Adaptive Control by Regulation-Triggered Batch Least Squares

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Abstract—The paper extends a recently proposed indirect, certainty-equivalence, event-triggered adaptive control scheme to the case of nonobservable parameters. The extension is achieved by using a novel batch least-squares identifier (BaLSI), which is activated at the time of the events. BaLSI guarantees the finite-time asymptotic constancy of the parameter estimates and the fact that the trajectories of the closed-loop system follow the trajectories of the nominal closed-loop system (nominal in the sense of the asymptotic parameter estimate, not in the sense of the true unknown parameter). Thus, if the nominal feedback guarantees global asymptotic stability and local exponential stability, then unlike conventional adaptive control, the newly proposed event-triggered adaptive scheme guarantees global asymptotic regulation with a uniform exponential convergence rate. The developed adaptive scheme is tested to a well-known control problem—the state regulation of the wing-rock model. Comparisons with other adaptive schemes are also provided for this particular problem.

Index Terms—Adaptive control, event-triggered control, least-squares estimation.

I. INTRODUCTION

ADAPTIVE control of linear and nonlinear finite-dimensional systems is an important topic in control literature. Classical and comprehensive references, such as [19], [26], [27], [39], are helpful for the understanding of existing approaches to adaptive control of finite-dimensional systems. Many existing approaches have also been extended to: 1) parabolic partial differential equations (PDEs) in one spatial dimension (see [42]) and 2) hyperbolic PDEs in one spatial dimension (see [1], [3], and references therein).

Event-triggered control has attracted considerable attention within the control systems community. Indeed, event-triggered control has been applied to difficult control problems that involve sampling, quantized measurements, output-feedback control, distributed networked control, and decentralized control; see [2], [5], [6], [8], [10]–[14], [28]–[30], [44]–[47], [50], [53]. In all cases, the system under event-triggered control becomes a hybrid dynamical system. Event-triggered direct adaptive control schemes have also appeared in the literature during the last two decades. Event-triggered adaptive control has been applied to globally Lipschitz continuous-time systems in the literature of neural networks (see [41], [48], [52], [54]). Direct adaptive control approaches for linear systems have been proposed in [31]–[34], where the proposed schemes either employ event-triggering or sampled-data techniques. Event-triggered adaptive control schemes for a special class of nonlinear systems, where the input is applied with zero-order-hold, were studied in [51]. Adaptive control design methodologies with logic-based switching for linear and nonlinear control systems have been developed in [4], [15]–[18], [36], [37], [49] (see also the references therein). The proposed direct supervisory adaptive control schemes in [4], [15]–[18], [36], [37] employ multimodel-based estimators of the performance of the “current” controller in conjunction with hierarchical hysteresis switching logic (which is the event-triggered element in the design). Therefore, this direct approach is based on an estimation error-triggered controller scheduling.

A different certainty-equivalence, regulation-triggered, indirect adaptive control scheme was proposed in [23] under a parameter observability assumption (but without any persistence of excitation assumption). The adaptive controller in [23] employed a dead-beat, least-squares identifier with delays, and allowed the constructive derivation of KL regulation estimates that guarantee the same convergence properties as that of the nominal feedback controller with known parameters. The approach was extended in [24] to the case of reaction-diffusion PDEs in one spatial dimension with constant coefficients.

In this paper, we consider nonlinear systems of the form

\[
\dot{x} = f(x, u) + g(x, u)\theta,
\]

\[
x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad \theta \in \Theta \subseteq \mathbb{R}^l,
\]

(1)

where \( f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n, g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{n \times l} \) are smooth mappings with \( f(0, 0) = 0, g(0, 0) = 0 \), and \( \theta \in \Theta \subseteq \mathbb{R}^l \) is a vector of constant but unknown parameters that takes values in a closed convex set \( \Theta \subseteq \mathbb{R}^l \). By modifying the identifier used in [23], we obtain a new identifier that can work even without any parameter observability assumption. The proposed identifier is a batch least-squares identifier (BaLSI) which is activated at the...
times of the events (similar ideas have also been used in [20], [38], [40] in the context of machine learning but not necessarily with the use of a least-squares methodology and not in a batch processing way) and guarantees the following. (referred to as “fact” hereunder)

1) The parameter estimates $\hat{\theta}(t) \in \Theta \subseteq \mathbb{R}^l$ change at most $l$ times, where $l$ is the number of unknown parameters. As a consequence, the parameter estimates remain constant after the time $\tau > 0$ of the last event for which a change in the parameter estimate occurs (finite-time asymptotic constancy of the parameter estimates). Moreover, the parameter estimation error $\dot{\theta}(t) - \theta$ satisfies $g(x(t), u(t))(\theta - \theta_s) = 0$ for all $t \geq \tau$, where $\theta_s$ is the constant value of the parameter estimate after the last event, i.e., $\dot{\theta}(t) = \theta_s$ for $t \geq \tau$.

Moreover, the BaLSI, when combined with a certainty-equivalence controller (as in [23]) achieves the following. (referred to as “fact” hereunder.)

2) When no change in the parameter estimate occurs, the closed-loop system follows the trajectories of the nominal closed-loop system (“nominal” in the sense of the asymptotic parameter estimate, not in the sense of the true unknown parameter).

To see this, notice that a certainty-equivalence controller $u = k(\hat{\theta}, x)$ applied to system (1) with $u = k(\hat{\theta}, x)$ being the nominal feedback, gives for $t \geq \tau$

$$\dot{x} = f(x, k(\hat{\theta}, x)) + g(x, k(\hat{\theta}, x))\theta$$

$$= f(x, k(\hat{\theta}, x)) + g(x, k(\hat{\theta}, x))\theta_s + g(x, k(\hat{\theta}, x))(\theta - \theta_s)$$

$$= f(x, k(\hat{\theta}, x)) + g(x, k(\hat{\theta}, x))\theta_s$$

e.g., we follow the trajectories of the nominal closed-loop system $\dot{x} = f(x, k(\hat{\theta}, x)) + g(x, k(\hat{\theta}, x))\theta$ with $\theta$ being replaced by $\theta_s$.

Thus, Fact 1 in conjunction with Fact 2 guarantees that the solution of the closed-loop system presents exactly the same convergence properties as the nominal closed-loop system. If the nominal feedback guarantees global asymptotic stability and local exponential stability, then the proposed event-triggered adaptive scheme guarantees global asymptotic regulation with a uniform exponential convergence rate.

The use of the regulation-triggered schedule of events (as in [23]) allows the following facts.

3) No finite-escape time occurs, even if the nonlinearity is arbitrary.

4) Useful bounds for the solution are obtained, which allow the derivation of $KL$ regulation estimates.

5) When no change in the parameter estimate occurs, then two consecutive events differ by a constant user-specified time.

Fact 1 in conjunction with Fact 5 guarantee that no Zeno behavior is possible. Finally, the BaLSI guarantees that for many cases, the parameter estimates will converge to the actual values of the parameters. To our knowledge, this collection of desirable features is not exhibited simultaneously by any other adaptive scheme. More specifically, the absence of $KL$ regulation estimates for the supervisory adaptive control schemes in [15]--[18], [36], and [37] is explained by the use of the estimation error-triggered policy (instead of our regulation-triggered policy) and the fact that the settling time of the parameter estimate cannot be estimated. This is also true for our scheme (i.e., the time $\tau > 0$ of the last event for which a change in the parameter estimate occurs cannot be estimated), but due to Fact 2 (which does not only hold for the last event but for all events), we are in a position to bound the solution of the closed-loop system by means of an appropriate $KL$ regulation estimate. However, it should be noticed that: 1) exponential $KL$ regulation estimates have been provided for direct adaptive schemes for specific cases (see [31] for the linear case as well as [4] for a specific nonlinear case); 2) exponential convergence of the regulation error for the linear case has been shown in [17]; and 3) important robustness results with respect to various errors have been provided in [4], [15]--[18], [36], [37], while here, we do not consider the possible effect of noise, disturbances, and unmodeled dynamics (with the exception of the numerical example in Section V).

In this way, we extend the results contained in [23] to linear and nonlinear finite-dimensional systems with nonobservable parameters. The present paper also generalizes the results contained in [23] to systems with parameters that take values in a closed, convex set of the parameter space and consequently, the scheme can work even with nonzero parameters (e.g., high-frequency gains).

The structure of the paper is as follows. Section II is devoted to the formulation of the problem and the presentation of the assumptions under which the adaptive regulator is constructed. Section III provides the detailed description of the event-triggered identifier and the adaptive controller. The main results of the present work are given in Section IV (Theorem IV.1, Theorem IV.2, and Corollary 4.3). Section V contains the numerical study of an important illustrative example—the wingrock model. Section VI contains the proofs of all main results. Finally, the concluding remarks are provided in Section VII.

Notation: The following notations are used throughout the paper.

