Robust adaptive regulation of dynamically positioned ships with unknown dynamics and unknown disturbances

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Summary
We develop a robust adaptive regulating control law for dynamically positioned ships subject to unknown dynamics and bounded unknown disturbances incorporating the radial basis function (RBF) neural network (NN), the dead zone adaptive technique, and a robust control term into the vectorial backstepping approach. The RBF NNs with the dead zone adaptive laws approximate the ship unknown dynamics. The adaptive law-based robust control term compensates for unknown disturbances, NN approximation errors, and undesirable errors arising from the design procedures. The developed dynamic positioning (DP) control law regulates the ship position and heading to the desired values with arbitrarily small errors, while guaranteeing the uniform ultimate boundedness of all signals in the DP closed-loop control system of ships. High-fidelity simulations on two supply ships and comparisons demonstrate the effectiveness and the superiority of the developed DP control law.

KEYWORDS
robust adaptive control, ship dynamic positioning, unknown dynamics, unknown time-varying disturbances, vectorial backstepping

1 | INTRODUCTION

With the ocean exploitation and exploration expanding toward the distant and deep sea, the ships installed with dynamic positioning (DP) systems have been increasingly used in the offshore industries. The DP systems of ships are intended to regulate the ships’ horizontal motions by exclusive means of their own propellers and thrusters.1 The DP systems make the ships work in the deep sea, have high positioning accuracy, and are flexible in operation. As such, they are the indispensable technology for many offshore operations, such as drilling, diving support, and offloading.2

The first DP system came to existence in the 1960s for offshore drilling applications adopting the proportional-integral-derivative control algorithm cascaded with the low-pass and/or the notch filter.3 More advanced DP control was developed in the 1970s based on multivariable optimal control and Kalman filter theory,4,6 which requires the linearization of the ship motion mathematical model. To overcome this problem, the DP nonlinear control methods emerged from the 1990s, such as backstepping control7,8 and Takagi-Sugeno (T-S) fuzzy control,9 however ignoring the ocean disturbances. Considering unknown constant disturbances, Godhavn et al10 proposed a nonlinear tracking control law for the ships using the adaptive backstepping method, where adaptive laws were employed to estimate unknown constant disturbances. Bertin et al11 developed a DP nonlinear control law for the underactuated ships using the feedback linearization technique; Fossen and Strand12 proposed a DP weather optimal positioning control using the adaptive backstepping method, where the ship heading automatically adjusts such that the minimum energy was used; Veksler et al13 presented a DP control strategy using model predictive control method. Considering wave- and low-frequency (LF) disturbances,
Fossen and Strand\textsuperscript{14} presented a passive nonlinear observer as a wave filter to estimate the LF position, velocities, and disturbances from noisy position measurements. Combining this passive observer with a proportional-derivative–type controller, Loria et al\textsuperscript{15} proposed a DP output-feedback controller. However, this passive observer requires a priori knowledge of ocean disturbances. Considering unknown disturbances, Hassani et al\textsuperscript{16} presented a multiple model adaptive wave filtering estimator for the DP system design of vessels; Nguyen et al\textsuperscript{17} proposed a DP hybrid output-feedback control strategy based on the supervisory control theory, where different controllers and observers can be online switched according to varying sea states; Hu et al\textsuperscript{18} developed a robust adaptive control law using an observer and vectorial backstepping method; Du et al\textsuperscript{19} proposed a DP robust control law for ships under input saturation incorporating an auxiliary dynamic system and a disturbance observer into the dynamic surface control method. The aforementioned DP nonlinear control schemes require a priori knowledge of the ship motion mathematical model parameters.

Considering unknown ship model parameters and unknown constant disturbances, Do\textsuperscript{20} designed a DP robust adaptive output-feedback control law using the adaptive vectorial backstepping method and an adaptive velocity observer. Under parameter perturbations and unknown time-varying disturbances in the ship motion mathematical model, Kjerstad et al\textsuperscript{21} proposed a DP disturbance rejection control scheme, where an observer provides the estimates of unknown time-varying disturbances and model parameter perturbations according to acceleration measurements; Tannuri et al\textsuperscript{22} developed a DP sliding mode control, whose performance was evaluated numerically and experimentally on a 1:150 scaled shuttle tanker; Hassani et al\textsuperscript{23} developed a DP robust control scheme using $H_\infty$ and mixed-$\mu$ technique, whose effectiveness was verified by experiments with a 1:30 scaled supply ship.

