omega\tau)$.

We obtain the average model of the pressure magnitude dynamics in the form

$$\dot{A}(t) = -\alpha_0 A(t) + H(A(t), k_c, \theta_c), \tag{7}$$

where $H(A) = \frac{k_2}{2\pi}\int_0^{2\pi} h(k_1 A \sin(\theta - \theta_1), k_c A \sin(\theta - \theta_c - \omega\tau)) \sin(\theta - \theta_2)\, d\theta$. The approximate limit cycle magnitudes A are found by solving the equation $-\alpha_0 A + H(A, k_c, \theta_c) = 0$. For a fixed control gain k_c, we assume that there is a unique stable equilibrium A of (7) of the form $A = g(\theta_c)$. Under this assumption Eq. (7) can be put in the form

$$\dot{A}(t) = -\alpha_0(A(t), \theta_c)(A(t) - g(\theta_c)) \tag{8}$$

for some $\alpha_0(A(t), \theta_c)$ nonnegative for all arguments. In Section 3 we will identify functions $g(\theta_c)$ and $\alpha_0(A(t), \theta_c)$ from an experiment. More precisely, we will obtain a local model of the growth rate coefficient $\alpha_0(A(t), \theta_c)$ under assumption that it only depends on θ_c. This model will be valid locally around $A = g(\theta_c)$. However, the model will not be valid globally, as it does not represent the fact that $A = 0$ is also an equilibrium of Eq. (8).

Experimentally obtained estimates of the pressure magnitude show a strong random component. In Jacobson et al. (2000), it was determined that the turbulent fluctuations of about 10% of the mean air flow lead to good agreement between the level of random pressure oscillations in the model and in the experimental data. Since the random component of the pressure magnitude will play a very important role in performance of the adaptive algorithm, we include it in the model. The physically motivated noise could be added to the velocity at nozzle exit in Eq. (2) at a white noise source. Rather than going through the difficulty of transformation of the input noise through all changes of variables described in this section, we will simply add a colored output noise component to the pressure magnitude obtained from Eq. (8). The output noise model will be identified from experimental data.

3. Identification of averaged pressure magnitude dynamics of a controlled combustion instability

Closed-loop identification experiments were conducted with the purpose of identifying the average model of pressure magnitude dynamics of the form represented by Eq. (8)

$$\dot{x_0} = -\alpha(\theta_c)(x_0 - g(\theta_c)), \tag{9}$$

$$A = x_0 + v, \tag{10}$$

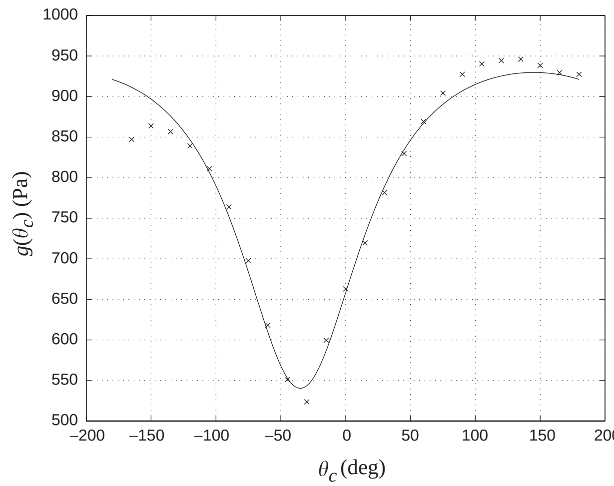

Fig. 1. Static map from control phase θ_c to oscillation amplitude A.

where $\alpha(\theta_c) > 0$ are the time constants of the exponential relaxation processes at the equilibria $A = g(\theta_c)$ of (9), A is the measured instantaneous pressure magnitude, and v is a colored noise modeling pressure magnitude fluctuation (due to turbulent velocity fluctuation in the nozzle). In experiments, the control phase input $\theta_c(t)$ is provided and response of the pressure magnitude estimate $A(t)$ from the frequency-tracking Extended Kalman Filter described in Banaszuk et al. (2000b) is recorded. The equilibrium map $g(\theta_c)$ is obtained by fitting a curve through the equilibria found from steady-state experiments, and decay rates $\alpha(\theta_c)$ at these equilibria are estimated from cumulative averaging of several step responses (the steps are in the controller parameter) to eliminate noise. A linear colored noise model (independent of θ_c) is obtained to fit the spectrum of the fluctuating component v in experimental data.[2]