1) For a vector $x \in \mathbb{R}^n$ we denote by $|x|$ its Euclidean norm, by its transpose. For a real matrix $A \in \mathbb{R}^{n \times m}$, $A' \in \mathbb{R}^{m \times n}$ denotes its transpose and $|A| := \sup \{ |Ax| : x \in \mathbb{R}^n, |x| = 1 \}$ is its induced norm. For a square matrix $A \in \mathbb{R}^{n \times n}$, $\det(A)$ denotes its determinant and $N(A)$ denotes the null space of $A$, i.e., $N(A) = \{ x \in \mathbb{R}^n : Ax = 0 \}$. For a subspace $S$ of $\mathbb{R}^n$, we denote by $\dim(S)$ its dimension.

2) $\mathbb{R}_+$ denotes the set of nonnegative real numbers. $Z_+$ denotes the set of nonnegative integers.

3) We say that a function $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ is positive definite if $V(x) > 0$ for all $x \neq 0$ and $V(0) = 0$. We say that a continuous function $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ is radially unbounded if the following property holds—“for every $M > 0$ the set $\{ x \in \mathbb{R}^n : V(x) \leq M \}$ is compact.”

4) Let $A \subseteq \mathbb{R}^n$ be an open set, let $U \subseteq \mathbb{R}^n$ be a set with $A \subseteq U \subseteq \text{cl}(A)$, where $\text{cl}(A)$ denotes the closure of $A$, and let $\Omega \subseteq \mathbb{R}^n$ be a set. By $C^0(U : \Omega)$ we denote the class of continuous mappings on $U$ which take values in $\Omega$. By $C^k(U : \Omega)$, where $k \geq 1$, we denote the class of continuous functions on $U$, which have continuous derivatives of order $k$ on $U$ and also take values in $\Omega$. 

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5) By $K$ we denote the class of strictly increasing $C^0$ functions $a: \mathbb{R}_+ \to \mathbb{R}_+$ with $a(0) = 0$. By $K_\infty$ we denote the class of strictly increasing $C^0$ functions $a: \mathbb{R}_+ \to \mathbb{R}_+$ with $a(0) = 0$ and $\lim_{s \to +\infty} a(s) = +\infty$. By $KL$ we denote the set of all continuous functions $\sigma: \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ with the properties: a) for each $t \geq 0$ the mapping $\sigma(\cdot, t)$ is of class $K$; b) for each $s \geq 0$, the mapping $\sigma(s, \cdot)$ is nonincreasing with $\lim_{s \to +\infty} \sigma(s, t) = 0$.

All stability notions used in this paper are the standard stability notions for time-invariant systems (see [25]).

II. PROBLEM FORMULATION

Consider system (1) and suppose that there exists a smooth mapping $k: \Theta \times \mathbb{R}^n \to \mathbb{R}^n$ with $k(\theta, 0) = 0$ for all $\theta \in \Theta$, a constant $\sigma > 0$ and a family of continuously differentiable, positive definite, and radially unbounded functions $V(\theta, \cdot) \in C^1(\mathbb{R}^n; \mathbb{R}_+)$ parameterized by $\theta \in \Theta$ with the mapping $\Theta \times \mathbb{R}^n \ni (\theta, x) \to V(\theta, x)$ being continuous, such that the following assumptions hold.

(H1) For each $\theta \in \Theta$, $t \in \mathbb{R}^n$ is globally asymptotically stable (GAS) for the closed-loop system

$$x = f(x, k(\theta, x)) + g(x, k(\theta, x)) \theta.$$  

More specifically, the following inequality holds:

$$\nabla V(\theta, x) (f(x, k(\theta, x)) + g(x, k(\theta, x)) \theta) \leq -2\sigma V(\theta, x)$$

for all $\theta \in \Theta$, $x \in \mathbb{R}^n$.

(H2) For every nonempty, compact set $\Theta \subseteq \Theta$, the following property holds—“for every $M \geq 0$ there exists $R > 0$ such that the implication $V(\theta, x) \leq M$, $\theta \in \Theta$ $\Rightarrow |x| \leq R$ holds.”

Assumption (H1) is a standard stabilizability assumption (necessary for all possible adaptive control design methodologies). For nonlinear systems, the design of a globally stabilizing state feedback law $u = k(\theta, x)$ is usually performed with the aid of a control Lyapunov function (CLF, see [7], [22], [27], [43], and references therein). Therefore, the knowledge of the functions $k: \Theta \times \mathbb{R}^n \to \mathbb{R}^n$ and $V: \Theta \times \mathbb{R}^n \to \mathbb{R}$ is not a demanding requirement. Assumption (H2) is a technical assumption, which requires a “uniform” coercivity property for $V(\theta, \cdot)$ on compact sets of $\Theta \subseteq \mathbb{R}^l$. Assumption (H2) holds automatically for arbitrary closed, convex sets $\Theta \subseteq \mathbb{R}^l$ and for functions of the form

$$V(\theta, x) = a_1(\theta, x)x_1^2 + a_2(\theta, x)(x_2 - \varphi_1(\theta, x))^2 + \cdots + a_n(\theta, x)(x_n - \varphi_{n-1}(\theta, x_1, \ldots, x_{n-1}))^2$$

where $a_i: \Theta \times \mathbb{R}^i \to (n, +\infty)$ $(i = 1, \ldots, n)$ are functions bounded from below by a positive constant $\kappa > 0$ and $\varphi_i: \Theta \times \mathbb{R}^{i-1} \to \mathbb{R}$ $(i = 1, \ldots, n - 1)$ are continuous functions with $\varphi_i(\theta, 0) = 0$ for all $\theta \in \Theta$ and $i = 1, \ldots, n - 1$. The above functional form is met frequently in the study of nonlinear triangular single-input systems of the form

$$x_1 = f_1(\theta, x_1, x_2), \ldots, x_{n-1} = f_{n-1}(\theta, x_1, x_2), x_n = f_n(\theta, x_1, x_2, u).$$

In [23], the following parameter observability assumption is used.

(H3) There exists a positive integer $N$ such that the following implication holds:

“For every set of $N$ times $0 = \tau_0 < \tau_1 < \cdots < \tau_N$ and for every vectors $\theta, d_0, \ldots, d_N \in \Theta$ with $d_i \neq 0$ for $i = 0, \ldots, N$, the only, right differentiable mapping $x \in C^d ([0, \tau_N]; \mathbb{R}^n) \setminus C^d ([0, \tau_N]; \{\tau_0, \ldots, \tau_N\}; \mathbb{R}^n)$ satisfying $\dot{x}(t) = f(x(t), k(\theta + d_i, x(t))) + g(x(t), k(\theta + d_i, x(t)))$ for $t \in [\tau_i, \tau_{i+1})$, $i = 0, \ldots, N - 1$, $g(x(t), k(\theta + d_i, x(t)))d_{i+1} = 0$ for all $t \in [\tau_i, \tau_{j+1})$, $i = 0, \ldots, N - 1$ and $j = 0, \ldots, i$, is the identically zero mapping, i.e., $x(t) = 0$ for all $t \in [0, \tau_N]$."

Assumption (H3) was used in [23] in order to guarantee finite-time identification of the parameters. However, in what follows we will not employ Assumption (H3) and consequently we will not be able to guarantee finite-time identification. Neither Assumption (H3) nor finite-time identification are encountered in conventional adaptive control.

III. EVENT-TRIGGERED IDENTIFIER FOR A CERTAINTY-EQUIVALENCE ADAPTIVE CONTROLLER

In this section, we introduce the adaptive control law. The reader interested in a quick access to the adaptive controller may immediately refer to (4), (5), (7), (8), (18), and then, resume reading the rest of this section for explanations.

The control action between two consecutive events is governed by the nominal feedback $u = k(\theta, x)$ with the unknown $\theta \in \Theta$ replaced by its estimate $\hat{\theta}$. Moreover, the estimate $\hat{\theta}$ of the unknown $\theta \in \Theta$ is kept constant between two consecutive events. In other words, we have

$$u(t) = k(\hat{\theta}(\tau_i), x(t)), \quad t \in [\tau_i, \tau_{i+1}), \quad i \in \mathbb{Z}_+$$

$$\dot{\hat{\theta}}(t) = \hat{\theta}(\tau_i), \quad t \in [\tau_i, \tau_{i+1}), \quad i \in \mathbb{Z}_+$$

where $\{\tau_i \geq 0\}_{i=0}^\infty$ is the sequence of times of the events that satisfy

$$\tau_{i+1} = \min(\tau_i + T, r_i), \quad i \in \mathbb{Z}_+, \quad \tau_0 = 0,$$

where $T > 0$ is a positive constant (one of the tunable parameters of the proposed scheme) and $r_i > \tau_i$ is a time instant determined by the event trigger. Let $a \in C^d (\mathbb{R}^n; \mathbb{R}_+)$ be a positive definite function (the second tunable parameter of the proposed scheme). The event trigger sets $r_i > \tau_i$ to be the smallest time $t > \tau_i$ for which

$$V(\hat{\theta}(\tau_i), x(t)) = V(\hat{\theta}(\tau_i), x(\tau_i)) + a(x(\tau_i)),$$

where $x(t)$ denotes the solution of (1) with $u(t) = k(\hat{\theta}(\tau_i), x(t))$. For the case that $t > \tau_i$ satisfying (6) does not exist, we set $r_i = +\infty$. For the case $x(\tau_i) = 0$, we set $r_i := \tau_i + T$. Formally, the event trigger is described by the equations

$$r_i := \inf \left\{ t > \tau_i : V(\hat{\theta}(\tau_i), x(t)) = V(\hat{\theta}(\tau_i), x(\tau_i)) + a(x(\tau_i)) \right\}$$

for $x(\tau_i) \neq 0$, and

$$r_i := \tau_i + T, \quad \text{for} \ x(\tau_i) = 0.$$
In order to estimate the unknown vector \( \theta \in \Theta \), we develop the BaLSI. Notice that (by virtue of (1)) for every \( t, s \geq 0 \), the following equation holds:
\[
x(t) - x(s) = \int_{s}^{t} f(x(r), u(r))dr + \left( \int_{s}^{t} g(x(r), u(r))dr \right) \theta.
\] (9)