In practice, there obviously exist uncertainties in both the ship dynamics and the environmental disturbances. In the presence of unknown dynamics and unknown time-varying disturbances, Chang et al\textsuperscript{24} proposed a DP fuzzy control law for ships with multiplicative noises under actuator saturation using the T-S fuzzy model, the linear matrix inequality method, and the concept of parallel distributed compensation. Du et al\textsuperscript{25} developed a DP robust adaptive control law, where the projection algorithm-based adaptive laws together with the constructed observer provide online estimates of unknown time-varying disturbances and the other projection algorithm-based adaptive laws update the unknown ship mathematical model parameters. Du et al\textsuperscript{26} proposed a DP robust adaptive neural controller, where the radial basis function (RBF) neural networks (NNs) were used to approximate ship unknown dynamics together with unknown time-varying disturbances. In the work of Du et al\textsuperscript{26}, the uniform ultimate boundedness of the ship positioning errors was guaranteed and the NN approximation errors were not addressed.

In this paper, the DP control problem of ships simultaneously with unknown dynamics and bounded unknown time-varying disturbances is solved. A robust adaptive control law is developed to regulate the ship to the desired position and heading with arbitrarily small errors, while the uniform ultimate boundedness of all signals in the DP closed-loop control system of ships is guaranteed. The main contributions of this paper are as follows.

1. Different from the work of Hu et al\textsuperscript{18}, the developed DP robust adaptive regulating control law does not require any priori knowledge of ship motion mathematical model parameters due to employing adaptive RBF NNs.
2. Compared with the work of Du et al\textsuperscript{26}, the dead zone is introduced in the adaptive laws of RBF NNs, so that the parameter drift and burst are prevented; and the adaptive law-based robust control term is inserted into the DP control design to compensate the bounded unknown disturbances, NN approximation errors, and undesirable errors arising from the design procedures, so that the robustness of the DP control law is improved.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation and preliminaries. Section 3 details the DP robust adaptive control design. Section 4 provides simulations involving two supply ships and simulation comparisons with an existing DP control law. Section 5 concludes this paper.

# PROBLEM FORMULATION AND PRELIMINARIES

## Problem formulation

Let the position $(x, y)$ and heading $\psi$ of the ship in the north-east frame be expressed in the vector $\eta = [x, y, \psi]^T$ and the velocities in the ship-fixed frame be expressed in the vector $\nu = [u, v, r]^T$. These three modes are referred to surge, sway, and yaw. The motion mathematical model of a dynamically positioned ship is given by\textsuperscript{14}:

$$\dot{\eta} = J(\psi)\nu$$

$$M\dot{\nu} = -D\nu + \tau + d(t),$$
where \( J(\psi) \) is the rotation matrix as follows

\[
J(\psi) = \begin{bmatrix}
\cos (\psi) & -\sin (\psi) & 0 \\
\sin (\psi) & \cos (\psi) & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

with the property \( J^T(\psi) = J^{-1}(\psi) \). \( M \in \mathbb{R}^{3 \times 3} \) being the inertia matrix including the added mass components, which is positive definite and symmetric. \( D \in \mathbb{R}^{3 \times 3} \) is the linear damping matrix. \( \tau = [\tau_1, \tau_2, \tau_3]^T \) is the control vector produced by the propulsion system, consisting of surge force \( \tau_1 \), sway force \( \tau_2 \), and yaw moment \( \tau_3 \). \( d(t) = [d_1(t), d_2(t), d_3(t)]^T \) denotes the LF disturbance forces and moment due to second-order waves, wind, currents in the ship-fixed frame.

**Assumption 1.** (1) The model parameter matrices \( M \) and \( D \) are unknown. (2) The LF disturbances \( d_i(t), i = 1, 2, 3 \) are bounded unknown, which means there exists

\[
\| d(t) \| \leq d^* < \infty
\]

with an unknown constant \( d^* > 0 \) and \( \| \cdot \| \) representing the two-norms a vector or a matrix.