3.1. Identification of equilibrium map

An experimental equilibrium map is obtained by varying θ_c in a phase ramp/staircase of discrete steps from 0° to 360°: $\theta_c(t) = \theta_{st}[t]$, where $[t]$ denotes the greatest integer less than time t, and $\theta_{st} = 15°$ is the discrete increment in θ_c in each step. The duration of steps is sufficiently long (1 s) to allow for the transients in pressure magnitude to settle down. The steady-state values are estimated by averaging the magnitude data after the transient is over. From experimental data at a low combustor power shown in Fig. 1, there does seem to be a smooth variation of the steady-state magnitudes of thermoacoustic instability with θ_c along a single curve, and there is a definite minimum oscillation magnitude at a certain phase. A parametrization for the static map, motivated in part by analysis in Section 2 and in part by the shape of the

[2] Youping Zhang fit the output noise model using combustion data and implemented the identified model in SIMULINK.

experimental static map itself, is

$$g(\theta_c) = \Gamma \left\{ \frac{1 + L\sin(\theta_c + \phi)}{1 + M\sin(\theta_c + \phi)} \right\}, \tag{11}$$

where θ_c is the phase of the phase-shifting controller, and $g(\theta_c)$ is the steady-state oscillation magnitude. Note that the map is parametrized with only four parameters. The parameters are obtained by fitting the parametrization to the experimental data by a nonlinear least-squares fit. For the experiment at a low combustor power, the parameters obtained were $\Gamma = 0.1246$, $L = 0.7659$, $M = 0.6286$, $\phi = -0.9614$.

3.2. Identification of decay rates

Experimentally, the magnitude transients are obtained by introducing a large square wave variation in the control phase. The duration of the pulses is long enough (2 s) to allow settling of the transients of each step, and the upper and lower values of θ_c in the pulses are chosen so as to ensure an observable difference in equilibrium magnitude $g(\theta_c)$, typically 90°.

To eliminate noise in the transient measurements, a cumulative averaging of the various step responses in a given square wave response is performed. We assume that the noise is zero mean, its autocorrelation function decays to zero within half-pulse period T, and that the magnitude transients settle within half-pulse period T. Time traces of transient responses are averaged cumulatively to obtain the Nth estimate of the magnitude time-trace:

$$\bar{A}(t) = \frac{1}{[t/T] + 1} \sum_{i=0}^{[t/T]} A(t - iT), \tag{12}$$

where $A(t)$ is the measurement of magnitude as in the previous section, $T = 2$ s is the time period of the pulses, and $[t/T]$ denotes the greatest integer less than t/T. The transients due to the upward and downward steps are averaged separately, since they represent transients at different equilibria. The corresponding final averages are shown in Fig. 2. In the figures, the smooth curves of the exponential fits are superimposed over the rough curves from cumulative averages.

In the case where we can measure the magnitude perfectly without noise, direct integration of Eq. (9) yields $\alpha(\theta_{\text{fin}})$ as

$$\alpha(\theta_{\text{fin}}) = \frac{A(t) - A(t+s)}{\int_t^{t+s} A(\sigma)\,\mathrm{d}\sigma - s g(\theta_{\text{fin}})} \tag{13}$$

$\forall t$ s.t. $\left[\frac{2t}{T}\right] = \left[\frac{2(t+s)}{T}\right]$, and $\forall s < T/2$, where θ_{fin} is the final phase of the phase step. However, since we do not have noise-free data, we estimate the exponent from the Nth cumulative average of the measured transients (Eq. (12)) as follows:

$$\hat{\alpha}_N(\theta_{\text{fin}}) = \frac{\bar{A}(NT) - \bar{A}(NT + T_{\text{settle}})}{\int_{NT}^{NT+T_{\text{settle}}} \bar{A}(\sigma)\,\mathrm{d}\sigma - T_{\text{settle}}\, g(\theta_{\text{fin}})}. \tag{14}$$

