Brief paper

Output feedback control of two-lane traffic congestion

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Abstract

This paper develops output feedback boundary control to mitigate traffic congestion of an unidirectional two-lane freeway segment. The macroscopic traffic dynamics are described by the Aw–Rascle–Zhang (ARZ) model respectively for both the fast and slow lanes. The traffic density and velocity of each of the two lanes are governed by coupled $2 \times 2$ nonlinear hyperbolic partial differential equations (PDEs). Lane-changing interactions between the two lanes lead to exchanging source terms between the two pairs second-order PDEs. Therefore, we are dealing with $4 \times 4$ nonlinear coupled hyperbolic PDEs. Based on driver’s preference for the slow and fast lanes, a reference system of lane-specific uniform steady states in congested traffic is chosen. To stabilize traffic densities and velocities of both lanes to the steady states, two distinct variable speed limits (VSLs) are applied at outlet boundary, controlling the traffic velocity of each lane. Using backstepping transformation, we map the coupled heterodirectional hyperbolic PDE system into a cascade target system, in which the traffic oscillations are damped out through actuation of the velocities at the downstream boundary. Two full-state feedback boundary control laws are developed. We also design a collocated boundary observer for state estimation with sensing of the densities at the outlet. Output feedback boundary controllers are obtained by combining the collocated observer and full-state feedback controllers. The finite time convergence to equilibrium is achieved and a performance improvement in fuel consumption, drivers’ comfort and total travel time is demonstrated with the proposed output feedback controllers for some parameter choice, compared with the open-loop system. The proposed output feedback design is also validated for different congested traffic scenarios and compared with one-lane backstepping and PI control designs.

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1. Introduction

Traffic congestion on freeways has been investigated intensively over the past decades. Motivations behind are to understand the formation of traffic congestion, and to further prevent or suppress instabilities of traffic flow. Macroscopic modeling of traffic dynamics is to describe evolution of aggregated traffic state values including the traffic density and velocity. Traffic dynamics are governed by hyperbolic PDEs, including the first-order model by Lighthill, Whitham and Richards (LWR) (Lighthill & Whitham, 1955; Richards, 1956), and the second-order Aw–Rascle–Zhang (ARZ) model (Aw & Rascle, 2000; Zhang, 2002). The LWR model is a conservation law of traffic density. It is simple yet powerful in understanding the formation and propagation of traffic shockwaves on freeway. But it fails to describe stop-and-go traffic (Flynn, Kasimov, Nave, Rosales, & Seibold, 2009). The oscillations of densities and velocities travel with traffic stream, causing unsafe driving conditions, increased consumption of fuel and delay of travel time.

In order to suppress the traffic congestion, freeway traffic regulations of traffic velocity and flow are implemented with traffic management infrastructures like variable speed limits (VSLs) and on-ramp metering. VSLs employ variable message signs to display driving velocity in response to the real-time traffic conditions. VSLs were employed to homogenize the velocities of individual vehicles (Abdel-Aty, Dilmore, & Dhindsa, 2006; Lee, Hellinga, & Saccomanno, 2006; Papageorgiou, Kosmatopoulos, & Papamichail, 2008) and were also used in Carlson, Papamichail, Papageorgiou, and Messmer (2010) to slow down the traffic and create controlled mainstream flow. VSL strategies are studied such that traffic congestion is alleviated, traffic flow efficiency is improved and traffic safety is enhanced.

Freeway traffic control problem by VSLs was addressed by model-based optimal control approaches in the previous works (Carlson et al., 2010; Hegyi, De Schutter, & Hellendoorn, 2005;
The employed models are spatially and temporally discretized macroscopic models including the cell-transmission model (CTM) discretized from the LWR model and the METANET model (Papageorgiou, Blosseville, & Haj-Salem, 1990) from the second-order macroscopic model. For the cell-transmission model, Zhang and Ioannou (2016) developed feedback linearization VSL controllers to homogenize the upstream traffic and to maximize the flow at bottleneck. A discrete-time link model was used in Carlson et al. (2010) which incorporated VSL control into the METANET and considers the optimal control problem by VSLs to minimize total time spent. An extended METANET model was considered in Hegi et al. (2005) and authors proposed a model predictive control approach to optimally coordinate VSLS to suppress shockwaves on freeway. The VSLS affect the static velocity–density relation by limiting the free speed. For each link, the states are spatially homogeneous and evolve in discrete time. For multi-lane traffic control, Festa and Göttlich (2018) considered mean field game approach to an optimal control problem of multi-lane traffic, based on the LWR model with exchanging flow rates between lanes. In Roncoli, Papageorgiou, and Papamichail (2015), authors developed a novel multi-lane model based on the CTM and formulated an optimal control problem by integrated control measures of ramp metering, VSLS, lane-changing control through vehicle-to-infrastructure communication. The previous works are based on the discretized model and mainly aimed at improving the traffic flow efficiency. However, control of the stop-and-go traffic oscillations remains as a challenging research problem since the control algorithms need to be directly designed for the second-order macroscopic ARZ model that are continuous in time and space. In addition, such type of design avoids the discretization errors and a detailed depiction of the traffic condition is provided by the model.

Many recent efforts (Bastin & Coron, 2016; Bekiaris-Liberis & Krstic, 2018; Karafyllis, Bekiaris-Liberis, & Papageorgiou, 2018; Yu & Krstic, 2018, 2019; Zhang, Prieur, & Qiao, 2019) were focused on developing control approaches directly for the macroscopic traffic PDE models instead of their approximated models. In Bastin and Coron (2016), is discussed the possibility of applying boundary feedback stabilization for the first-order and second-order traffic PDE system. The first-order traffic model with lane-drop was investigated in Bekiaris-Liberis and Krstic (2018). The authors developed predictor feedback control law to regulate the bottleneck density to a desired value. In Zhang et al. (2019), PI boundary controllers were developed for VSL and ramp metering to stabilize the linearized second-order homogeneous ARZ model. For the second-order traffic model, Karafyllis et al. (2018) provided a nonlinear boundary feedback law that achieved global stabilization by controlling the inlet flow. The PDE backstepping control design was firstly applied to stabilize the stop-and-go traffic of the one-lane ARZ model with ramp metering (Yu & Krstic, 2019) or with VSL (Yu & Krstic, 2018). Compared with the PI controllers applied to the ARZ problem in Zhang et al. (2019), backstepping controller in Yu and Krstic (2018, 2019) requires only one controller whereas PI controllers employ both VSL and ramp metering. In addition, sensing and actuation are at the same location which facilitate the implementation in practice and avoid long-distance communication.