**Remark 1.** The ship dynamic model parameters depend on the ship’s mass, hydrodynamic derivatives, inertia moment, etc., which are difficult to be accurately identified. On the other hand, the ocean environment is often changing and unpredicted as well as has finite energy. As such, Assumption 1 is reasonable.

The control objective in this paper is to develop a DP robust adaptive control law for the ship (1)-(2) under Assumption 1, so that the ship position \((x, y)\) and heading \( \psi \) are regulated to the desired value \( \eta_d = [x_d, y_d, \psi_d]^T \) with arbitrarily small errors, while all signals in the DP closed-loop control system are uniformly ultimately bounded.

### 2.2 RBF neural network

In the RBF NN, the hidden layer completes a fixed nonlinear transformation and the output layer linearly combines the outputs of the hidden layer. Therefore, the output of the RBF NN is as follows \(^{27,28}\):

\[
f_m(\chi) = w^T \phi(\chi),
\]

where \( \chi = [\chi_1, \chi_2, \ldots, \chi_m]^T \in \Omega_\chi \subset \mathbb{R}^m \) is the input vector and \( w = [w_1, w_2, \ldots, w_l]^T \) is the NN weight vector with \( l > 1 \) being the node number. \( \phi(\chi) = [\phi_1(\chi), \phi_2(\chi), \ldots, \phi_l(\chi)]^T \) is the basis function vector. In this paper, \( \phi_j(\chi) (j = 1, 2, \ldots, l) \) is taken as the Gaussian function

\[
\phi_j(\chi) = \exp \left[ -\frac{\| \chi - \kappa_j \|^2}{h_j^2} \right],
\]

where \( \kappa_j = [\kappa_{j1}, \kappa_{j2}, \ldots, \kappa_{jm}]^T \) and \( h_j > 0 \) denote the center and the width of the Gaussian function, respectively.

Universal approximation results show that if the node number \( l \) is sufficiently large, the NN \( w^T \phi(\chi) \) can approximate any continuous function \( f(\chi) \) defined on a compact set \( \Omega_\chi \subset \mathbb{R}^m \). This is described by

\[
f(\chi) = w^* w^T \phi(\chi) + e(\chi), \quad \forall \chi \in \Omega_\chi \subset \mathbb{R}^m
\]

where \( e(\chi) \) is the NN approximation error and \( w^* \in \mathbb{R}^l \) is the ideal NN weight vector defined by

\[
w^* := \text{arg min}_{w \in \mathbb{R}^l} \left\{ \sup_{\chi \in \Omega_\chi} \left| f(\chi) - w^T \phi(\chi) \right| \right\}.
\]

Over the compact set \( \Omega_\chi \), the ideal NN weight vector \( w^* \) and the NN approximation error \( e(\chi) \) are bounded, ie,

\[
\| w^* \| \leq w_m, \quad | e(\chi) | \leq e_m, \quad \forall \chi \in \Omega_\chi,
\]

where \( w_m \) and \( e_m \) are positive constants.

### 3 DP ROBUST ADAPTIVE CONTROL DESIGN

A robust adaptive control law is derived incorporating the RBF NNs, the dead zone adaptive technique, and a robust control term into the vectorial backstepping method \(^{29}\) for the DP of ships. The RBF NNs with the dead zone adaptive
laws approximate the ship unknown dynamics. The adaptive law-based robust control term compensates for bounded unknown disturbances, NN approximation errors, and undesirable errors arising from the design procedures.

The DP control design consists of two steps based on the following change of coordinates:

\[ z_1 = \eta - \eta_d \]  
\[ z_2 = \nu - \alpha_1, \]  

where \( \alpha_1 \in \mathbb{R}^3 \) is the intermediate control function vector to be designed later.

**Step 1:** Differentiating (10) along (1) obtains

\[ \dot{z}_1 = J(\psi)\nu, \]  

where \( \nu \) is viewed as the virtual control vector.

Choose the Lyapunov function candidate for (12)

\[ V_1 = \frac{1}{2}z_1^Tz_1. \]  

According to (12) and (11), the time derivative of (13) is

\[ \dot{V}_1 = z_1^T J(\psi)(z_2 + \alpha_1). \]  

Design the intermediate control function vector \( \alpha_1 \) for \( \nu \) as follows:

\[ \alpha_1 = -J^T(\psi)K_1z_1, \]  

where \( K_1 = K_1^T \) is the 3 x 3 positive-definite design matrix.