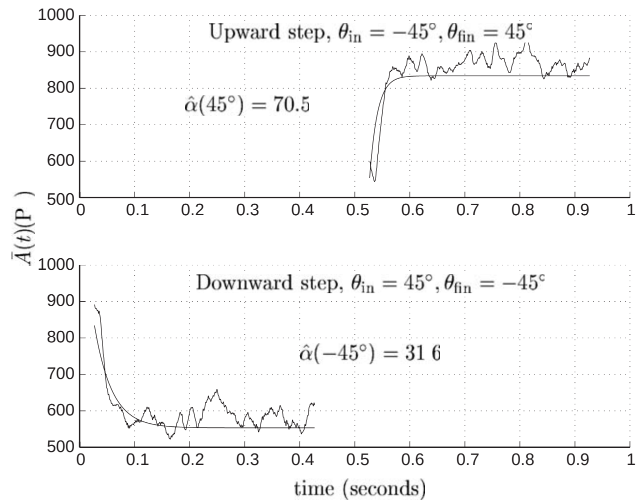

Fig. 2. Estimate of the decay rate.

Here, we approximate $\bar{A}(NT) \approx g(\theta_{\text{in}})$, and $\bar{A}(NT + T_{\text{settle}}) \approx g(\theta_{\text{fin}})$, where θ_{in} is the starting phase of the phase step, NT is a step time instant, and T_{settle} is such that the magnitude transients settle within it. The exponents $\hat{\alpha}(\theta_c)$ (θ_c in degrees) thus calculated are indicated on Fig. 2. The identified pressure magnitude dynamics and the colored noise model have been implemented in SIMULINK for simulation studies of the adaptive algorithm to narrow the range of adjustable parameters and thereby minimize experimental time and expense.

4. Controller phase tuning using extremum-seeking algorithms

In Section 2, we derived through analysis, the dynamical dependence of pressure oscillation amplitude A upon control phase θ_c; in Section 3, we identified this model from closed-loop experiments. The optimal phase shift, being a function of the operating conditions, and being dependent upon several unknown parameters that are difficult to estimate (like heat release time delay τ) is tuned online by extremum seeking for the following reasons:

1. The stable combustion process and actuator dynamics (around 200 Hz) are much faster than the pressure magnitude dynamics ($\approx$ 10 Hz) as verified by experiment in Section 3, and therefore permit the problem to be reduced to simply the reduction of the pressure amplitude (the dynamics of the frequency tracking observer are also as fast as the combustion process dynamics it observes).
2. Direct availability of the phase-shift θ_c for tuning, and the magnitude $A(t) = \sqrt{p_c(t)^2 + p_q(t)^2}$ for measurement as an objective to minimize, through the frequency tracking observer.

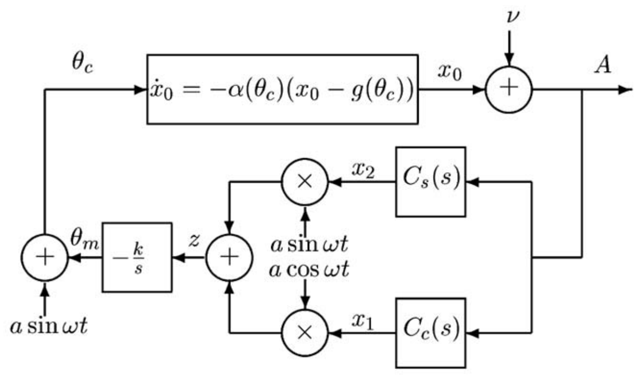

Fig. 3. Extremum-seeking scheme.

3. The equilibrium map $g(\theta_c)$ of pressure amplitude versus control phase is smooth, unique, and has a unique minimum.
4. Perturbation-based extremum seeking provides guarantees of stability and convergence of θ_c to its optimum; this can be proved as in Krstic (2000) and Krstic and Wang (2000) using the separation of time-scales between the slow update of θ_c and the faster magnitude dynamics.