The aforementioned results treated multi-lane freeway traffic cumulatively as a single lane by assuming averaged velocity and density over cross section of all lanes. The individual dynamics of each lane and inter-lane interactions were neglected. In fact, the different velocity equilibria in the multi-lane problem give rise to lane-changing interactions and further lead to traffic congestion (Laval & Daganzo, 2006). To address the phenomenon, a number of macroscopic multi-lane models (Helbing & Greiner, 1997; Herty & Klar, 2003; Klar, Greenberg, & Rascle, 2003; Klar & Wegener, 1998; Michalopoulos, Beskos, & Yamauchi, 1984) have been developed from microscopic, then kinetic to macroscopic descriptions. In this paper, we adopt the multi-lane ARZ traffic model by Herty and Klar (2003) and Klar and Wegener (1998) to describe a two-lane freeway traffic with lane-changing between the two lanes. Lane interactions appear as interfering source terms in the system, leading to more involved couplings and a higher order of PDEs. The complexity of the PDE model for multi-lane problem is greatly increased compared to the one-lane problem.

Feedback boundary control design for a general class of hyperbolic PDEs using backstepping method has been studied in Anfinsen and Aamo (2017), Auriol and Di Meglio (2016, 2020), Coron, Vazquez, Krstic, and Bastin (2013), Di Meglio, Vazquez, and Krstic (2013), Hu, Di Meglio, Vazquez, and Krstic (2016) and Yu and Krstic (2019). In Di Meglio et al. (2013), stabilization of a $n + 1$ counter connecting hyperbolic PDES was achieved with a single boundary. The result was further extended in Hu et al. (2016) by proposing output feedback for a fully general case of hetero directional $n + m$ first-order linear coupled hyperbolic PDEs. Actuation of all the $m$ PDEs from the same boundary is required to stabilize the system in finite time. A shorter convergence time was obtained in Auriol and Di Meglio (2016) by modifying the target system structure. To guarantee the robustness of the design, Anfinsen and Aamo (2017) solved the disturbance rejection problem for such class system when there is boundary disturbance. For two coupled hyperbolic PDE system, Auriol and Di Meglio (2020) obtained robust output feedback design in presence of a general class of disturbances and noise. In this paper, a two-lane ARZ model presents as the heterodirectional $4 \times 4$ coupled nonlinear hyperbolic PDEs, governing the traffic densities and velocities of the fast and slow lanes. We aim to stabilize the two-lane traffic using the PDE backstepping method, inspired by the stabilization results in Hu et al. (2016). The actuations of traffic velocities at the outlet boundary are realized by two VSLS.

The contribution of this paper lies in the following aspects. The two-lane stop-and-go traffic PDE model is an open control problem. This paper applies the PDE backstepping control design methodology to the two-lane ARZ model which is state-of-art in macroscopic modeling of congested traffic. Furthermore, we complete the theoretical results in Auriol and Di Meglio (2016) and Hu et al. (2016) by proposing a collocated boundary observer along with the controller design whereas only the anti-collocated ones are considered previously. The control algorithms for VSLS utilize the boundary sensing information. If sensors and VSLS are installed at the same location, then possible delays and errors of long-distance communication are avoided. Therefore, it is important to design collocated boundary observer and controllers for practical consideration. We demonstrate that not only the finite-time stabilization of the closed-loop system is achieved but also some common performance indices are improved for some prior parameters. The proposed controllers are also tested for different congested traffic scenarios. Comparison with one-lane backstepping and PI controllers shows that the proposed two-lane controllers outperform them and are thus valuable and possibly even necessary to stabilize the two-lane congested traffic with lane-changing.

The paper is organized as follows: in Section 2 we introduce the two-lane ARZ traffic model and then linearize it around uniform steady states. In Section 3 backstepping transformation is derived for the linearized model in Riemann coordinates. We present full-state feedback control for outlet boundary actuation of velocities. In Section 4, we design collocated boundary observers and obtain output feedback control. In Section 5, control design is validated with numerical simulation and evaluated with some common performance indices.
2. Problem statement

The two-lane traffic on unidirectional roads is described with the following two-lane traffic ARZ model by Herty and Klar (2003) and Klare and Wegener (1998). There are VSLs at the outlet regulating the exiting velocities of the fast lane and slow lane traffic. The two-lane traffic ARZ model is given by

\[
\frac{\partial \rho_1}{\partial t} + \frac{\partial (\rho_1 v_1)}{\partial x} = \frac{1}{T_f} \rho_1 - \frac{1}{T_f} \rho_i,
\]

\[
\frac{\partial (\rho_i v_i)}{\partial t} + \frac{\partial (\rho_i v_i^2)}{\partial x} - (\gamma \rho_i) \frac{\partial v_i}{\partial x} = \frac{1}{T_s} \rho_i v_i - \frac{1}{T_s} \rho_1 v_1 + \rho_i (V(\rho_i) - v_i),
\]

\[
\frac{\partial \rho_s}{\partial t} + \frac{\partial (\rho_s v_s)}{\partial x} = \frac{1}{T_f} \rho_s - \frac{1}{T_f} \rho_i,
\]

\[
\frac{\partial (\rho_i v_i)}{\partial t} + \frac{\partial (\rho_i v_i^2)}{\partial x} - (\gamma \rho_i) \frac{\partial v_i}{\partial x} = \frac{1}{T_s} \rho_i v_i - \frac{1}{T_s} \rho_s v_s + \rho_i (V(\rho_i) - v_i).
\]