Substituting (15) into (14), according to the property \( J^T(\psi) = J^{-1}(\psi) \), we obtain

\[ \dot{V}_1 = -z_1^T K_1 z_1 + z_2^T J(\psi) z_2. \]  

**Step 2:** The time derivative of (11) along (2) is

\[ \dot{z}_2 = M^{-1}(\tau + d - D\nu - M\alpha_1). \]  

Choose the Lyapunov function candidate for (12) and (17)

\[ V_2 = V_1 + \frac{1}{2}z_2^TMz_2. \]  

The time derivative of (18) along the solutions of (16) and (17) is

\[ \dot{V}_2 = -z_1^T K_1 z_1 + z_2^T J(\psi) z_2 + z_2^T (\tau + d - D\nu - M\alpha_1). \]  

Since the dynamic model parameters \( M \) and \( D \) are unknown, the term \( D\nu + M\alpha_1 \) in (19) is unknown and is expressed with (7) as follows:

\[ D\nu + M\alpha_1 = \Theta^T \beta(\zeta) + \bar{e}(\zeta), \]  

where \( \zeta = [\nu^T, \alpha_1^T]^T \) is the input vector, \( \Theta^* = \begin{bmatrix} \theta_1^T & \theta_1^{lx} & \theta_1^{lx} \\ \theta_2^{lx} & \theta_2^{lx} & \theta_2^{lx} \\ \theta_3^{lx} & \theta_3^{lx} & \theta_3^{lx} \end{bmatrix} \in \mathbb{R}^{3 \times 3} \) with \( \theta_i^* = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{i3}]^T \) \( i = 1, 2, 3 \) is the ideal neural-network weight matrix, \( l > 0 \) is the node number, \( \beta(\zeta) = [\beta_1(\zeta), \beta_2(\zeta), \beta_3(\zeta)]^T \in \mathbb{R}^3 \) with \( \beta_i(\zeta) = [\beta_{i1}(\zeta), \beta_{i2}(\zeta), \ldots, \beta_{i3}(\zeta)]^T \) \( i = 1, 2, 3 \) is the basis function vector, and \( \bar{e}(\zeta) \in \mathbb{R}^l \) is the approximation error vector. From (9), it follows that \( \|\bar{e}(\zeta)\| \leq \bar{e}^* \) with \( \bar{e}^* > 0 \) being an unknown constant.

For the convenience of the DP control design later, notate the unknown parameter

\[ \delta = d^* + \bar{e}^* + c^*, \]  

where \( c^* \) is any positive constant representing the upper bound of the undesirable errors arising from the design procedures.

Substituting (20) into (19) obtains

\[ V_2 = -z_1^T K_1 z_1 + z_2^T J(\psi) z_2 + z_2^T [\tau + d - \Theta^T \beta(\zeta) - \bar{e}(\zeta)]. \]
Based on the RBF NNs and the robust control term, we design the DP robust adaptive control law

\[
\tau = -J^T(\psi)z_1 - K_2z_2 + \hat{\Theta}^T \beta(\zeta) + h_r,
\]

(23)

where \( K_2 = K_2^T \) is the 3 \times 3 positive definite design matrix, \( \hat{\Theta} = \begin{bmatrix} \hat{\Theta}_1^T & 0_{1 \times d} & 0_{1 \times d} \\ 0_{1 \times d} & \hat{\Theta}_2^T & 0_{1 \times d} \\ 0_{1 \times d} & 0_{1 \times d} & \hat{\Theta}_3^T \end{bmatrix} \in \mathbb{R}^{3 \times 3} \) with \( \hat{\Theta}_i = [\hat{\theta}_{i1}, \ldots, \hat{\theta}_{il}] \) \((i = 1, 2, 3)\) is the estimate of \( \Theta^* \), and \( h_r \) is the robust control term.

Motivated by Gutman,\(^3\) we design the adaptive law-based robust control term

\[
h_r = -\frac{z_2 \hat{\delta}^2}{\hat{\delta} \| z_2 \| + \rho}
\]

(24)

with

\[
\hat{\delta} = \gamma_1(\| z_2 \| - \gamma_2 \hat{\delta}), \quad \hat{\delta}(0) > 0,
\]

(25)

where \( \hat{\delta} \) is the estimate of \( \delta \), \( \rho \) is any positive constant, and \( \gamma_1 \) and \( \gamma_2 \) are positive design constants.