A modified version of the classical extremum-seeking scheme (Krstic, 2000) implemented for phase-shift tuning is shown in Fig. 3. The extremum-seeking algorithm used in this paper relies on a small sinusoidal variation of θ_c with frequency ω and amplitude a to obtain a measure of the gradient of the map $g(\theta_c)$.[3] Instead of a simple washout filter, it uses a magnitude observer to extract the in-phase and quadrature components of the magnitude estimate of A at the frequency ω. The magnitude observer decomposes the magnitude estimate signal into constant, in-phase, and quadrature component (the last two relative to the phase perturbation signal $\sin(\omega t)$). The transfer functions from the magnitude estimate to the in-phase and quadrature components are given below:

$$C_s(s) = \frac{l_2 s + \omega l_1}{s^3 + (l_2 + l_3)s^2 + (\omega^2 + l_1\omega)s + l_3\omega^2}, \tag{15}$$

$$C_c(s) = \frac{l_1 s - \omega l_2}{s^3 + (l_2 + l_3)s^2 + (\omega^2 + l_1\omega)s + l_3\omega^2}, \tag{16}$$

where $l_1 = -0.016$, $l_2 = 1.996$ and $l_3 = 1.996$ were chosen for stable observation. The sine and cosine components of A are demodulated by $a \sin \omega t$ and $a \cos \omega t$, respectively, and then summed up and passed through the integrator which has a gain k.

[3] A different approach in Zhang (2000) uses the triangular search algorithm, which uses three past-sampled average magnitude values to determine the new control phase.

4.1. Averaged linearized models

To aid selection of extremum-seeking parameters ω, a and k to ensure stable extremum-seeking, we derive averaged linearized models of system in Fig. 3 as in Krstic (2000). One of them relates tracking error in controller parameter $\tilde{\theta} = \theta_c^* - \theta_c + a \sin \omega t$ with change in minimizer θ_c^* which minimizes $g(\theta_c)$:

$$\frac{\tilde{\theta}(s)}{\theta_c^*(s)} = \frac{1}{1 + L(s)}. \tag{17}$$

The other relates output A to output noise n:

$$\frac{A(s)}{n(s)} = \frac{1}{1 + M(s)}, \tag{18}$$

where

$$\begin{aligned} L(s) = \frac{ka^2}{2} g''(\theta_c^*)[&F_o(s + j\omega)(s + j\omega) \\ &\times (C_s(s + j\omega) - jC_c(s + j\omega)) \\ &+ F_o(s - j\omega)(s - j\omega)(C_s(s - j\omega) \\ &+ jC_c(s - j\omega))] \frac{1}{s}, \end{aligned} \tag{19}$$

$$M(s) = ka^2 g''(\theta_c^*) s F_o(s) \left(\frac{sC_s(s) + \omega C_c(s)}{s^2 + \omega^2} \right), \tag{20}$$

where $F_o(s) = \frac{\alpha}{s+\alpha}$ approximates the magnitude dynamics. An assumption made in the derivation of the transfer functions is $\alpha(\theta_c) = \alpha$ is a constant. This is partially justified by the fact that the local behavior is dominated by frequencies (<1 rad/s) of an order of magnitude less than $\alpha(\theta_c) \in [15, 45]$ rad/s in the identified models. Asymptotic stability/instability of $\frac{1}{1+L(s)}$ in Eq. (17) gives local asymptotic stability/instability in tracking near the minimum $g(\theta_c^*)$. Similarly, $\frac{1}{1+M(s)}$ in Eq. (18) reveals noise sensitivity and steady-state output performance. The analysis presented can be used for parameter selection in extremum seeking with the aid of tools for linear time invariant systems such as root-locus, Bode, and Nyquist. It can especially be used a priori to rule out destabilizing designs, or designs that are sensitive to noise frequencies in the operating environment.

5. Experiments with adaptive algorithm in combustion rig

A cost-effective alternative to both engine and full annular combustor testing is to test a sector cut from the full combustor annulus containing one or several fuel nozzles. In this section, we present results of experiments in United Technologies Research Center (UTRC) conducted on 4 MW Single Nozzle Rig in August 1998 using full-scale engine fuel nozzle at realistic operating conditions. Rig schematics are presented in Fig. 4. About 10% of the net fuel was modulated for control purposes using a linear proportional