The traffic density \(\rho_i(x, t)\) and velocity \(v_i(x, t)\) \((i = f, s)\) are defined in \(x \in [0, L]\) for position, \(t \in [0, \infty)\) for time, where \(L\) is the length of the freeway segment. The above nonlinear hyperbolic PDEs consist of two subsystems of second-order nonlinear hyperbolic PDEs, each describing one-lane traffic dynamics. Lane-changing interactions and drivers’ behavior adapting to the traffic appear as source terms on the right hand side of PDEs. The variable \(p_i(\rho_i)\) is defined as the traffic density pressure \(p_i(\rho_i) = v_m \left( \frac{\rho_i}{\rho_m} \right)\), which is an increasing function of density \(\rho_i, v_m\) is the maximum traffic velocity, \(\rho_m\) is the maximum traffic density and the constant coefficient \(\gamma \in \mathbb{R}_+\) reflects the aggressiveness of drivers on the road. The parameter \(T_i\) is defined as relaxation time that reflects driver’s behavior adapting to the traffic equilibrium velocity in the lane \(i\). The parameter \(T_i \in \mathbb{R}_+\) describes the driver’s preference for remaining in lane \(i\), which relates to the both lanes’ density and velocity. The equilibrium velocity–density relationship \(V(\rho_i)\) is given in the form of the Greenshield’s model, \(V(\rho_i) = v_m \left( 1 - \left( \frac{\rho_i}{\rho_m} \right) \right)\). The model is chosen due to its simplicity but the control design presented later is not limited by this choice. Note that the equilibrium velocity–density model is for cumulative single lane traffic. Distinct velocity equilibrium does exist in each of the two lanes (Klare et al., 2003).

2.1. Driver’s preference over two lanes

We consider to linearize the nonlinear hyperbolic system \((\rho_i, v_i)\) around uniform steady states \((\rho_i^*, v_i^*)\). We obtain the following equations that need to be satisfied by the steady states \(\frac{1}{T_f} \rho_i^* - \frac{1}{T_i} \rho_i^* v_i = 0, \frac{1}{T_f} \rho_i v_i^* - \frac{1}{T_i} \rho_i^* v_i^* + \frac{\rho_i^*(V(\rho_i^*) - v_i^*)}{T_i} = 0,\) and \(\frac{1}{T_f} \rho_i^* v_i^* - \frac{1}{T_i} \rho_i v_i^* + \frac{\rho_i (V(\rho_i) - v_i)}{T_i} = 0.\) The steady state density–velocity relations are defined based on the Greenshield’s model. Thus the steady states \((\rho_i^*, v_i^*, v_s^*, \rho_s^*)\) need to satisfy

\[
\rho_i^* = \sigma \rho_i^*,
\]

\[
v_i^* = v_m \left( 1 - r_i \left( \frac{\rho_i}{\rho_m} \right) \right),
\]

where \(v_i^*\) and \(v_s^*\) differ from single-lane \(V(\rho_i).\) The ratio coefficients \(r_i\) and \(r_s\) are defined as

\[
r_i = \frac{1 + \left( \frac{1}{\sigma} \right)^Y + \left( \frac{1}{\gamma} \right)^Y}{1 + \left( \frac{1}{\gamma} \right)^Y + \left( \frac{1}{\sigma} \right)^Y}, \quad r_s = \frac{1 + \frac{1}{\gamma} + \frac{1}{\gamma} (\sigma)^Y}{1 + \frac{1}{\gamma} + \frac{1}{\gamma} (\sigma)^Y}.
\]

The parameter \(\sigma\) defines driver’s preference for the fast lane over slow lane according to (5), \(\sigma = T_f/T_i.\) Compared with the single-lane Greenshield’s model, the relations of steady state traffic velocities \(v_i^*\) and densities \(\rho_i^*\) depend on the drivers lane-changing preference parameter \(\sigma.\) Assuming that overall drivers prefer fast lane over slow lane, we use Fig. 1 \((\sigma > 1, \gamma = 1)\) to show the equilibrium velocity–density relation and the fundamental diagrams of the single-lane, the fast and slow lane. \(\rho_m\) represents the equivalent maximum density of the single-lane. The actual maximum density in the fast lane \(\rho_f^m\) and the slow lane \(\rho_s^m\) are related to \(\rho_m\) by

\[
\rho_f^m = \sqrt{\rho_m}, \quad \rho_s^m = \sqrt{\rho_m}.
\]

If drivers prefer the fast lane, the decrease of velocity gets steeper in the slow lane and less steep in the fast lane, then we have \(\sigma > 1 \implies r_f < 1 < r_s.\) At the same density, the fast lane traffic is “more tolerant to risk” of high density than in the single-lane case, and the slow lane traffic is “less tolerant to risk” than in the single-lane case. As a result, the traffic flow of the fast lane is higher than the slow lane at the same density in the fundamental diagram shown in Fig. 1. If drivers prefer the slow lane, it holds that \(\sigma < 1 \implies r_f > 1 > r_s.\) In general, the activities of lane changing segregate the drivers into the more “risk-tolerant” ones in the fast lane and the more “risk-averse” ones in the slow lane. The risk-tolerant drivers prefer to drive with a faster speed at the same density, compared with the risk-averse drivers.

![Fig. 1. Steady states of one lane, fast and slow lane.](image)

2.2. VSLs control of the linearized two-lane ARZ model

For boundaries, we assume constant traffic flow entering from the inlet boundary \(x = 0\) of the two lanes. Two VSLs implemented at the outlet \(U_f(t)\) and \(U_s(t)\) actuate the traffic velocity variations for the fast and slow lanes respectively.

\[
\rho_f(0, t) = \frac{\rho_f^0 v_f^0}{v_f(0, t)}, \quad \rho_s(0, t) = \frac{\rho_s^0 v_s^0}{v_f(0, t)},
\]

\[
v_f(L, t) = U_f(t) + v_f^*, \quad v_s(L, t) = U_s(t) + v_s^*.
\]

The control objective is to stabilize and to homogenize the traffic upstream with the VSLs. Observers for state estimation are also designed by measurements of boundary values around the steady states

\[
y_i(t) = \rho_i(L, t), \quad y_i(t) = \rho_i(L, t).
\]

Each line’s exiting traffic density is measured, which in practice could be obtained by video camera.