To prevent the parameter drift and burst, the adaptive laws with the dead zone are designed as follows:

\[
\hat{\theta}_i = \begin{cases} -G_i[\beta_i(\zeta)z_2 + \sigma_i \hat{\theta}_i], & \text{if } |z_2| \geq b_i \\ 0_{1 \times d}, & \text{if } |z_2| < b_i, \quad i = 1, 2, 3,
\end{cases}
\]

(26)

where \( G_i = G_i^T \) is the \( l \times l \) positive definite design matrix, \( \sigma_i \) is the positive design constant, and \( b_i > 0 \) is the dead zone break point.

**Remark 2.** The effect of the dead zone is that the parameter adaptive laws ignore the smaller errors resulting from the disturbances or the measurement noises and respond to the larger errors being related to the transient response from large initial conditions. We introduce the dead zone in the adaptive laws to prevent the parameter drift and burst due to the second-order wave disturbances, the measurement noises, etc. A much larger value would stop adaptation, which brings the system to unstable, while a much smaller value would lead to bursting. The choice of dead zone break point (threshold) depends on the characteristics of wave disturbances and measurement noises.

The main results are given in the following theorem.

**Theorem 1.** Consider the closed-loop system consisting of the ship (1)-(2) under Assumption 1 and the DP robust adaptive control law (23) with the robust control term (24)-(25) and the adaptive laws (26). The ship position \((x, y)\) and heading \(\psi\) are maintained at the desired value \(\eta_d = [x_d, y_d, \psi_d]^T\) with arbitrarily small errors, while all signals in the DP closed-loop control system are guaranteed to be uniformly ultimately bounded through appropriately choosing the positive design constants \(\rho, \gamma_1, \gamma_2, \sigma_i (i = 1, 2, 3)\) and the positive-definite symmetric design matrices \(G_i, K_1, K_2\).

**Proof.** Choose the augmented Lyapunov function candidate

\[
V_{2u} = V_2 + \frac{1}{2} \sum_{i=1}^{3} \hat{\theta}_i^T G_i^{-1} \hat{\theta}_i + \frac{1}{2} \hat{\delta}^2
\]

(27)

where \( \hat{\theta}_i = \hat{\theta}_i - \Theta_i^* \) \((i = 1, 2, 3)\) and \( \hat{\delta} = \hat{\delta} - \delta \).

In the light of (22), (23), and (25), the time derivative of (27) is

\[
\dot{V}_{2u} = \dot{V}_2 + \frac{1}{\gamma_1} \hat{\delta} \dot{\hat{\delta}}
\]

\[
= -z_1^T K_1 z_1 - z_2^T K_2 z_2 + z_2^T [\hat{\Theta}^T \beta(\zeta) + h_r + d - \tilde{e}(\zeta)] + \sum_{i=1}^{3} \hat{\theta}_i^T G_i^{-1} \hat{\theta}_i + \hat{\delta} \| z_2 \| - \gamma_2 \hat{\delta}.
\]

(28)

Subsequently, the stability analysis of the DP closed-loop control system is conducted in the following two cases.

**Case 1.** \(|z_2| \geq b_i (i = 1, 2, 3)\). Then, \( \hat{\theta}_i = -G_i[\beta_i(\zeta)z_2 + \sigma_i \hat{\theta}_i] \) from (26).
Using (24), (4), \( \| \mathbf{\tilde{e}}(\zeta) \| \leq \mathbf{\tilde{e}}^* \), (21) and \( \mathbf{\delta} = \hat{\mathbf{\delta}} - \delta \), we have