Define for every \( i \in Z_+ \) the function \( h_i : \mathbb{R}^l \rightarrow \mathbb{R}_+ \) by the formula
\[
h_i(\theta) := \int_{0}^{\tau_i+1} \int_{0}^{\tau_i+1} |p(t, s) - q(t, s)\theta|^2 ds dt,
\] where
\[
p(t, s) := x(t) - x(s) - \int_{s}^{t} f(x(r), u(r))dr, \quad q(t, s) := \int_{s}^{t} g(x(r), u(r))dr.
\] (10)

It follows from (9) and (10) that for every \( i \in Z_+ \), the function \( h_i(\theta) \) has a global minimum at \( \theta = 0 \) with \( h_i(0) = 0 \). Consequently, we get from Fermat’s theorem for extrema that the following equation holds:
\[
Z(\tau_{i+1}) = G(\tau_{i+1})\theta,
\] (13)
where
\[
G(\tau) := \left( \int_{0}^{\tau} \int_{0}^{\tau} q'(t, s)q(t, s)ds dt \right) \in \mathbb{R}^{l \times l},
\] and
\[
Z(\tau) := \left( \int_{0}^{\tau} \int_{0}^{\tau} q'(t, s)p(t, s)ds dt \right) \in \mathbb{R}^l.
\] (15)

It should be noticed that the matrix \( G(\tau) \) is symmetric and positive semidefinite. Consequently, if \( G(\tau_{i+1}) \in \mathbb{R}^{l \times l} \) is invertible (i.e., \( \det (G(\tau_{i+1})) \neq 0 \)), then \( G(\tau_{i+1}) \in \mathbb{R}^{l \times l} \) is positive definite with \( \det (G(\tau_{i+1})) > 0 \) and
\[
\theta = (G(\tau_{i+1}))^{-1} Z(\tau_{i+1}).
\] (16)

The estimate (16) is nothing else but the least-squares estimate of the unknown vector \( \theta \in \mathbb{R}^l \) on the interval \([0, \tau_{i+1}]\). In the general case, the following convex optimization problem with linear equality constraints:
\[
\min_{\theta \in \Theta} \left\| \theta - \hat{\theta}(\tau_i) \right\|^2
\] s.t. \( Z(\tau_{i+1}) = G(\tau_{i+1})\theta \) (17)
has a unique solution. We can, therefore, define the following parameter update law:
\[
\hat{\theta}(\tau_{i+1}) = \arg \min \left\{ \left\| \theta - \hat{\theta}(\tau_i) \right\|^2 : \theta \in \Theta, Z(\tau_{i+1}) \right\} = G(\tau_{i+1})\theta
\] (18)
which is the BaLSI. It should also be noticed that the operator involved in (18) is not a continuous operator. However, in practice an accurate continuous approximation of the parameter update law (18) may be used.

**Remark III.1:** (a) The parameter update law (18) (the BaLSI) is the key difference of the proposed scheme and the scheme in [23]. More specifically, in [23], the least-squares identifier used a parameter update law of the form (18) for which the lower limits of the integrals appearing in (18) were not necessarily zero. This feature enables the identifier to use all available information up to the current value of time for the estimation of the parameters. On the other hand, in [23], only the most recent information were fed to the identifier.

(b) In order to avoid the discontinuity that arises at \( x(\tau_i) = 0 \) in the application of the event trigger, the following formula can be used for the practical implementation of the event trigger:
\[
r_i := \inf \left\{ t > \tau_i : V(\hat{\theta}(\tau_i), x(t)) > V(\hat{\theta}(\tau_i), x(\tau_i)) + a(x(\tau_i)) + \epsilon \right\}
\] (19)
for all \( x(\tau_i) \in \mathbb{R}^n \), where \( \epsilon > 0 \) is a constant. Selecting \( \epsilon > 0 \) in an appropriate way may exclude the possibility of Zeno behavior even in the presence of perturbations (larger perturbations will require larger \( \epsilon > 0 \)). However, this paper focuses only on the presentation of the adaptive scheme for nonlinear systems without any external perturbations.

(c) The BaLSI can be implemented by a set of ODEs. It should be noticed that if the parameter \( \theta \) appears only in some differential equations, say \( j \leq n \) equations, then there is no point of integrating the differential equations that contain no information for \( \theta \). If \( \theta \) appears in the differential equations that give \( \dot{x}_k, \ldots, \dot{x}_k \), where \( k_1, \ldots, k_j \in \{1, \ldots, n\} \), let \( C = \{c_{p,q} : p = 1, \ldots, j, q = 1, \ldots, n\} \in \mathbb{R}^{j \times n} \) be the matrix with \( c_{1,k_1} = c_{2,k_2} = \cdots = c_{j,k_j} = 1 \) and \( c_{p,q} = 0 \) if otherwise. In this case, an implementation of the parameter update law (18) is given by the following ODEs:
\[
\dot{z} = C_\theta x, \quad z \in \mathbb{R}^l
\]
\[
B = t (Cg(x,u))', \quad B \in \mathbb{R}^{l \times j}
\]
\[
\dot{w} = (Cg(x,u))' (z + \phi) + \bar{B} \bar{C} f(x,u), \quad w \in \mathbb{R}^l
\]
\[
\dot{\phi} = t\bar{C} f(x,u), \quad \phi \in \mathbb{R}^j
\]
\[
\dot{Y} = 2(B \bar{C} x - w), \quad Y \in \mathbb{R}^l
\]
\[
\dot{Q} = 2(B \bar{C} g(x,u))' + 2B \bar{C} g(x,u), \quad Q \in \mathbb{R}^{l \times l}
\] (20)
with initial conditions \( z(0) = \phi(0) = 0 \), \( Q(0) = \dot{Q}(0) = 0 \), \( B(0) = 0 \), \( Y(0) = w(0) = 0 \). The parameter update law (18) is given by
\[
\hat{\theta}(\tau_{i+1}) = \arg \min \left\{ \left\| \theta - \hat{\theta}(\tau_i) \right\|^2 : \theta \in \Theta, Y(\tau_{i+1}) \right\} = Q(\tau_{i+1})\theta
\] (21)
and notice that
\[
Y(t) = \int_{0}^{t} \int_{0}^{t} q'(r,s)\bar{C'} p(r,s)ds dr
\]
\[
Q(t) = \int_{0}^{t} \int_{0}^{t} q'(r,s)\bar{C'} \bar{C} q(r,s)dr ds
\]
Moreover, using the fact that $Q \in \mathbb{R}_{+}^{l \times l}$ is symmetric, it is possible to use only $l(l+1)/2$ from the $l^2$ second-order ODEs $\hat{Q} = 2(\hat{C}g(x,u))B' + 2B\hat{C}g(x,u)$. However, in many cases, the structure of the control system (1) allows a large reduction of the number of ODEs that are needed for the implementation of the parameter update law given by (18), by selecting the matrix $\hat{C} \in \mathbb{R}^{l \times n}$ in an appropriate way.

It is straightforward to show (using (20)) that if local exponential regulation of $x$ is achieved, then the variables $z \in \mathbb{R}^l$, $B \in \mathbb{R}_{+}^{l \times l}$, $w \in \mathbb{R}^l$, $\phi \in \mathbb{R}^l$ remain bounded for all $t \geq 0$. Moreover, in this case, it may be shown that the entries of $Y$ and $Q$ are bounded by a linear function of time. For practical operation, the system (20) may be reinitiated frequently in order to keep the entries of the matrix $Q \in \mathbb{R}_{+}^{l \times l}$ small (and consequently, the components of the vector $Y \in \mathbb{R}^l$, since we always have $Y(t) = Q(t)\theta$ when $Q \in \mathbb{R}_{+}^{l \times l}$ is nonsingular; in this case, the exact value of $\theta \in \mathbb{R}^l$ has been found and the updates are used only in case that a change of the parameter values occurs.