Then we linearize the above nonlinear hyperbolic system \((\rho_i, v_i, \rho_s, v_s)\) around steady states \((\rho_i^*, v_i^*, \rho_s^*, v_s^*)\) that satisfy (5)–(6). The deviations from the steady states are defined as \(\delta_i = \rho_i - \rho_i^*, \delta_i = v_i - v_i^*, i = f, s.\) In order to diagonalize the spatial derivatives on the left hand side of the equations, we write the
above linearized hyperbolic system in the Riemann coordinates \((\tilde{u}_i, \tilde{v}_i)\) as

\[
\tilde{u}_i = \frac{\gamma \rho_i^*}{p_i^*} \tilde{v}_i + \tilde{v}_i, \quad \tilde{v}_i = \tilde{v}_i.
\]  

To apply the backstepping approach and to design boundary control, we scale the state variables \(\tilde{v}_i\) and \(\tilde{v}_i\) to be the same. The scaled variables \(\tilde{v}_i\) and \(\tilde{v}_i\) are defined as \(\tilde{v}_i = \exp\left(\frac{\gamma \rho_i^*}{p_i^*} x\right)\tilde{v}_i\) and \(\tilde{v}_i = \exp\left(\frac{\gamma \rho_i^*}{p_i^*} x\right)\tilde{v}_i\). Then we obtain a coupled 4 \times 4 first-order hetero-directional hyperbolic system in \((\tilde{u}_i, \tilde{v}_i, \tilde{v}_i, \tilde{v}_i)\),

\[
\begin{align*}
\partial_t \tilde{u}_i + v_i^* \partial_x \tilde{u}_i &= \tilde{a}_{11}^{uv} \tilde{u}_i + \tilde{a}_{12}^{uv} \tilde{u}_i + \tilde{a}_{11}^{uv} \tilde{v}_i, \\
\partial_t \tilde{v}_i + v_i^* \partial_x \tilde{v}_i &= \tilde{a}_{21}^{uv} \tilde{u}_i + \tilde{a}_{22}^{uv} \tilde{v}_i + \tilde{a}_{21}^{uv} \tilde{v}_i, \\
\partial_t \tilde{v}_i - (\gamma \rho_i^* - v_i^*) \partial_x \tilde{v}_i &= \tilde{a}_{11}^{uv} \tilde{u}_i + \tilde{a}_{12}^{uv} \tilde{u}_i + \tilde{a}_{11}^{uv} \tilde{v}_i, \\
\tilde{u}_i(0, t) &= \kappa \tilde{v}_i(0, t), \quad \tilde{v}_i(0, t) = \kappa \tilde{v}_i(0, t), \\
\tilde{v}_i(L, t) &= L \tilde{u}_i(t), \quad \tilde{v}_i(L, t) = L \tilde{u}_i(t),
\end{align*}
\]

where the constant boundary coefficients \(k_i\) are defined as \(k_i = -\gamma \rho_i^* - v_i^*\), the constant coefficients \(t_i\) and \(l_i\) are defined as \(l_i = (\gamma \rho_i^* - v_i^*)^{-1}. It is straightforward to derive the in-domain coefficient matrix \([\tilde{A}(x)]\) from the spatial transformation of a constant matrix \([A]\). The transformation matrix \(T(x)\) is defined as \(T(x) = \begin{bmatrix} 1 & \frac{x}{L} \\ 0 & 1 \end{bmatrix}\), and

\[
A = TAT^{-1}, \quad T^{uv} = \begin{bmatrix} \exp\left(\frac{\gamma \rho_i^*}{p_i^*} x\right) & 0 \\ 0 & \exp\left(\frac{\gamma \rho_i^*}{p_i^*} x\right) \end{bmatrix}.
\]

Among the transformed sub-matrices, the elements of \([A^{uv}]\) are constant and the elements of \([A^{uv}(x)], [A^{uv}(x)]\) and \([A^{uv}(x)]\) are spatially-varying coefficients. The constant parameter block matrix \([A]\) is denoted as, \(A = \begin{bmatrix} A^{uv} & A^{uv} \\ A^{uv} & A^{uv} \end{bmatrix}\), where the elements of sub-matrices of \([A]\) are defined as,

\[
\begin{align*}
A^{uv} &= \begin{bmatrix} -\frac{1}{\gamma} & \frac{1}{\gamma} & 0 & 0 \\ -\frac{1}{\gamma} & \frac{1}{\gamma} & 0 & 0 \\ 0 & 0 & -\frac{1}{\gamma} & \frac{1}{\gamma} \\ 0 & 0 & -\frac{1}{\gamma} & -\frac{1}{\gamma} \end{bmatrix}, \\
A^{uv} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
A^{uv} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\end{align*}
\]

We consider the congested regime in Yu and Krstic (2019) where steady state traffic density disturbances convex downstream and the velocity disturbances travel upstream. Therefore the conditions \(v_i^* - \gamma \rho_i^* < 0\), \(v_i^* - \gamma \rho_i^* < 0\) hold for the characteristic speeds of \(\tilde{v}_i\). States \(\tilde{u}_i\) convect downstream while states \(\tilde{v}_i\) propagate upstream. We denote the transports speeds as \(\epsilon_1 = v_i^*, \epsilon_2 = v_i^*, \mu_1 = (\gamma \rho_i^* - v_i^*), \mu_2 = (\gamma \rho_i^* - v_i^*)\). Note that the steady velocity of the fast lane is larger than that of the slow lane, the constant transport speeds satisfy the following inequalities, \(-\mu_1 < -\mu_2 < 0 < \epsilon_1 < \epsilon_2\).

\[
3. \text{ Full-state feedback control design}
\]