\[
\mathbf{z}_2^T [\mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) + \mathbf{h}_r + \mathbf{d} - \mathbf{\tilde{e}}(\zeta)] + \sum_{i=1}^{3} \mathbf{\tilde{\Theta}}_i^T \mathbf{1}_i^{-1} \mathbf{\hat{\theta}}_i + \tilde{\mathbf{\delta}} \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
= \mathbf{z}_2^T [\mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) + \mathbf{h}_r + \mathbf{d} - \mathbf{\tilde{e}}(\zeta)] - \sum_{i=1}^{3} \mathbf{\tilde{\Theta}}_i^T \mathbf{\beta}(\zeta) \mathbf{z}_2 - \sum_{i=1}^{3} \mathbf{\sigma}_i \mathbf{\tilde{\Theta}}_i \mathbf{\hat{\theta}}_i + \tilde{\mathbf{\delta}} \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
= - \frac{\mathbf{z}_2^T \mathbf{z}_2 \tilde{\mathbf{\delta}}^2}{\delta \| \mathbf{z}_2 \| + \rho} + \mathbf{z}_2^T [\mathbf{d} - \mathbf{\tilde{e}}(\zeta)] - \sum_{i=1}^{3} \mathbf{\sigma}_i \mathbf{\tilde{\Theta}}_i \mathbf{\hat{\theta}}_i + \tilde{\mathbf{\delta}} \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
\leq - \frac{\mathbf{z}_2^T \mathbf{z}_2 \tilde{\mathbf{\delta}}^2}{\delta \| \mathbf{z}_2 \| + \rho} + \frac{\mathbf{\delta} \| \mathbf{z}_2 \| + (\hat{\delta} - \delta) \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}}{\delta \| \mathbf{z}_2 \| + \rho}
\]

\[
= - \frac{\mathbf{\delta} \| \mathbf{z}_2 \|^2}{\delta \| \mathbf{z}_2 \| + \rho} + \frac{\mathbf{\delta} \| \mathbf{z}_2 \|^2 + \rho \mathbf{\delta} \| \mathbf{z}_2 \|}{\delta \| \mathbf{z}_2 \| + \rho} - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
= \rho \frac{\mathbf{\delta} \| \mathbf{z}_2 \|}{\delta \| \mathbf{z}_2 \| + \rho} - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
\leq \rho - \frac{\gamma_2}{2} (\tilde{\mathbf{\delta}}^2 + \delta^2 - \delta^2) - \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 + \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2
\]

\[
\leq - \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 - \frac{\gamma_2}{2} \mathbf{\tilde{\mathbf{\delta}}^2} + C_1,
\]

where \( C_1 = \rho - \frac{\gamma_2}{2} \mathbf{\tilde{\mathbf{\delta}}^2} + \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 \) with \( \| \mathbf{\tilde{\Theta}} \| \) being a constant.

Case 2. \( | \mathbf{z}_2 | < b_i \) (i = 1, 2, 3). Then, \( \mathbf{\hat{\theta}}_i = \mathbf{0}_{b \times 1} \) from (26).

Since \( \mathbf{\hat{\theta}}_i \) is the constant vector due to \( \mathbf{\hat{\theta}}_i = \mathbf{0}_{b \times 1} \) and \( \mathbf{\beta}(\zeta) \) is the bounded basis function vector, \( \mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) \) is bounded. \( c^* \) in (21) is viewed as the upper bound of \( \| \mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) \| \) in Case 2.

Using (24), (4), \( \| \mathbf{\tilde{e}}(\zeta) \| \leq \mathbf{\tilde{e}}^* \), (21) and \( \mathbf{\delta} = \hat{\mathbf{\delta}} - \delta \), we have

\[
\mathbf{z}_2^T [\mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) + \mathbf{h}_r + \mathbf{d} - \mathbf{\tilde{e}}(\zeta)] + \sum_{i=1}^{3} \mathbf{\tilde{\Theta}}_i^T \mathbf{1}_i^{-1} \mathbf{\hat{\theta}}_i + \tilde{\mathbf{\delta}} \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
= - \frac{\mathbf{z}_2^T \mathbf{z}_2 \tilde{\mathbf{\delta}}^2}{\delta \| \mathbf{z}_2 \| + \rho} + \mathbf{z}_2^T [\mathbf{\tilde{\Theta}}^T \mathbf{\beta}(\zeta) + \mathbf{d} - \mathbf{\tilde{e}}(\zeta)] + \tilde{\mathbf{\delta}} \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
\leq - \frac{\mathbf{z}_2^T \mathbf{z}_2 \tilde{\mathbf{\delta}}^2}{\delta \| \mathbf{z}_2 \| + \rho} + \frac{\mathbf{\delta} \| \mathbf{z}_2 \| + (\hat{\delta} - \delta) \| \mathbf{z}_2 \| - \gamma_2 \tilde{\mathbf{\delta}}}{\delta \| \mathbf{z}_2 \| + \rho}
\]