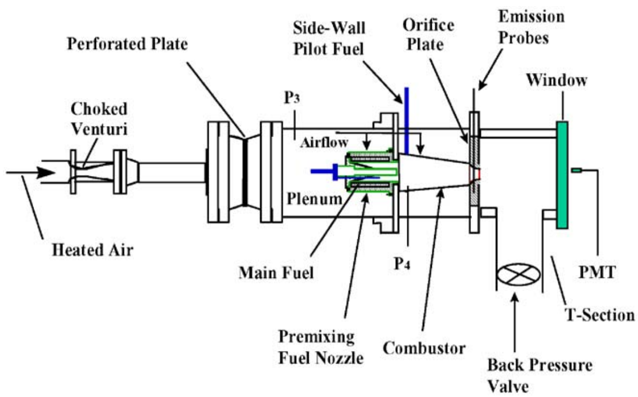

Fig. 4. UTRC single-nozzle combustion rig.

valve (For more details on the UTRC experimental rigs see Cohen et al. (1998)). The control gain is fixed and only the control phase is updated using the algorithm described in Section 4.

Performance specifications for the adaptive algorithm have been defined for algorithm initialization transients and engine acceleration transients: when initialized with a phase corresponding to amplification of oscillations, the algorithms should quickly produce and maintain phases corresponding to suppression of the oscillations; during engine acceleration transients the algorithms should be able to suppress oscillations relative to uncontrolled levels.

The dependence of the mean pressure magnitude and frequency of the corresponding mode on the control phase has been determined experimentally at several power conditions, so that the optimal control phase θ_c^* was known a priori. This information let us check the performance of the extremum-seeking algorithm. To test the transient performance of the adaptive algorithm, initialization transients are introduced, where the initial control phase $\theta_c(0)$ differs significantly from θ_c^*. We show two time traces of control phase and pressure magnitude (instantaneous and low-pass filtered) as functions of time during initialization transients in Fig. 5. The horizontal lines in the top figures represent the optimal control phase θ_c^*. The horizontal lines in the middle figures show the mean values of pressure magnitude for the uncontrolled case and for the closed-loop optimal control phase. The bottom figures show dependence of low-pass filtered pressure magnitude on the control phase overlaid over a sketch of the map representing the mean