The full-state feedback design is a special case of Hu et al. (2016). We introduce the backstepping transformation to the scaled \((\tilde{u}_i, \tilde{v}_i)\)-system in (13)–(18),

\[
\begin{align*}
\alpha_i(x, t) &= \tilde{u}_i(x, t), \\
\beta_i(x, t) &= \tilde{v}_i(x, t), \\
\tilde{v}_i(x, t) &= \tilde{v}_i(x, t) - \int_0^x \left[ K_{11} \quad K_{12} \right] \tilde{u}_i(x, t) \, \text{d}x \\
&- \int_0^x L_{21} \quad L_{22} \tilde{v}_i(x, t) \, \text{d}x.
\end{align*}
\]

The kernel variables \([K(x, \xi)]\) and \([L(x, \xi)]\) evolve in a triangular domain \(\mathcal{D} = \{(x, \xi) : 0 \leq \xi \leq x \leq 1\}. Taking derivative with respect to time and space on both sides of (20)–(21) along the solution of a target system, we obtain the kernel equations that govern the kernels \([K(x, \xi)]\) and \([L(x, \xi)]\). The well-posedness of the kernel equations is proved using the method of characteristics and the successive approximations, following the result for a general class of kernel system in Hu et al. (2016). There exists a unique solution \(K, L \in L^\infty(\mathcal{D})\). Therefore, we establish the invertibility of the backstepping transformation (20), (21) and can study the stability of the target system due to its equivalence to the \((\tilde{u}_i, \tilde{v}_i)\)-system. More details of the target system and the kernel equations can be referred to in Hu et al. (2016). With the backstepping transformation and the kernel equations, we map the \((\tilde{u}_i, \tilde{v}_i)\)-system to a cascade target system where the transformed boundary conditions (18) and backstepping transformation (21) yield the following full-state feedback control laws. We then reach the main stabilization result.

**Theorem 1.** Consider the linearized two-lane traffic ARZ model with the boundary conditions in (13)–(18), the initial conditions \(\tilde{u}_i(x, 0), \tilde{v}_i(x, 0) \in L^\infty((0, 1))\) and the following control laws

\[
U_i(t) = \exp\left(-\frac{\rho_i^*}{\mu_1} L\right) \int_0^t \gamma \rho_i^* \bar{K}_{11}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi + \gamma \rho_i^* \bar{K}_{12}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi, \\
&+ \int_0^t \gamma \rho_i^* \bar{K}_{11}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi + \int_0^t \gamma \rho_i^* \bar{K}_{12}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi,
\]

\[
U_i(t) = \exp\left(-\frac{\rho_i^*}{\mu_2} L\right) \int_0^t \gamma \rho_i^* \bar{K}_{21}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi + \gamma \rho_i^* \bar{K}_{22}(L, \xi) \left(\rho_i(\xi, t) - \rho_i^*\right) \, \text{d}\xi,
\]

where the kernels \([K]\) and \([L]\) are obtained by solving the kernel equations. The steady states \((\rho_i^*, v_i^*, \rho_i^*, v_i^*)\) are finite-time stable and the convergence is reached in

\[
t_i = \frac{L}{v_i^*} + \frac{L}{\gamma \rho_i^* - v_i^*} + \frac{L}{\gamma \rho_i^* - v_i^*}.
\]
4. Collocated observer and output feedback control

In this section, we develop a collocated observer by taking measurement of density states at the outset of the segment defined in (11). Using the state estimates obtained from the observer design and the full-state feedback control laws, we construct output feedback controllers. Note that the anti-collocated observer can be designed here by taking measurement of velocity states \( \hat{v}_i(0, t) \) and \( \hat{v}_j(0, t) \) at the inlet. The antimcollocated observer design is trivial which presents as a copy of the \((\hat{w}_i, \hat{w}_j, \hat{v}_i, \hat{v}_j)\)-system. More importantly, collocated observer design is practical in implementation along with the full-state feedback control design.

4.1. Collocated observer design

For state estimation of the scaled system in (13)–(18), we obtain the measurement of \( \hat{w}_i(L, t) \) and \( \hat{w}_j(L, t) \) from (12), \( Y_i(t) = \hat{w}_i(L, t) = \frac{\gamma P_{i}^{x}}{\rho_{i}^s} y_{i}(t) + \hat{v}_i(t), \) \( Y_j(t) = \hat{w}_j(L, t) = \frac{\gamma P_{j}^{x}}{\rho_{j}^s} y_{j}(t) + \hat{v}_j(t), \) respectively. The values of \( Y_i(t) \) and \( Y_j(t) \) are obtained from \( y_i(t), y_j(t) \) and control inputs \( U_i(t), U_j(t), \) respectively:

\[
Y_i(t) = \frac{\gamma P_{i}^{x}}{\rho_{i}^s} y_{i}(t) + U_i(t), \quad Y_j(t) = \frac{\gamma P_{j}^{x}}{\rho_{j}^s} y_{j}(t) + U_j(t). \tag{25}
\]

The observer equations \((\hat{u}_i, \hat{u}_j, \hat{u}_s, \hat{u}_t)\) that estimate \((u_i, u_s, u_t, \hat{v}_i)\) read as follows:

\[
\begin{align*}
\partial_t \hat{u}_s &= - v_i^* \partial_t \hat{w}_t \hat{a}_s^{uw} \hat{w}_t + \hat{a}_{12t} \hat{w}_i + \hat{a}_{21t} \hat{w}_s + \partial_{x} \hat{u}_s(t), \quad \text{(26)} \\
\partial_t \hat{u}_t &= - v_i^* \partial_t \hat{w}_t \hat{a}_s^{uw} \hat{w}_t + \hat{a}_{22t} \hat{w}_t + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \partial_{x} \hat{u}_t(t), \quad \text{(27)} \\
\partial_t \hat{u}_s &= \{ \gamma P_i^x \} \hat{u}_s(t) + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \partial_{x} \hat{u}_s(t), \quad \text{(28)} \\
\partial_t \hat{u}_t &= \{ \gamma P_j^x \} \hat{u}_t(t) + \hat{a}_{22t} \hat{w}_t + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \partial_{x} \hat{u}_t(t), \quad \text{(29)} \\
\hat{w}_i(0, t) &= k_i \hat{u}_i(0, t), \quad \hat{w}_s(0, t) = k_i \hat{u}_s(0, t), \quad \text{(30)} \\
\hat{u}_i(L, t) &= U_i(t), \quad \hat{u}_s(L, t) = U_s(t). \quad \text{(31)}
\end{align*}
\]