\[
= - \frac{\mathbf{\delta} \| \mathbf{z}_2 \|^2}{\delta \| \mathbf{z}_2 \| + \rho} + \frac{\mathbf{\delta} \| \mathbf{z}_2 \|^2 + \rho \mathbf{\delta} \| \mathbf{z}_2 \|}{\delta \| \mathbf{z}_2 \| + \rho} - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
= \rho \frac{\mathbf{\delta} \| \mathbf{z}_2 \|}{\delta \| \mathbf{z}_2 \| + \rho} - \gamma_2 \tilde{\mathbf{\delta}}
\]

\[
\leq \rho - \frac{\gamma_2}{2} (\tilde{\mathbf{\delta}}^2 + \delta^2 - \delta^2) - \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 + \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2
\]

\[
\leq - \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 - \frac{\gamma_2}{2} \mathbf{\tilde{\mathbf{\delta}}^2} + C_2,
\]

where \( C_2 = \rho - \frac{\gamma_2}{2} \mathbf{\tilde{\mathbf{\delta}}^2} + \sum_{i=1}^{3} \mathbf{\sigma}_i \| \mathbf{\tilde{\Theta}}_i \|^2 \) with \( \| \mathbf{\tilde{\Theta}} \| \) being a constant.
Synthesizing (29) and (30), we rewrite (28) as follows:

\[
V_{2a} \leq -\zeta_1^T K_1 \zeta_1 - \zeta_2^T K_2 \zeta_2 - \sum_{i=1}^{3} \frac{\sigma_i}{2} \| \tilde{\theta}_i \|^2 - \frac{\gamma_2^2}{2} \delta^2 + C
\]

\[
\leq -2 \mu V_{2a} + C, \tag{31}
\]

where \( \mu = \min\{\lambda_{\text{min}}(K_1), \lambda_{\text{min}}(K_2M^2)\} \), \( \min_{i=1,2,3}(\frac{\sigma_i}{2} \lambda_{\text{min}}(I_i)) \), and \( \frac{\gamma_2^2}{2} \) with \( \lambda_{\text{min}}(\cdot) \) denoting the minimum eigenvalue of a matrix and \( C = \max\{C_1, C_2\} \).

Solving (31), we have

\[
0 \leq V_{2a}(t) \leq \frac{C}{2\mu} + \left[ V_{2a}(0) - \frac{C}{2\mu} \right] e^{-2\mu t}. \tag{32}
\]

It is obviously seen from (32) that \( V_{2a}(t) \) is uniformly ultimately bounded. Then, it is known from (27) that \( \| \zeta_1 \|, \| \zeta_2 \|, \| \tilde{\theta}_i \| (i = 1, 2, 3) \) and \( | \delta | \) are bounded; hence, \( \| \tilde{\theta}_i \| (i = 1, 2, 3) \) and \( | \delta | \) are bounded. From (10) and the boundedness of \( \| \eta_i \|, \| \eta \| \) is bounded; \( \| \alpha_i \| \) is then bounded from (15); hence, \( \| p \| \) is bounded from (11). Therefore, uniform ultimate boundedness of all signals in the DP closed-loop control system is achieved.

Furthermore, according to (27), (18), (13), and (32), we have

\[
\| \zeta_1 \| \leq \sqrt{\frac{C}{\mu} + 2 \left[ V_{2a}(0) - \frac{C}{2\mu} \right] e^{-2\mu t}}. \tag{33}
\]

It is obvious from (33) that, for any positive constant \( \zeta_2 > \sqrt{C/\mu} \), there exists a constant \( T_\zeta > 0 \) such that \( \| \zeta_1 \| \leq \zeta_2 \) for all \( t > T_\zeta \). Thus, \( \zeta \) settles within the compact set \( \zeta_\zeta = \{ \zeta_1 \in \mathbb{R}^1 \| \zeta_1 \| \leq \zeta_2 \} \), which can be made arbitrarily small by appropriately selecting the positive design constants \( \rho, \gamma_1, \gamma_2, \sigma_i (i = 1, 2, 3) \) and the positive-definite symmetric design matrices \( I_i, K_1, K_2 \). Therefore, the ship position \( (x, y) \) and heading \( \psi \) are regulated to the desired value \( \eta_d = [x_d, y_d, \psi_d]^T \) with arbitrarily small errors. Thus, Theorem 1 is proved.