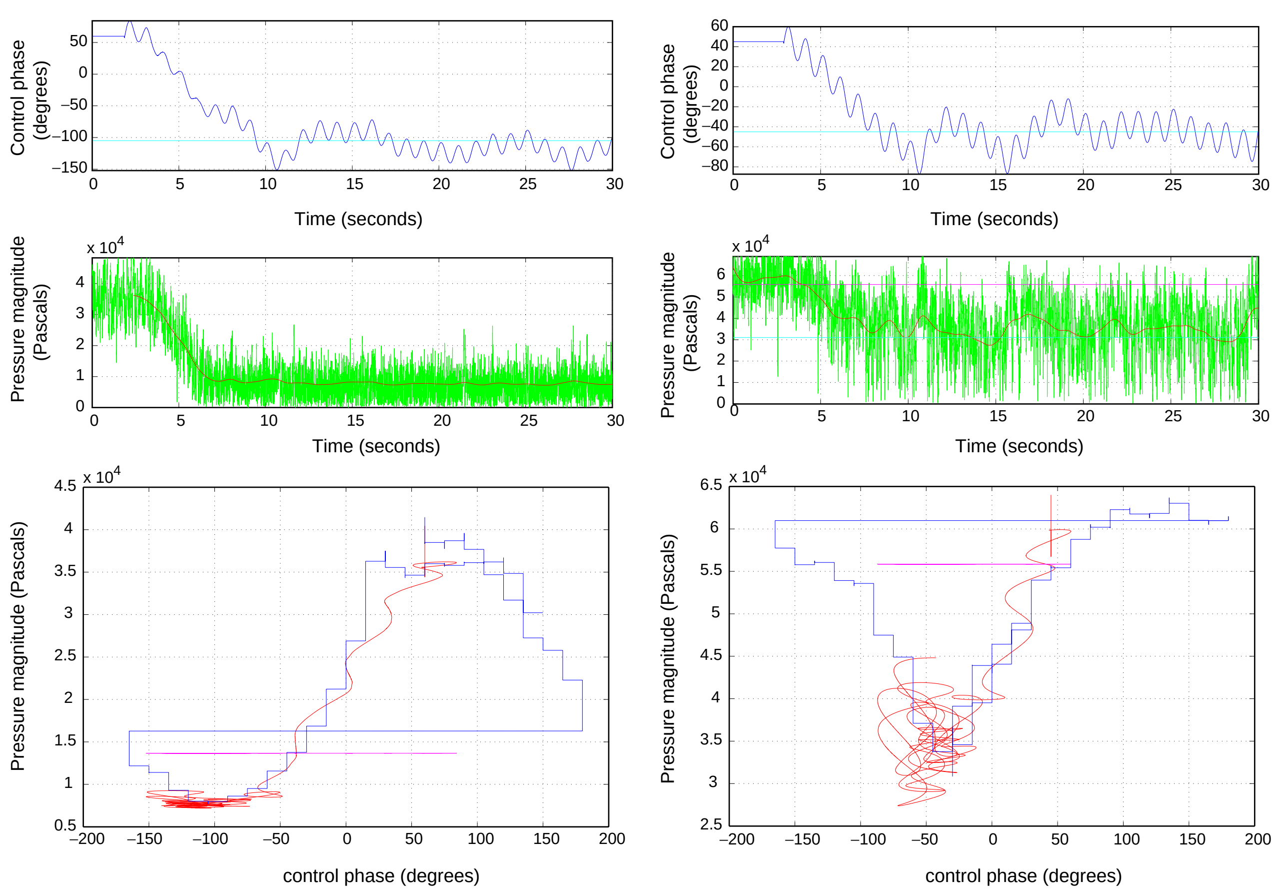

Fig. 5. Initialization transients: time traces of pressure amplitude (instantaneous and average) and control phase (top) and pressure magnitude as function of control phase (bottom). Parameters: higher power, left-$f = 1$ Hz, $a = 15°$, $k = 1000$, $\theta_m(0) = 60°$; lower power, right-$f = 1$ Hz, $a = 15°$, $k = 150$, $\theta_m(0) = 115°$.

pressure magnitude from control phase ramp experiments. The horizontal lines represent the mean value of pressure magnitude for the uncontrolled case. For the frequency $f = \omega/2\pi$ of sinusoidal variation introduced in the control phase below 10 Hz (corresponding to a separation of time-scales), integrator gain k ranging from 150 to 1000, and amplitude of forcing $a = 10°, 15°$, the algorithm behaved very well at high-power condition (medium noise and small pressure oscillations) and reasonably well at low-power conditions (large noise and pressure oscillations). On reaching a neighborhood of the optimal value, the control phase usually stayed in a reasonably small neighborhood of that value, rarely produced control phases corresponding to level higher than uncontrolled levels, and always provided better average pressure oscillations levels than uncontrolled levels. For further details on the experimental attenuation of pressure oscillations using the extremum-seeking algorithm described in this paper see Banaszuk, Zhang, & Jacobson (2000a).

It has been inferred that the major factor affecting the performance of the extremum-seeking schemes is the "noise" present in the pressure magnitude. This noise component (denoted by v) was introduced in the model in Section 3. As we mentioned in Section 2, the noise can be attributed to an effect of turbulent flow in the nozzle. The changes in operating conditions appearing during engine acceleration and deceleration are likely to resemble the transients between different power levels on single nozzle rig. In experiments, the frequencies of the pressure modes, the mean pressure magnitude levels, and noise levels varied significantly between power levels.

It was determined that in order to work in a simulated transient from low- to high-power conditions, the classical algorithm would have to be modified to allow for adaptive gain change (by a factor of five). One fixed gain k would not work at both low- and high-power conditions.

Ability of an extremum-seeking algorithm to track the fast-changing θ_c^* during the engine acceleration and deceleration transient conditions in the presence of disturbance driving the system can be studied by simulation as in Banaszuk et al. (2000a). However, there is a need to study stability, robustness, and performance of the algorithms *during fast engine transients* using analytical tools. Traditional methods based on simplistic time-scale separation are not applicable as the *time scale of change of operating conditions during engine transients* is not well separated from the time scale of the transients in the pressure magnitude dynamics.