The output injections in (26)–(29) are defined as

\[
\hat{u}_i(L, t) = Y_i(t) - \hat{u}_i(L, t), \quad \text{(32)}
\]

The observer output injection gains matrices \([P(x)]\) and \([Q(x)]\) are to be designed so that the estimation error system is driven to converge to zero in finite time. The error system \((\hat{w}_i, \hat{v}_i, \hat{v}_s, \hat{v}_t)\) is straightforward to derive by subtracting the observers (26)–(31) from the original linearized system (13)–(18), where \( \hat{w}_i = \hat{w}_i - \hat{w}_i, \hat{v}_s = \hat{v}_s - \hat{v}_s, \hat{v}_t = \hat{v}_t - \hat{v}_t. \) We then apply the backstepping transformation to the error system given by

\[
\begin{align*}
\dot{\hat{w}}_i(x, t) &= \hat{a}_s \hat{u}_i(x, t) + \int_x^l M_{11}(x, \xi) \hat{a}_s(\xi, t) \, d\xi, \\
\dot{\hat{v}}_i(x, t) &= \hat{a}_s \hat{u}_i(x, t) + \int_x^l M_{12}(x, \xi) \hat{a}_s(\xi, t) \, d\xi, \quad \text{(34)}
\end{align*}
\]

where the kernels \([M(x, \xi)], [N(x, \xi)]\) evolve in the triangular domain \( \mathcal{T} = \{(x, \xi) : 0 \leq x \leq \xi \leq L\} \) are defined later.

Then we obtain the following cascade target system

\[
\begin{align*}
\dot{\tilde{\hat{a}}}_s &= - v_i^* \tilde{\hat{a}}_s \partial_t \hat{w}_t + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \hat{a}_{12t} \hat{w}_t \partial_t \hat{w}_t + \partial_x \hat{a}_s(t) \partial_t \hat{w}_t, \\
\dot{\tilde{\hat{a}}}_s &= \gamma P_i^x \hat{a}_s(t) + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \partial_x \hat{a}_s(t) \partial_t \hat{w}_t, \quad \text{(36)}
\end{align*}
\]

\[
\begin{align*}
\dot{\tilde{\hat{a}}}_s &= \frac{\gamma P_i^x}{\rho_i^s} \hat{a}_s(t) + \hat{a}_{12t} \hat{w}_i + \hat{a}_{12t} \hat{w}_s + \partial_x \hat{a}_s(t) \partial_t \hat{w}_t, \quad \text{(36)}
\end{align*}
\]
Considering the following variables by defining $\bar{x} = L - x$, $\bar{\xi} = L - \xi$, and new kernels $\bar{M}(\bar{x}, \bar{\xi}) = M(L - x, L - \xi)$ and $\bar{N}(\bar{x}, \bar{\xi}) = N(L - x, L - \xi)$, defined in the triangular domain $\mathcal{D} = [\bar{x}, \bar{\xi}]: 0 \leq \bar{x} \leq \bar{\xi} \leq L$. We find that the following kernel equations obtained for $\bar{M}(\bar{x}, \bar{\xi})$ and $\bar{N}(\bar{x}, \bar{\xi})$ have the same structure with the controller kernel system. The well-posedness are obtained from the kernel matrices $\bar{M}(\bar{x}, \bar{\xi})$ and $\bar{N}(\bar{x}, \bar{\xi})$. Therefore, there exists a unique solution $M, N \in L^\infty(\mathcal{D})$. The stability of target system (36)–(41) is equivalent to the error system. The observer gains matrices $P(x)$ and $Q(x)$ are obtained from the kernel matrices

$$P(x) = M(x, L) \begin{bmatrix} u_s^f & 0 \\ 0 & u_t^f \end{bmatrix}, \quad Q(x) = N(x, L) \begin{bmatrix} u_s^s & 0 \\ 0 & u_t^s \end{bmatrix}. \quad (51)$$

Note that the states estimation of the original traffic flow variables $(\rho_s, v_s, \rho_f, v_f)$ are obtained by the inverse transformation of (12). The following conclusion is reached for the observer design.

**Theorem 3.** Consider the linearized two-lane traffic ARZ model with the boundary conditions in (13)–(18), the initial conditions $\bar{w}_s(x, 0), \bar{v}_s(x, 0) \in L^\infty(0, L)$, state estimates are obtained from the collocated observer design (26)–(31) for $(\hat{\rho}_s, \hat{\rho}_f, \hat{v}_s, \hat{v}_f)$. The output injection gains $P$ and $Q$ are obtained in (51) by solving the kernels $[M]$ and $[N]$ from (46)–(50). The finite-time convergence of estimation errors to zero equilibrium is reached in $t_o$ given by (42).

**Proof.** Lemma 2 with the existence of the backstepping transformation for the observer in (34), (35) yields the convergence of estimation errors $(\hat{w}_s, \hat{w}_f, \hat{v}_s, \hat{v}_f)$ to zero for $t > t_o$.

### 4.2. Output feedback controller

The output feedback controllers are constructed by employing the states estimates in the full-state feedback laws which yield the finite-time stability of the closed-loop system to zero equilibrium. Combining the collocated observer design (26)–(31) and full-state feedback controllers (22), (23), we obtain the output feedback controllers.

**Theorem 4.** Consider the linearized two-lane traffic ARZ model with the boundary conditions in (13)–(18), the initial conditions $\bar{w}_s(x, 0), \bar{v}_s(x, 0) \in L^\infty(0, L)$, and the output feedback laws combining the full state feedback (22), (23) and the collocated observer (26)–(31), where the output injection gains obtained by solving the kernels $[M]$ and $[N]$ in (46)–(47). The steady states $(\rho^*_s, v^*_s, \rho^*_f, v^*_f)$ are finite-time stable and the convergence is reached in $t_{out}$ defined as

$$t_{out} = t_o + t_f, \quad (52)$$

where $t_o$ is given in (42) and $t_f$ in (24).