In practice, it is better to avoid the implementation of the parameter update law (18); due to the presence of modeling and measurement errors, the equation $Y(\tau_{i+1}) = Q(\tau_{i+1})\theta$ (guaranteed by (13) when modeling and measurement errors are absent) may not hold. Therefore, there is no guarantee that the set $\{ \theta \in \Theta : Y(\tau_{i+1}) = Q(\tau_{i+1})\theta \}$ is nonempty. Consequently, we may need to relax the minimization problem (21) and use the following parameter update law instead of (21):

$$\hat{\theta}(\tau_{i+1}) = \arg \min \left\{ \| \theta - \hat{\theta}(\tau_i) \|^2 + \beta |Y(\tau_{i+1}) - Q(\tau_{i+1})\theta|^2 : \theta \in \Theta \right\}$$

where $\beta > 0$ is a large positive constant. The parameter update law (22) has additional advantages compared to (21), since (22) introduces a regularization effect. To see this, notice that when $\Theta = \mathbb{R}^l$, (22) gives $\hat{\theta}(\tau_{i+1}) = (\beta^{-1}I + Q^2(\tau_{i+1}))^{-1}(\beta^{-1}\hat{\theta}(\tau_i) + Q(\tau_{i+1})Y(\tau_{i+1}))$ where (due to the fact that $Q(\tau_{i+1})$ is a symmetric and positive semidefinite matrix) the matrix $\beta^{-1}I + Q^2(\tau_{i+1})$ is always positive definite.

\section{IV. Statements of Stability Results}

We consider the plant (1) with the controller (4), (5), (7), (8), and the parameter estimator (18). The first main result guarantees global regulation of $x$ to zero.

\textbf{Theorem IV.1:} Consider the control system (1) under assumptions (H1), (H2). Let $T > 0$ be a positive constant and let $a : \mathbb{R}^n \to \mathbb{R}^+$ be a continuous, positive definite function. Then, there exists a family of $KL$ mappings $\omega_{\theta, \hat{\theta}} \in KL$ parameterized by $\theta, \hat{\theta} \in \Theta$ such that for every $\theta \in \Theta$, $x_0 \in \mathbb{R}^n$, $\bar{\theta} \in \Theta$, the solution of the hybrid closed-loop system (1) with (4), (5), (7), (8), (18), and initial conditions $x(0) = x_0$, $\hat{\theta}(0) = \bar{\theta}$ is unique, is defined for all $t \geq 0$, and satisfies $|x(t)| \leq \omega_{\theta, \hat{\theta}}(|x_0|, t)$ for all $t \geq 0$. Moreover, there exist $\tau \geq 0$, $\bar{\theta} \in \Theta$ (both depending on $\Theta$, $x_0 \in \mathbb{R}^n$, $\bar{\theta} \in \Theta$) such that $\hat{\theta}(t) = \bar{\theta}$ for all $t \geq \tau$ and the equation $g(x(t), u(t)) = 0$ holds for all $t \geq 0$. Furthermore, the estimate $\tau_i \geq (i - l)T$ holds for all $i \geq l$. Finally, under assumption (H3) and $x_0 \neq 0$, then $\hat{\theta}(t) = \theta$ for all $t \geq NT$.

\textbf{Remarks on Theorem IV.1:} (a) Theorem IV.1 guarantees that there is a finite settling time $\tau \geq 0$ for the parameter estimate $\hat{\theta}(t) \in \Theta$. Unfortunately, an upper bound of the settling time cannot be provided. (b) It is important to notice that no assumption for persistency of excitation is made in Theorem IV.1. (c) The proof of Theorem IV.1 shows that at most $l$ switchings of the parameter estimate $\hat{\theta}(t)$ occur. Moreover, the estimate $\tau_i \geq (i - l)T$ for all $i \geq l$, indicates that the times of the events cannot have a finite accumulation point (Zeno behavior).

The second main result guarantees local and global exponential regulation of $x$ to zero under the assumption that nominal feedback law $u = k(\theta, x)$ achieves local and global exponential stabilization, respectively.

\textbf{Theorem IV.2:} Consider the control system (1) under assumptions (H1), (H2). Moreover, suppose that for every nonempty, compact set $\Theta \subseteq \Theta$, there exist constants $R \in (0, +\infty)$, $K_1 > K_2 > 1$ such that

$$K_1|x|^2 \leq V(\theta, x) \leq K_2|x|^2$$

for all $x \in \mathbb{R}^n$, $\theta \in \Theta$ with $|x| \leq R$. Suppose also that for each $\theta \in \Theta$, $0 \in \mathbb{R}^n$ is locally exponentially stable (LES) for the closed-loop system (2). Let $T > 0$ be a positive constant and let $a : \mathbb{R}^n \to \mathbb{R}^+$ be a continuous, positive definite function that satisfies $\sup \{|x|^{-2}a(x) : x \in \mathbb{R}^n, x \neq 0, |x| \leq \delta \} < +\infty$ for certain $\delta \in (0, +\infty)$. Then, there exists a family of constants $M_{\theta, \hat{\theta}} > 0$, $R_{\theta, \hat{\theta}} \in (0, +\infty)$, parameterized by $(\theta, \hat{\theta}) \in \Theta \times \hat{\Theta}$, such that, for every $\theta \in \Theta$, $x_0 \in \mathbb{R}^n$, $\hat{\theta} \in \Theta$ with $|x_0| < R_{\theta, \hat{\theta}}$, the solution of the hybrid closed-loop system (1) with (4), (5), (7), (8), (18), and initial conditions $x(0) = x_0$, $\hat{\theta}(0) = \hat{\theta}$ satisfies the estimate $|x(t)| \leq M_{\theta, \hat{\theta}}(\exp(-\sigma t)|x_0|$ for all $t \geq 0$, with $\sigma > 0$ being the constant involved in (3). If $R = +\infty$, $\delta = +\infty$, and $0 \in \mathbb{R}^n$ is globally exponentially stable (GES) for the closed-loop system (2), then $R_{\theta, \hat{\theta}} = +\infty$.

It should be noticed that Theorem IV.2 guarantees that the local exponential stability estimate $|x(t)| \leq M_{\theta, \hat{\theta}}(\exp(-\sigma t)|x_0|$ holds when $|x_0| \leq R_{\theta, \hat{\theta}}$ and for arbitrary initial condition $\hat{\theta}_0 \in \Theta$. In other words, the adjective “local” refers only to $x$ and not to $\theta$. Moreover, as pointed out in Section I, the reader should notice that the event-triggered adaptive scheme (4), (5), (7), (8), (18) guarantees convergence with the same convergence rate $\sigma > 0$ as the nominal feedback controller with known parameter values. This is not possible for conventional adaptive control; in conventional adaptive control, there is no uniform exponential convergence rate for all initial conditions. The uniform exponential convergence rate is achieved by estimating the parameter vector in an appropriate
way using the least-squares identifier. Therefore, the solution of the adaptive closed-loop system coincides with the solution of (2) with \( \theta \) replaced by \( \theta_{\ast} \), i.e., the solution of the closed-loop system (1) with \( u = k(\theta, x) \) and known parameter values.

Finally, it should be emphasized that in addition to the exponential regulation estimate \( |x(t)| \leq M_{\theta, \delta_{0}} \exp(-\sigma t) |x_{0}| \), Theorem IV.2 guarantees all the conclusions of Theorem IV.1 (because all assumptions of Theorem IV.1 are fulfilled).

Finally, the following corollary deals with the case of controllable linear time-invariant (LTI) single-input systems with unknown parameters.

**Corollary IV.3**: Consider the system

\[
\dot{x}_i = \sum_{j=1}^{n} \theta_{i,j} x_j + \theta_{i,i+1} x_{i+1}, \quad i = 1, \ldots, n
\]

\[
x = (x_1, \ldots, x_n) \in \mathbb{R}^n, \quad u = x_{n+1} \in \mathbb{R}
\]

\[
\theta = (\theta_{1,1}, \theta_{1,2}, \ldots, \theta_{n,n-1})' \in \mathbb{R}^n
\]

\[
\Theta = \{ \theta \in \mathbb{R}^n : \theta_{i,i+1} \geq \kappa, \quad i = 1, \ldots, n \}
\]

where \( l = \frac{n(n+3)}{2} \) and \( \kappa > 0 \) is a constant. Then, for every \( \sigma > 0, \theta \in \Theta \), there exists a symmetric, positive definite matrix \( P(\theta) = \{ p_{i,j}(\theta) : i, j = 1, \ldots, n \} \in \mathbb{R}^{n \times n} \) and a vector \( k(\theta) \in \mathbb{R}^n \) such that the inequality

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} p_{i,j}(\theta) x_i x_j \leq -\sigma \sum_{i=1}^{n} \sum_{j=1}^{n} p_{i,j}(\theta) x_i x_j
\]

with \( u = k(\theta, x) \) holds for all \( x \in \mathbb{R}^n \). Moreover, the mappings \( \Theta \ni \theta \rightarrow P(\theta) \in \mathbb{R}^{n \times n}, \Theta \ni \theta \rightarrow k(\theta) \in \mathbb{R}^n \) are continuous. Finally, for every \( T > 0, A > 0 \), there exists a family of constants \( M_{\theta, \delta} > 0 \) parameterized by \( \theta \in \Theta, \theta \in \Theta \), such that for all \( \theta \in \Theta, x_{0} \in \mathbb{R}^n, \theta_{0} \in \Theta \) the solution of the hybrid closed-loop system (1) with \( f(x, u) = 0 \)

\[
g(x, u) \theta := \left[ \begin{array}{c}
\theta_{1,1} x_1 + \theta_{1,2} x_2 \\
\vdots \\
\theta_{n,1} x_1 + \cdots + \theta_{n,n} x_n + \theta_{n,n+1} u
\end{array} \right]
\]

\[
a(x) := A|x|^2, \quad V(\theta, x) := x' P(\theta) x, \quad k(\theta, x) = k(\theta, x) \]

(4), (5), (7), (8), (18), and initial conditions \( x(0) = x_{0}, \theta(0) = \theta_{0} \) satisfies the estimate \( |x(t)| \leq M_{\theta, \delta} \exp(-\sigma t) |x_{0}| \) for all \( t \geq 0 \).