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Andrzej Banaszuk is a project leader in the area of modeling and control of unsteady combustion processes in jet engines at United Technologies Research Center. Since joining UTRC in 1997, Andrzej Banaszuk has conducted research in control-oriented modeling, nonlinear model validation, parameter identification, and linear, nonlinear and adaptive control design for unsteady flow and combustion phenomena affecting operations of aeroengines. Specifically, he worked on modeling, identification, and control of turbomachinery flutter, rotating stall, combustion instability, flow separation, and mixing. Andrzej Banaszuk is an author of 32 journal papers and 48 conference papers. He holds 2 patents. Andrzej Banaszuk holds Ph.D. in Electrical Engineering from Warsaw University of Technology and Ph.D. in Mathematics from Georgia Institute of Technology. Before coming to United Technologies Research Center he has held academic positions at Warsaw University of Technology, Georgia Institute of Technology, University of Colorado at Boulder, and University of California at Davis performing research in linear and nonlinear control theory. From 1999 to 2002 he was an Associate Editor of IEEE Transactions of Controls Systems Technology. He was appointed to serve on the Board of Governors of IEEE Control Systems Society in 2004.

Kartik Ariyur received his Ph.D. and M.S. degrees from the University of California, San Diego in 2002 and 1999 respectively, and his BTech degree in 1996 from the Indian Institute of Technology, Madras, all in Mechanical Engineering. He has been with Honeywell Laboratories in Minneapolis from 2002 as a Research Scientist. He is currently working on problems in vehicle health management, stabilization of nonlinear hybrid systems, rapid trajectory generation and obstacle avoidance for unmanned aerial vehicles, and in wireless signal processing. Previously, he has worked on problems ranging from attenuation of blade-vortex interaction in helicopters, to formation flight to combustion and compressor control, and control of pulsed detonation engines. He is co-author, with Miroslav Krstic, of the book, "Real-Time Optimization by Extremum Seeking Control," published by John Wiley & Sons in 2003. While a student, he performed research on gas turbine problems at United Technologies Research Center in East Hartford, CT, and on adaptive filtering problems at Qualcomm Inc., in San Diego, CA. Dr. Ariyur is a member of the IEEE.

Miroslav Krstic is Professor and Vice Chair in the Department of Mechanical and Aerospace Egineering at University of California, San Diego. Prior to moving to UCSD, he was Assistant Professor in the Department of Mechanical Engineering and the Institute of Systems Research at University of Maryland (1995-97). He got his Ph.D. in Electrical Engineering from University of California at Santa Barbara, under Petar Kokotovic as his advisor, and received the UCSB Best Dissertation Award. Krstic is an IEEE Fellow and has received the National Science Foundation Career Award, the Office of Naval Research Young Investigator Award, the Presidential Early Career Award for Scientists and Engineers (PECASE), the George S. Axelby Outstanding Paper Award of IEEE Transactions on Automatic Control, and the O. Hugo Schuck Award for the best paper at American Control Conference. Krstic is a co-author of the books Nonlinear and Adaptive Control Design (Wiley, 1995), Stabilization of Nonlinear Uncertain Systems (Springer-Verlag, 1998), Flow Control by Feedback (Springer-Verlag, 2002), and Real Time Optimization by Extremum Seeking Control (Wiley, 2003). He is a co-author of two patents on control of aeroengine compressors and combustors. He has served as Associate Editor for the IEEE Transactions on Automatic Control, International Journal of Adaptive Control and Signal Processing, Systems and Control Letters, and Journal for Dynamics of Continuous, Discrete, and Impulsive Systems. Krstic is a Vice President for Technical Activities and a member of the Board of Governors of the IEEE Control Systems Society. His research interests include nonlinear, adaptive, robust, and stochastic control theory for finite dimensional and distributed parameter systems, and applications to fluid flows and fusion.

Dr. Clas A. Jacobson is currently Director of the Systems Department at the United Technologies Research Center (UTRC) in East Hartford, Connecticut. He is responsible for building capability involving systems engineering at UTRC. The mission of UTRC is to provide innovation in product development in partnership with the UTC business units. The operating units of UTC include Pratt & Whitney, Sikorsky Aircraft, Otis Elevators, Carrier, Hamilton-Sundstrand and UT Power. Dr. Jacobson received the B.S. degree in computer and systems engineering in 1982, and the M.Eng. and Ph.D. degrees in electrical engineering, in 1984 and 1986, respectively, all from Rensselaer Polytechnic Institute, Troy, New York. Prior to coming to UTRC he worked at Northeastern University in Boston from 1986-1995 as an Associate Professor. He taught and did research in the area of control systems. He has published over fifty papers and holds two patents.