**Proof.** Theorem 3 yields that state estimates $(\hat{\rho}_s, \hat{\rho}_f, \hat{v}_s, \hat{v}_f)$ converge to $(\rho^*_s, \rho^*_f, v^*_s, v^*_f)$ locally after $t = t_o$. Applying Theorem 1, one has that $(\rho^*_s, \rho^*_f, v^*_s, v^*_f)$ converge to $(\rho^*_s, \rho^*_f, v^*_s, v^*_f)$ after $t = t_f$ locally. Therefore, after $t = t_o + t_f$, we have the local convergence of the state variables to the steady states.

### 5. Numerical simulation

The control design presented in the previous sections are validated by numerical simulation. The simulation is performed for the linearized two-lane ARZ PDE model and the two-step Lax Wendroff method is used to approximate the solution. Both the fast-lane and slow-lane are considered in the congested regime.

Steady states density $\rho^*_s = 180 \text{ veh/km, } v^*_s = 28.5 \text{ km/h}$ and $\rho^*_f = 90 \text{ veh/km, } v^*_f = 33 \text{ km/h}$ are chosen given the maximum density $\rho^m_s = 220 \text{ veh/km, } \rho^m_f = 142 \text{ veh/km, } \gamma = 0.8$ and maximum velocity $v^m = 140 \text{ km/h}$ so that the traffic of both lanes are lightly congested. We consider the situation that overall drivers prefer remaining in the slow lane rather than changing to the fast lane so that $T_f = 15 \text{ s}$ is chosen to be smaller than $T_s = 30 \text{ s}$. Therefore, higher density traffic appears in the slow lane and it can contain higher rate of the traffic flow. The relaxation time is chosen as $T_f = 100 \text{ s}$ and $T_f = 200 \text{ s}$.

#### 5.1. Output feedback stabilization and performance

We consider the initial traffic states are oscillating around the equilibrium states and thus implement sinusoid initial conditions which are highlighted with color blue. The constant incoming flow and outgoing flow are considered for the open-loop simulation as shown in Fig. 2. Considering the steady state velocity as the averaged traffic velocity, it takes around 2 min for both the fast-lane and slow-lane vehicles to leave the considered freeway segment. But the oscillations sustain for more than 12 min. The full state feedback stabilization results are shown in Fig. 3. The finite-time convergence of the density and velocity to the spatially constant steady states is achieved in $t_f = 5 \text{ min}$, which is highlighted with color green.

Combining the observer design and the full-state feedback controllers, we derive the output feedback controllers and then simulate the closed-loop system in Fig. 4. The finite convergence time of the closed-loop with output feedback controllers is $t = t_o + t_f = 10 \text{ min}$, as illustrated in the figures with green highlighted lines. We can see that the states converge to the steady state values before the green highlighted lines. The velocity control inputs that are displayed on the VSLs for fast and slow lanes are plotted in Fig. 5. Compared with the slow lane velocity control input, the variation of the fast lane control input around the steady state velocity is larger. For example, the output feedback driving speed advisory for vehicles in the fast lane to leave the segment varies in a range of 25 km/h to 44 km/h while that of the slow lane varies in a range of 26 km/h to 34 km/h.
For practical consideration, it is not feasible to directly implement the continuous time-varying velocity control inputs as shown in Fig. 5, since drivers need reaction time to follow the driving speed displayed by VSLs. The digital post processing of the continuous in time control signals is thus required in the implementation of VSLs and we briefly discuss it in the next section. It should also be noted that if VSL cannot be changed discretely on the time scale of the stop-and-go oscillations, then such oscillations are not stabilizable by any VSL-implemented algorithm. For example, in Fig. 5, if VSL can only be changed every 2 min on the time scale exhibited by the stop-and-go oscillations in Fig. 2, no VSL control can stabilize such congested traffic.

The performance of the output feedback controllers is evaluated for three performance indices which are total travel time (TTT), drivers’ comfort and fuel consumption. The considered performance indices are defined as

\[
J_{\text{fuel}} = \int_0^{t_{\text{sim}}} \int_0^L \max[0, b_0 + b_1 v(x, t) + b_2 v(x, t) a(x, t)] \rho(x, t) \, dx \, dt
\]

(53)

\[
J_{\text{comfort}} = \int_0^{t_{\text{sim}}} \int_0^L (a(x, t)^2 + a_i(x, t)^2) \rho(x, t) \, dx \, dt
\]

(54)

\[
J_{\text{TTT}} = \int_0^{t_{\text{sim}}} \int_0^L \rho(x, t) \, dx \, dt
\]

(55)

according to Treiber and Kesting (2013), where \(a(x, t)\) is defined as the local acceleration \(a(x, t) = v_t(x, t) + v(x, t) v_o(x, t)\) and the parameters for fuel consumption are taken from Treiber and Kesting (2013, chapter 20) as \(b_0 = 25 \cdot 10^{-3} \text{l/s}\), \(b_1 = 24.5 \cdot 10^{-6} \text{l/m}\), \(b_2 = 32.5 \cdot 10^{-9} \text{ls}^2/\text{m}^2\), \(b_4 = 125 \cdot 10^{-6} \text{ls}^2/\text{m}^2\) for a \(L = 1 \text{ km}\), \(t_{\text{sim}} = 12 \text{ min}\). We denote \(J_{X, \text{open}}\) as the performance indices with respect to the open-loop results, whereas \(J_{X, \text{output}}\) as the indices with respect to the closed-loop results, where \(X = \{\text{fuel, comfort, TTT}\}\). The improved performance indices are \(J_{\text{fuel, output}} = 92.74\%\), \(J_{\text{comfort, output}} = 82.63\%\), and \(J_{\text{TTT, output}} = 97.79\%\). The fuel consumption of total traffic reduces 7.26% with driver’s comfort improved 17.37% and the total travel time reduced 7.21% for the closed-loop system with the output feedback VSL controllers in 12 min. The numerical simulation demonstrates that the proposed output feedback control design not only stabilizes the two-lane traffic flow system in the finite time but also improves all the performance indices.