**V. ADAPTIVE CONTROL OF THE WING-ROCK MODEL**

The wing-rock model proposed and used in [27] and [35] with zero torque at equilibrium is given by the system

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = \theta_1 x_1 + \theta_2 x_2 + \theta_3 |x_1| x_2 + \theta_4 |x_2| x_2 + \theta_5 x_3
\]

\[
\dot{x}_3 = -\mu x_3 + \mu u
\]

\[
x = (x_1, x_2, x_3) \in \mathbb{R}^3, \quad u \in \mathbb{R}
\]

\[
\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)' \in \Theta = \mathbb{R}^4 \times [\kappa, +\infty)
\]

where \( \kappa > 0 \) and \( \mu > 0 \) are known parameters. A locally Lipschitz nonlinear feedback law that achieves local exponential stabilization and global asymptotic stabilization of the equilibrium point \( 0 \in \mathbb{R}^3 \) is given by the formula

\[
k(\theta, x) = x_3 - \mu^{-1} \theta_5 (x_2 + L x_1) + \mu^{-1} \frac{\partial \varphi}{\partial x_1} (\theta, x_1, x_2) x_2
\]

\[
+ \mu^{-1} \frac{\partial \varphi}{\partial x_2} (\theta, x_1, x_2) (\theta_1 x_1 + \theta_2 x_2 + \theta_3 |x_1| x_2
\]

\[
+ \theta_4 |x_2| x_2 + \theta_5 x_3)
\]

\[
- \mu^{-1} L (x_3 - \varphi(\theta, x_1, x_2))
\]

(26)

where \( L > 1 \) is a constant and

\[
\varphi(\theta, x_1, x_2) := -\theta_5^{-1} ((1 + \theta_1) x_1 + \theta_2 x_2)
\]

\[
- \theta_5^{-1} \left( L + \vec{\beta}(\theta, x_1, x_2) + \frac{L^2}{4} \left( \vec{\beta}(\theta, x_1, x_2) \right)^2 \right)
\]

\[
\times (x_2 + L x_1)
\]

(27)

\[
\vec{\beta}(\theta, x_1, x_2) := 1 + \frac{1}{2} \theta_3^2 x_1^2 + \frac{1}{2} \theta_2^2 x_2^2
\]

(28)

More specifically, the feedback law \( u = k(\theta, x) \) guarantees the differential inequality

\[
\dot{V}(\theta, x) \leq -2(L - 1) V(\theta, x)
\]

(29)

for all \( \theta \in \Theta, x \in \mathbb{R}^3 \) for the control Lyapunov function

\[
V(\theta, x) := \frac{1}{2} x_2^2 + \frac{1}{2} (x_2 + L x_1)^2 + \frac{1}{2} (x_3 - \varphi(\theta, x_1, x_2))^2.
\]

(30)

Therefore, Assumption (H1) and Assumption (H2) hold with \( \sigma := L - 1 \). Moreover, for every nonempty, compact set \( \Theta \subset \Theta \), there exist constants \( R > 0, K_2 > K_1 > 0 \) such that (23) holds. Therefore, all assumptions of Theorem IV.2 hold for system (25). Thus, for every \( \kappa, \mu, T > 0, A > 0 \), \( \alpha \in C^0(\mathbb{R}^3, \mathbb{R}_+) \) being a positive definite function that satisfies \( \sup_{x \in \mathbb{R}^3} |x|^2 a(x) : x \in \mathbb{R}^3, \) \( x \neq 0, |x| \leq \delta < +\infty \) for certain \( \delta > 0 \), there exists a family of constants \( M_{\theta, \delta} > 0 \) parameterized by \( \theta \in \Theta \times \Theta \), such that for every \( \theta \in \Theta, x_{0} \in \mathbb{R}^3, \theta_{0} \in \Theta \) with \( |x_{0}| \leq R(\theta, \theta_{0}) \) the solution of the hybrid closed-loop system (25) with (4), (5), (7), (8), (18), and initial conditions \( x(0) = x_{0}, \theta(0) = \theta_{0} \) satisfies the estimate \( |x(t)| \leq M_{\theta, \delta} \exp(-\sigma t) |x_{0}| \) for all \( t \geq 0 \), with \( \sigma := L - 1 > 0 \) and \( \Theta = \mathbb{R}^4 \times [\kappa, +\infty) \).

We studied system (25) numerically with

\[
\theta_1 = -26.67, \quad \theta_2 = 0.76485, \quad \theta_3 = -2.9225, \quad \theta_4 = 0, \quad \theta_5 = 1.5
\]

\[
\mu = 15, \quad \kappa = 1
\]

which are exactly the same parameter values used in [27]. Notice that \( 0 \in \mathbb{R}^3 \) is unstable for the open-loop system (25) with \( u \equiv 0 \) and the solution is attracted by a limit cycle (the wing-rock phenomenon; see [27]). The controller parameters were selected as follows:

\[
L = 1.5, \quad T = 0.4, \quad a(x) := 2 \cdot 10^5 |x|^2 + |x|^4
\]

for \( x \in \mathbb{R}^3 \) and following Remark III.1(b) the hybrid adaptive controller was implemented by using (22) with \( \beta = 10^{17} \) and the set of
ODEs
\[
\dot{z} = x_2
\]
\[
\dot{Y}_i = 2(b_i x_2 - w_i), \quad i = 1, \ldots, 5
\]
\[
\dot{b}_i = t \zeta_i(x)
\]
\[
\dot{w}_i = z \zeta_i(x), \quad i = 1, \ldots, 5
\]
\[
\dot{Q}_{i,j} = 2(b_j \zeta_i(x) + b_i \zeta_j(x)), \quad j = 1, \ldots, i, i = 1, \ldots, 5
\]
with initial conditions \(z(0) = Y_i(0) = b_i(0) = w_i(0) = Q_{i,j}(0) = 0\) for \(i, j = 1, \ldots, 5\), where
\[
\zeta_1(x) := x_1, \quad \zeta_2(x) := x_2, \quad \zeta_5(x) := x_3,
\]
\[
\zeta_3(x) := |x_1| x_2, \quad \zeta_4(x) := |x_2| x_2
\]
for \(x \in \mathbb{R}^3\).

In order to compare the performance of the closed-loop system (25) with (4), (5), (7), (8), (22), we also used the adaptive controller based on the extended matching design studied for general nonlinear systems in [27, Chs. 3 and 4]. More specifically, the extended matching design (combined with the projection schemes explained in [27, Appendix E]) gives the following adaptive controller:
\[
\frac{d}{dt} \hat{\theta}_i = \gamma \frac{\partial V}{\partial x_2}(\hat{\theta}, x) \zeta_i(x), \quad i = 1, \ldots, 4
\]
\[
\frac{d}{dt} \hat{\theta}_5 = \gamma \frac{\partial V}{\partial x_2}(\hat{\theta}, x) \zeta_5(x)
\]
\[
\begin{cases}
1, \text{ if } \hat{\theta}_5 \geq \kappa \text{ or } \frac{\partial V}{\partial x_2}(\hat{\theta}, x) \zeta_5(x) \geq 0 \\
1 - \min \left(1, e^{-1}(\kappa - \hat{\theta}_5)\right), \text{ if otherwise}
\end{cases}
\]
\[
u = k(\hat{\theta}, x) + \mu \gamma \frac{\partial V}{\partial x_2}(\hat{\theta}, x) \frac{\partial \varphi}{\partial \theta}(\hat{\theta}, x_1, x_2) \zeta(x)
\]
where \(V\) is given by (30), \(k\) is given by (26), (27), (28), with \(L = 1.5, \gamma > 0\) is a constant (the adaptation gain)
\[
\zeta(x) := (\zeta_1(x), \zeta_2(x), \zeta_3(x), \zeta_4(x), \zeta_5(x))^T
\]
for \(x \in \mathbb{R}^3\) and \(\varepsilon \in (0, \kappa)\) is the constant for which the inequality \(\hat{\theta}_5(t) \geq \kappa - \varepsilon\) is guaranteed to hold for all \(t \geq 0\). The adaptation gain was selected \(\gamma = 10\) and \(\varepsilon \in (0, \kappa)\) was selected to be equal to 0.001.

We compared the performance for many initial conditions and the representative results are presented next. Although, we are not able to verify Assumption (H3) for system (25) with nominal controller given by (26), finite-time exact estimation of the parameters was achieved for all tested initial conditions in a few (two or three) triggers.

The results shown in Figs. 1–9 are obtained by using two different initial conditions for system (25):
1) first initial condition: \(x_0 = (-0.35, -0.5, 0.05)^T\);
2) second initial condition: \(x_0 = (0.4, 0, 0)^T\).

The second initial condition is exactly the initial condition for which numerical results are presented in [27]. In all cases, the initial condition for the parameter estimates is (as in [27])
\[
\hat{\theta}_0 = 1.35\theta.
\]

Figs. 1 and 2 show the projected trajectories of the solutions on the plane for the first and second initial condition, respectively, when no measurement errors are present. It is clearly shown that the trajectories of the solutions of the closed-loop system (25) with the event-triggered adaptive scheme (4), (5), (7), (8), (22) differ significantly from the solutions of the closed-loop system with the extended-matching design, mainly because the parameter estimates for the extended-matching design fail to converge to the actual values of the parameters, as shown in Fig. 8. In contrast, the trajectories of the closed-loop system with the event-triggered adaptive scheme are similar to the trajectories of the nominal closed-loop system (25) with (26) due to the fact that the exact values of the parameters are found after a short initial transient period, as indicated by Fig. 7.

The design based on the extended matching presents the smallest overshoot in the \(x_1, x_2\) state components for both initial conditions as shown in Figs. 1 and 2, when no measurement
errors are present. However, this is not the case for the entire state vector. Figs. 3 and 4 present the evolution of the norm of the state vector \( |x(t)| \) for \( t \in [0, 4] \) and for both initial conditions, when no measurement errors are present. It is clear that the closed-loop system with the extended matching design exhibits very sharp overshoots for an initial transient period. Figs. 5 and 6 show the reason that explains this phenomenon (for the second initial condition but this holds for both initial conditions); the spikes occur at the times when \( \hat{\theta}_4(t) \) takes values close to the lowest allowable limit \((\kappa - \varepsilon = 0.999)\). During this transient period, the value of \( \hat{\theta}_4(t) \) changes and assumes a stabilizing value (around 0.15), which allows the termination of the transient period.

We also studied the effect of measurement errors, i.e., the case where we measure

\[
\hat{x}(t) = x(t) + e(t) \tag{38}
\]

where \( e(t) \in \mathbb{R}^3 \) is the measurement error. In this case the nominal feedback becomes \( u(t) = k(\theta, \hat{x}(t)) \), the classical extended-matching adaptive controller is given by (36) with \( x \) replaced by \( \hat{x} \) and the proposed event-triggered adaptive scheme is given by (4), (5), (7), (8), (22)–(34) with \( x \) replaced by \( \hat{x} \). We used \( e(t) = 0.01 \sin(14\pi t) \cdot (1, 1, 1)' \tag{39} \) for \( t \geq 0 \) and the results are shown in Figs. 9–12. It is clearly seen by Fig. 11 that the parameter estimation process by the BaLSI presents robustness with respect to measurement errors;
the identifier manages to bring the parameter estimates very close to the exact parameter values in a few triggers even in the presence of measurement errors. However, Figs. 9 and 10 show that the overshoot exhibited by the proposed event-triggered adaptive scheme given by (4), (5), (7), (8), (22), (31)–(34) is larger when measurement errors are present due to the delayed convergence of the parameter estimates. 

Figs. 9, 10, and 12 show that the behavior of the closed-loop system with the classical extended-matching adaptive controller in the presence of measurement errors does not differ significantly from the behavior in the absence of measurement errors.

The numerical results allow us to conclude that the proposed event-triggered adaptive scheme (4), (5), (7), (8), (22) exhibits exponential convergence of the state to zero and exact finite-time estimation of the unknown parameters (at least for all tested initial conditions and when measurement errors are absent) as well as robustness with respect to small measurement errors. However, these features come at a cost—the computational effort and the memory requirements for the implementation of the event-triggered adaptive scheme (4), (5), (7), (8), (22) are significantly larger than those of the extended-matching design. The implementation of the proposed event-triggered adaptive scheme (4), (5), (7), (8), (22) requires 46 additional first-order ODEs to be solved in parallel to the three ODEs of the system, while the extended matching design requires only five additional first-order ODEs.

VI. PROOF OF RESULTS

We start this section with the proof of Theorem IV.1.

Proof of Theorem IV.1: The first claim is a direct consequence of the event trigger given by (7) and (8). The proof of the first claim is straightforward and is omitted.

Claim 1: If a solution \((x(t), \hat{\theta}(t)) \in \mathbb{R}^n \times \Theta\) of the closed-loop system (1) with (4), (5), (7), (8), and (18) is defined on \(t \in [0, \tau_i]\) for certain \(i \in \mathbb{Z}_+\), then the solution is defined on \(t \in [0, \tau_{i+1}]\). Moreover, it holds that

\[
V(\hat{\theta}(\tau_i), x(\tau_i)) \leq V(\hat{\theta}(\tau_i), x(\tau_i)) + a(x(\tau_i))
\]

for all \(t \in [\tau_i, \tau_{i+1}]\).

It follows from Claim 1 that for every initial condition, the corresponding solution \((x(t), \hat{\theta}(t)) \in \mathbb{R}^n \times \Theta\) of the closed-loop system (1) with (4), (5), (7), (8), and (18) is defined for
Claim 3 and the fact that $N(G(0)) = \mathbb{R}^{l}$ (recall definition (14)) implies that the following inclusion holds for all $i \geq 0$:

$$N(G(\tau_{i+1})) \subseteq N(G(\tau_{i})).$$

(41)

The following claim clarifies what happens when a switching in the value of the parameter estimate $\hat{\theta}(t)$ occurs.

**Claim 4:** If $\hat{\theta}(\tau_{i+1}) \neq \hat{\theta}(\tau_{i})$ then $\dim(N(G(\tau_{i+1}))) < \dim(N(G(\tau_{i})))$.

**Proof of Claim 4:** If $\hat{\theta}(\tau_{i+1}) \neq \hat{\theta}(\tau_{i})$, then it follows from (18) that $G(\tau_{i+1})(\hat{\theta}(\tau_{i+1}) \neq Z(\tau_{i+1}))$, where $Z(\tau_{i+1})$ is defined by (15). It follows from (13) and definitions (14), (18) that the vector $\xi = \theta(\tau_{i}) - \theta$ satisfies $G(\tau_{i+1})\xi \neq 0$. Moreover, it follows from (13) and definitions (14), (18) that $G(\tau_{i})\xi = 0$. Therefore, there exists a vector $\xi \in N(G(\tau_{i}))$ with $\xi \notin N(G(\tau_{i+1}))$.

We next prove by contradiction that $\dim(N(G(\tau_{i+1}))) < \dim(N(G(\tau_{i})))$. Suppose that $\nu = \dim(N(G(\tau_{i+1}))) \geq \bar{\nu} = \dim(N(G(\tau_{i}))) > 0$ (the case $\bar{\nu} = 0$ can be excluded by Claim 2 and (13), (18) which would imply that $\hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_{i}) = \theta$. Let $\{\xi_{1}, \ldots, \xi_{\bar{\nu}}\}$ be a basis for $N(G(\tau_{i}))$, and let $\{w_{1}, \ldots, w_{\nu}\}$ be a basis for $N(G(\tau_{i+1}))$. At least one of the vectors $\xi_{1}, \ldots, \xi_{\bar{\nu}}$ must not belong to $N(G(\tau_{i+1}))$ because otherwise we would have $N(G(\tau_{i})) \subseteq N(G(\tau_{i+1}))$, which contradicts the existence of a vector $\xi \in N(G(\tau_{i}))$ with $\xi \notin N(G(\tau_{i+1}))$. Let $j \in \{1, \ldots, \bar{\nu}\}$ with $\xi_{j} \notin N(G(\tau_{i+1}))$. Notice that the set of $\nu + 1 \geq \bar{\nu} + 1$ vectors $\{w_{1}, \ldots, w_{\nu}, \xi_{j}\}$ is a linearly independent set of vectors in $N(G(\tau_{i}))$, contradicting the assumption that $\bar{\nu} = \dim(N(G(\tau_{i})))$. The proof of Claim 4 is complete.

A direct consequence of Claim 4 is the fact that at most $l$ switchings of the value of the parameter estimate $\hat{\theta}(t)$ can occur in the time interval $[\hat{\theta}(0), \hat{\theta}(\tau_{i})]$. Moreover, Assumption (H2) guarantees that for each fixed $s \geq 0$, $\theta \in \Theta$, and the mapping $x \rightarrow \tilde{V}(x; \theta, s)$ is radially unbounded. Consequently, [22, Prop.
2.2, p. 107) implies that for each fixed \( s \geq 0, \theta \in \mathbb{R}^l \), there exist functions \( a_{\theta,s} \in K_{\infty}, \beta_{\theta,s} \in K_{\infty} \) such that
\[
a_{\theta,s}(|x|) \leq V(\theta;x,s), \beta_{\theta,s}(|x|) \geq W(\theta;x,s)
\]
(46)
for all \( x \in \mathbb{R}^n \). For any given \( s \geq 0, \theta \in \Theta \), define the KL function
\[
\omega_{\theta,s}(r,t) := a_{\theta,s}^{-1}(\exp(-2\sigma t)\beta_{\theta,s}(r))
\]
(47)
for \( t, r \geq 0 \). The following claim clarifies what happens when \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \).