5.2. Different traffic scenarios, one-lane backstepping and PI controllers

To demonstrate the effect and relevance of the proposed VSL controllers, we compare the stabilization result for different traffic scenarios and simulate one-lane backstepping and PI controllers.

For different traffic scenarios, we evaluate the overall closed-loop stabilization result with the state variables in the \(L^\infty\)-norm, defined as \(S(t) = S_i(t) + S_f(t)\), where

\[
S_i(t) = \left\| \rho_i(x, t) - \rho_i^* \right\|_{L^\infty} + \left\| v_i(x, t) - v_i^* \right\|_{L^\infty}
\]

(56)

The stabilization result is demonstrated with the temporal evolution of the stabilization result \(S(t)\), whose convergence to zero is guaranteed by Theorem 4. Four different traffic scenarios that are
represented by different steady states \((\rho^\star_s, \rho^\star_f, v^\star_s, v^\star_f)\) are plotted in Fig. 6. The same sinusoid initial conditions around the steady states are considered. The red dashed line represents the output feedback stabilization result that is shown from Figs. 2–8. We can see traffic becomes less dense from the blue dashed line to the black line. As the traffic density becomes smaller and velocity becomes bigger, the closed-loop system goes through larger transient before being stabilized. This is because the faster velocity in the congested traffic requires larger VSL control efforts and induce more oscillated system behavior. On the other hand, comparing the red dashed line and green dotted line, it is found out that the green dotted line shows a bigger transient than the red dashed line since it has larger density and velocity discrepancies between the fast and slow lane than that of the red dashed line. More lane-changing activities exist when the speed difference is larger between lanes.

For comparison, we consider the two-lane traffic as a general one-lane and impose the same displayed VSL values for the two lanes at the outlet. Then the lane-changing activities are ignored.

Two existing control strategies for one-lane ARZ model are employed including the PDE backstepping VSL in Yu and Krstic (2018) and PI control by VSL and ramp in Zhang et al. (2019). The general one-lane density is defined as the lateral spatial summation and the averaged velocity is

\[
\rho_a(x, t) = \rho_s(x, t) + \rho_f(x, t) \\
v_a(x, t) = \frac{\rho_s(x, t) v_s(x, t) + \rho_f(x, t) v_f(x, t)}{\rho_a(x, t)}
\]

The control objective is to regulate the one-lane traffic to averaged steady states \(\rho^\star_a = 270\) \(\text{veh/m}, v^\star_a = 29.8\) \(\text{km/h}\). We use the previous model parameters and conduct the simulation under the same initial conditions. The two-lane traffic evolution is shown in Fig. 7. There are still oscillations when the backstepping controller is applied same for the two lanes. Meanwhile, the steady state velocity of the fast lane and slow lane slowly adapt to the general one-lane velocity \(v^\star_a\).

PI boundary feedback controllers are developed in Zhang et al. (2019) for the linearized ARZ model. Boundary values of velocity at the inlet are measured to construct one PI VSL at outlet controlling the exiting velocity of the two lane, given by

\[
v_o(L, t) =
\]

Fig. 8. Density and velocity of slow and fast lane closed-loop system with one-lane PI controller. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
\[ v_1 + k_2^2 v_2(0,t) + k_1^2 \int_0^t (v_2(0,s) - v_2^*) ds. \]
where \( k_2^2 = -0.1 \), \( k_1^2 = -0.02 \) are tuning gains. The density and velocity in the fast and slow lanes still oscillate, as shown in 8. The slow lane traffic increases velocity and fast lane traffic decreases velocity, following the same VSL. The backstepping and PI velocity control inputs are plotted in Fig. 9 and PI has some offset to the steady state \( v_2^* \), in addition to oscillations. Both the one-lane VSL controls do not obtain satisfactory stabilization results when applied for the two-lane traffic. This is because the inter-lane activities and distinct fast and slow lane steady states are not considered by the one-lane control design.

6. Conclusion and future work

This paper solves the output feedback stabilization of a two-lane traffic congestion problem with lane-changing. The finite-time convergence of the linearized two-lane ARZ model is achieved with two VSLs actuating velocities at the outlet of a freeway segment. By taking the collocated boundary measurement of density variations, observers are designed for state estimation, which is theoretically novel and practically sound. The control designs are simulated for different traffic scenarios, evaluated with several performance indices and compared with two one-lane control approaches. This result paves the way for applying PDE backstepping techniques for multi-lane traffic with inter-lane activities.

For future work, it is of authors’ interest to consider the impact of VSLs on the downstream traffic while regulating the upstream traffic. To avoid congestion for downstream, we need to design new algorithms that control simultaneously the upstream and downstream traffic through the VSLs located in the middle. In Yu, Auriol, and Krstic (2020), this type of problem is solved for the one-lane traffic with ramp-metering. The design approach developed in Yu et al. (2020) can be adapted for the two-lane traffic considered in this paper. On the other hand, the implementation of VSLs could cause additional lane-changing behaviors in the controlled segment and this is not reflected in the model we use as the lane-changing parameters are constant. The limitation needs to be addressed by future study.

For practical consideration, the robustness of the proposed output feedback controllers to perturbed model parameters, boundary disturbances is also important to discuss in future research. We may prove the input-to-state stability for the closed-loop system using the controllers in this paper as the nominal ones and designing additional components to enable disturbances rejection, as in Auriol and Di Meglio (2020), Karafyllis and Krstic (2019) and Lamare, Auriol, Di Meglio, and Aarsnes (2018). Then the robustness of the output feedback controllers is guaranteed in presence of the disturbances such as on-ramp or off-ramp flow, the varying incoming flow from the inlet that is relatively small compared to the mainline flow. In addition, the digital post processing of the continuous velocity control signals needs to be studied. The problem was solved in Espitia, Yu, and Krstic (2020) for one-lane ARZ model by developing the Lyapunov-based event-triggered strategies, which could be potentially extend to the two-lane ARZ model.

References


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