**Claim 5:** If \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) then
\[
\nabla V(\hat{\theta}(\tau_i), x(t)) \dot{x}(t) \leq -2\sigma V(\hat{\theta}(\tau_i), x(t))
\]
(48)
for all \( t \in [\tau_i, \tau_{i+1}] \). Using Claim 1, the fact that \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) follows from the differential inequality (48) and the fact that the mapping \( t \to V(\hat{\theta}(\tau_i), x(t)) \) is continuous. Finally, (7) and the fact that the event trigger is not activated and thus, (5), (8) give \( \tau_{i+1} = \tau_i + T \). The proof of Claim 5 is complete.

Using estimate (44) and definitions (45), we are in a position to rephrase the above statements as follows.

1) If \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) then
\[
V(\hat{\theta}(\tau_i), x(t)) \leq \exp(-2\sigma(t - \tau_i))V(\hat{\theta}(\tau_i), x(\tau_i))
\]
for all \( t \in [\tau_i, \tau_{i+1}] \), with \( \tau_{i+1} = \tau_i + \mu T \).

2) If \( \hat{\theta}(\tau_{i+1}) \neq \hat{\theta}(\tau_i) \) then
\[
V(\hat{\theta}(\tau_i), x(t)) \leq e^{-\sigma(t-\tau_i)}e^{2\sigma \mu T} \left( V(\hat{\theta}(\tau_i), x(\tau_i)) + a(x(\tau_i)) \right)
\]
for all \( t \in [\tau_i, \tau_{i+1}] \).

Using estimate (44) and definitions (45), we are in a position to rephrase the above statements as follows.

1) If \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) then
\[
\nabla V(\hat{\theta}(\tau_i), x(t)) \dot{x}(t) \leq e^{-2\sigma(t-\tau_i)}W(\hat{\theta}(\tau_i), x(t), \hat{\theta}(0) - \theta)
\]
for all \( t \in [\tau_i, \tau_{i+1}] \), with \( \tau_{i+1} = \tau_i + \mu T \).

2) If \( \hat{\theta}(\tau_{i+1}) \neq \hat{\theta}(\tau_i) \) then
\[
\nabla V(\hat{\theta}(\tau_i), x(t)) \dot{x}(t) \leq e^{-2\sigma(t-\tau_i)}W(\hat{\theta}(\tau_i), x(t), \hat{\theta}(0) - \theta)
\]
for all \( t \in [\tau_i, \tau_{i+1}] \).

Finally, using (46), the above statements and definition (47), we are in a position to guarantee the following facts.

**F1** If \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) for \( j = 1, \ldots, \mu, \mu \geq 1 \), then
\[
|\dot{x}(t)| \leq \omega_{\theta,s}(x(\tau_i), t - \tau_i) \quad \text{for} \quad t \in [\tau_i, \tau_{i+1}]
\]
and
\[
\tau_{i+\mu} = \tau_i + \mu T
\]
(49)

**F2** If \( \hat{\theta}(\tau_{i+1}) \neq \hat{\theta}(\tau_i) \) then
\[
|x(t)| \leq \omega_{\theta,s}(x(\tau_i), t - \tau_i) \quad \text{for} \quad t \in [\tau_i, \tau_{i+1}]
\]
with \( s = \hat{\theta}(0) - \theta \).

Since at most \( l \) switchings of the parameter estimate value \( \hat{\theta}(t) \) can occur, Fact (F1) implies that \( \tau_i \geq (i - l)T \) for all \( i \geq 1 \). Therefore, we conclude that \( \sup_{\theta \in \Theta} \| (\tau_i) \| = +\infty \) and that every solution \( x(t), \hat{\theta}(t) \) for all \( t \in \mathbb{R}^n \times \Theta \) of the closed-loop system (1) with (4), (5), (7), (8), and (18) is defined for all \( t \geq 0 \).

Let \( H(x(0), \hat{\theta}(0), 0) \subseteq \{ \tau_1, \tau_2, \ldots \} \) be the set of all times \( \tau_i \) with \( \hat{\theta}(\tau_i) \neq \hat{\theta}(\tau_{i-1}) \). The set \( H(x(0), \hat{\theta}(0), 0) \) may be empty and can have at most \( l \) members (since at most \( l \) switchings of the value of the parameter estimate \( \hat{\theta}(t) \) can occur). By virtue of [21, Fact VI], for any given \( s \geq 0, \theta \in \Theta \), we are in a position to find a function \( \Omega_{\theta,s} \in KL \), such that
\[
\sup_{0 \leq t \leq \tau} (\omega_{\theta,s}(\hat{\theta}(\tau, r), t - \tau)) \leq \Omega_{\theta,s}(r, t)
\]
(50)
for all \( t, r \geq 0 \). Inequality (50) and Facts (F1), (F2) imply the following fact.

**F3** If \( \tau_j \in H(x(0), \hat{\theta}(0), \theta) \) and \( (\tau_i, \tau_j) \cap H(x(0), \hat{\theta}(0), \theta) \) then \( |\dot{x}(t)| \leq \Omega_{\theta,s}(x(\tau_i), t - \tau_j) \) for all \( t \in [\tau_i, \tau_j] \) with \( s = |\hat{\theta}(0) - \theta| \).

We next show the following claim.

**Claim 6:** If the cardinal number of the set \( H(x(0), \hat{\theta}(0), \theta) \) is \( \eta \in \{0, 1, \ldots, l\} \), then \( |\dot{x}(t)| \leq R_{\theta,s}(x(0), t) \) for all \( t \geq 0 \) with \( s = |\hat{\theta}(0) - \theta| \).

**Proof of Claim 6:** By virtue of Fact (F1), the claim holds if \( \eta = 0 \). Indeed, if the cardinal number of the set \( H(x(0), \hat{\theta}(0), \theta) \) is zero, i.e., if \( \hat{\theta}(\tau_{i+1}) = \hat{\theta}(\tau_i) \) for all \( i \geq 0 \), then Fact (F1) implies that \( |\dot{x}(t)| \leq \omega_{\theta,s}(x(0), t) \) for all \( t \geq 0 \) with \( s = |\hat{\theta}(0) - \theta| \). Definition (52) and the fact that \( \omega_{\theta,s}(r, t) \leq \Omega_{\theta,s}(r, t) \) for all \( t, r \geq 0 \) (a consequence of (50) and the fact that definitions (45), (47) guarantee that \( r \leq \omega_{\theta,s}(r, 0) \)) shows that the claim holds if \( \eta = 0 \).

Next, suppose that \( \eta > 0 \). Let \( T_1, \ldots, T_\eta \) be such that \( H(x(0), \hat{\theta}(0), \theta) = \{ T_1, \ldots, T_\eta \} \). By virtue of Facts (F1) and (F3) we get the following.

1) \( |\dot{x}(t)| \leq \omega_{\theta,s}(x(0), t) \) for \( t \in [0, T_1] \) with \( s = |\hat{\theta}(0) - \theta| \).

2) \( |\dot{x}(t)| \leq \omega_{\theta,s}(x(T_1), t - T_1) \) for \( t \in [T_1, T_1 + 1] \), \( i = 1, \ldots, \eta - 1 \) with \( s = |\hat{\theta}(0) - \theta| \) (this case applies only when \( \eta > 1 \)).

3) \( |\dot{x}(t)| \leq \omega_{\theta,s}(x(T_\eta), t - T_\eta) \) for \( t \geq T_\eta \) with \( s = |\hat{\theta}(0) - \theta| \).

Combining all the above cases with inequalities (51) and using the fact that \( R_{\theta,s}(r, \tau) \leq R_{\theta_{\eta+1},\theta_{\eta+1}}(r, \tau) \), for all \( t, r \geq 0 \)
0 and \(i = 0, \ldots, l - 1\), we get the desired estimate \(|x(t)| \leq R_{\theta|\theta,s}(x_0(t), t)\) for all \(t \geq 0\) with \(s = |\tilde{\theta}(0) - \theta|\). The proof of Claim 6 is complete.

Using the fact that the cardinal number of the set \(H(x(0), \tilde{\theta}(0), \theta)\) is at most \(l\) and the fact that \(R_{\theta|\theta,s}(r, t) \leq R_{\theta|\theta,s}(r, t)\) for all \(t \geq 0\) and \(i = 0, \ldots, l - 1\), we conclude that the required estimate \(|x(t)| \leq \omega_{\theta_0}(x_0, t)\) for all \(t \geq 0\) holds for the solution of the hybrid closed-loop system (1), (4), (5), (7), (8), (18), and initial conditions \(x(0) = x_0, \dot{\theta}(0) = \dot{\theta}_0\) with \(\omega_{\theta_0}(r, t) := R_{\theta|\theta,s}(r, t)\) and \(s = |\tilde{\theta}(0) - \theta|\).

Let \(\tau > 0\) be the maximum time in the set \(H(x(0), \tilde{\theta}(0), \theta)\) when \(H(x(0), \tilde{\theta}(0), \theta) \neq \emptyset\) and \(\tau = 0\) when \(H(x(0), \tilde{\theta}(0), \theta) = \emptyset\). Then, if \(\tau \neq \emptyset\), then it follows from (13), (14), and (18) that \(\xi = (\tilde{\theta}(\tau) - \theta) \in N(G(\tau_{i+1}))\) for all \(i \geq 0\) with \(\tau_{i+1} > \tau\). It follows from Claim 3 that \(q(x(t), u(t))\xi = 0\) for all \(t \geq 0\).

Finally, if the parameter observability assumption (H3) holds then we can repeat all arguments in the proof of Theorem IV.1 in [23] and show that \(\tilde{\theta}(t) = \theta\) for all \(t \geq NT\) when \(x(0) \neq 0\). The proof is complete.

We next continue with the proof of Theorem IV.2.

Proof of Theorem IV.2: Case (i): \(R, \delta < +\infty\). Since all assumptions of Theorem IV.1 hold for Theorem IV.2, the proof of Theorem IV.2 starts at the point where the proof of Theorem IV.1 ended. Therefore, all relations and everything written in the proof of Theorem IV.1 holds. Define

\[ A := \sup \left\{ |x|^2 a(x) : x \neq 0, |x| \leq \delta \right\}. \]  

Due to (23), (53), (44), and definitions (45) for every \(\theta \in \Theta\), \(s \leq 0\), there exist constants \(R^{\theta,s}_0 > 0, K_1^{\theta,s} > K_2^{\theta,s} > K_3^{\theta,s} > 0\) such that

\[ K_1^{\theta,s}|x|^2 \leq V(x; \theta, s), \quad e^{2\sigma T} \left(K_2^{\theta,s} + A\right) |x|^2 \geq W(x; \theta, s) \]  

for all \(x \in \mathbb{R}^n\) with \(|x| \leq \min(R^{\theta,s}_0, \delta)\). It follows from (54) and Facts (F1), (F2) that the following statements hold.

(S1) If \(\tilde{\theta}(\tau_{i+1}) = \tilde{\theta}(\tau_i)\) for \(j = 1, \ldots, \mu, \mu \geq 1\) and \(|x(\tau_i)| \leq \Gamma_{\theta,s}^{\text{min}}(R^{\theta,s}_0, \delta)\), where \(\Gamma_{\theta,s}^{\text{min}} := \exp(\sigma T) \left(K_1^{\theta,s} - 1 \right) K_2^{\theta,s} + A\), then \(|x(t)| \leq \Gamma_{\theta,s}^{e^{-\sigma(-\tau)}} |x(\tau_i)|\) for \(t \in [\tau_i, \tau_{i+1}]\) with \(s = |\tilde{\theta}(0) - \theta|\).

(S2) If \(\tilde{\theta}(\tau_{i+1}) \neq \tilde{\theta}(\tau_i)\) and \(|x(\tau_i)| \leq \Gamma_{\theta,s}^{\text{min}}(R^{\theta,s}_0, \delta)\), then \(|x(t)| \leq \Gamma_{\theta,s}^{e^{-\sigma(-\tau)}} |x(\tau_i)|\) for \(t \in [\tau_i, \tau_{i+1}]\) with \(s = |\tilde{\theta}(0) - \theta|\).

Facts (S1), (S2) allow us to state the following fact.

(S3) If \(\tau_i \in H(x(0), \tilde{\theta}(0), \theta, \tau_{i+1}) \cap H(x(0), \tilde{\theta}(0), \theta) = \emptyset\) and \(|x(\tau_i)| \leq \Gamma_{\theta,s}^{\text{min}}(R^{\theta,s}_0, \delta)\), then \(|x(t)| \leq \Gamma_{\theta,s}^{e^{-\sigma(-\tau)}} |x(\tau_i)|\) for \(t \in [\tau_i, \tau_{i+1}]\) with \(s = |\tilde{\theta}(0) - \theta|\).

Using the fact that the cardinal number of the set \(H(x(0), \tilde{\theta}(0), \theta)\) is at most \(l\) and facts (S1), (S3), we conclude that \(|x(t)| \leq \Gamma_{\theta,s}^{e^{-\sigma(-\tau)}} |x(0)|\) for all \(t \geq 0\), provided that \(|x(t)| \leq \Gamma_{\theta,s}^{e^{-\sigma(-\tau)}} |x(\tau_i)|\) for \(t \in [\tau_i, \tau_{i+1}]\) with \(s = |\tilde{\theta}(0) - \theta|\).

It follows that the desired estimate \(|x(t)| \leq M_{\theta|\theta,s} \exp(-\sigma t) |x_0|\) for all \(t \geq 0\) and for all \(\theta \in \Theta, x_0 \in \mathbb{R}^n, \tilde{\theta}_0 \in \Theta\) with \(|x_0| \leq \tilde{R}_{\theta|\theta,s}\) holds for the solution of the hybrid closed-loop system (1) with (4), (5), (7), (8), (18), and initial conditions \(x(0) = x_0, \dot{\theta}(0) = \dot{\theta}_0\) with \(M_{\theta|\theta,s} := \Gamma_{\theta,s}^{e^{-\sigma T}} \left(K_1^{\theta,s} - 1 \right) K_2^{\theta,s} + A\), and \(s = |\tilde{\theta}(0) - \theta|\).

Case (ii): The proof of the case \(R, \delta = +\infty\) is almost identical with the proof of Case (i), except for the fact that no restrictions in the magnitude of \(|x|\) are needed for the derivation of all estimates. The proof is complete.

Proof of Corollary IV.3: Let \(\sigma > 0\) be given (arbitrary). The existence of continuous mappings \(\Theta \ni \theta \rightarrow P(\theta) \in \mathbb{R}^{n \times n}\), \(\Theta \ni \theta \rightarrow k(\theta) \in \mathbb{R}^n\) for which \(P(\theta) = \{ p_{i,j}(\theta) : i, j = 1, \ldots, n\} \in \mathbb{R}^{n \times n}\) is a symmetric, positive definite matrix and for which the inequality \(\sum_{j=1}^n \sum_{j=1}^n p_{i,j}(\theta) \geq 2\sigma \sum_{j=1}^n \sum_{j=1}^n p_{i,j}(\theta) x_j x_j \geq 0\) for all \(x \in \mathbb{R}^n\) can be proved by induction on \(n\). For \(n = 1\), the statement holds with \(p_{1,1}(\theta) = \frac{1}{2}\) and \(k(\theta) = -\theta\). Assuming that the statement holds for certain integer \(n \geq 1\), then, straightforward manipulations show that the statement is also true for \(n + 1\) with

\[ p_{n+1,j}(\theta) = p_{n+1,i}(\theta) + \frac{1}{2} k_i(\theta) k_j(\theta), \quad i, j = 1, \ldots, n \]

\[ p_{n+1,n+1}(\theta) = -\frac{1}{2} k_i(\theta), \quad i = 1, \ldots, n \]

\[ p_{n+1,n+1}(\theta) = \frac{1}{2} \]

for \(i = 2, \ldots, n\).

The rest of the proof is a direct consequence of Theorem IV.2, (in case \(R, \delta = +\infty\), with \(f(x, u) \equiv 0, a(x) := A|x|^2, V(\theta, x) := x^T P(\theta) x, k(\theta, x) = k(\theta) x\), and \(g(x, u) := \{ (\theta_1 x_1 + \theta_2 x_2), \ldots, (\theta_n x_1 + \cdots + \theta_n x_n) \}^T\)). More specifically, inequality (23) is a consequence of continuity of the mapping \(\Theta \ni \theta \rightarrow P(\theta) \in \mathbb{R}^{n \times n}\), which implies that the function \(V(\theta, x) = x^T P(\theta) x\) is continuous on the compact set \(\Theta \times \{ x \in \mathbb{R}^n : |x| = 1 \}\).

The proof is complete.

VII. CONCLUDING REMARKS

This paper showed that regulation-triggered, adaptive schemes can guarantee exponential regulation even in the

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absence of persistency of excitation and parameter observability. The proposed adaptive scheme guarantees that the closed-loop system follows the trajectories of the nominal closed-loop system as well as a $KL$ estimate for the state component $x$. However, a number of issues remain open:

1) the use of weighting functions in the BaLSI;
2) the numerical implementation of the BaLSI.

Both issues may be important in practice. An additional issue that should be addressed in future research is the study of sensitivity with respect to modeling and measurement errors. Although preliminary studies (see [23]) have shown important robustness properties with respect to various perturbations (vanishing and nonvanishing), the issue requires further (both theoretical and numerical) study. To this purpose, similar ideas such as those used in [9] can be useful for the analysis of the properties of the closed-loop system under the effect of measurement noise. Finally, future research may also involve the study of the output feedback case.

REFERENCES


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