4 | SIMULATION AND COMPARISON STUDIES

In this section, to verify the effectiveness and the superiority of the developed DP robust adaptive regulating control law, we carry out the high-fidelity simulations using the Marine Systems Simulator (MSS) toolbox,\(^3\) which is a MATLAB/Simulink-based modular simulator developed in the Department of Marine Technology, Norwegian University of Science and Technology. The module MSS\|Vessel Templates\|DP MotionRAO Model, which is a ship motion simulator for station-keeping and low-speed maneuvering in six degrees of freedom, is adopted. Furthermore, the simulation comparisons with an existing DP robust adaptive neural control law from the work of Du et al\(^2\) are presented.

4.1 | Performance of proposed DP control law

In the following, to show the adaptability to the ship model parameter changes and the robustness against bounded unknown time-varying disturbances of our proposed control law, the simulations on two ship examples are carried out in two different disturbance cases.

Example 1. The ship example is a supply ship of 82.8 m in length, whose data is included in MSS.

In the simulations, the wave spectrum-type is JONSWAP, the peak frequency is 1.07 rad/s, the significant wave height is 0.15 m, and the mean wave direction is (35°/180°) × π rad in the north-east frame, corresponding to the sea state 2. The current speed is 0.1 m/s and the current direction is (35°/180°) × π rad. The mean wind angle is (30°/180°) × π rad and the wind speed is 2.65 m/s. Then, the corresponding LF disturbance curves are shown in Figure 1A. The desired ship position in the north-east frame is taken as the coordinate origin, that is, \( \eta_d = [0 \text{ m}, 0 \text{ m}, 0 \text{ rad}]^T \). The dead zone breakpoints are taken as \( b_1 = 0.025, b_2 = 0.008, \) and \( b_3 = 0.0005 \). Let the NN \( \tilde{\theta}_i^T \beta(\zeta) (i = 1, 2, 3) \) contain 20 nodes with centers \( \kappa_j (j = 1, 2, \ldots, 20) \) evenly spaced in \([-2, 20] \times [-2, 20] \times [-0.1, 0.2] \times [-2, 1] \times [-0.02, 0.01] \) and widths \( h_j = 5 \). The design parameters are chosen as follows: \( \rho = 1 \times 10^5, \gamma_1 = 3 \times 10^4, \gamma_2 = 5 \times 10^{-8}, \sigma_1 = \sigma_2 = 1 \times 10^{-5}, \sigma_3 = 1 \times 10^{-8}, I_1 = I_3 = 1 \times 10^3 I_{20 \times 20}, I_2 = 1 \times 10^4 I_{20 \times 20}, K_1 = \text{diag}(1.2 \times 10^{-2}, 1.5 \times 10^{-2}, 2.0 \times 10^{-2}) \) and \( K_2 = \text{diag}(2 \times 10^6, 2 \times 10^6, 2 \times 10^6) \). The initial conditions are taken as \( \eta(0) = [20 \text{ m}, 20 \text{ m}, (10^° /180°) \times \pi \text{ rad}]^T \),
FIGURE 1 Simulation results on Example 1 under $\tau$ and $\tau_{\text{com}}$. A, Low-frequency disturbances $d_1$, $d_2$, and $d_3$ acting on the ship; B, Ship position ($x$, $y$) and heading $\psi$; C, Ship surge velocity $u$, sway velocity $v$, and yaw rate $r$; D, Two-norms $||\hat{\theta}_1||$, $||\hat{\theta}_2||$, $||\hat{\theta}_3||$ and $||\hat{\theta}_{\text{com}}||$, $||\hat{\theta}_{\text{com}1}||$, $||\hat{\theta}_{\text{com}2}||$, $||\hat{\theta}_{\text{com}3}||$; E, Control inputs $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_{\text{com}1}$, $\tau_{\text{com}2}$, $\tau_{\text{com}3}$ [Colour figure can be viewed at wileyonlinelibrary.com]
ν(0) = [0 m/s, 0 m/s, 0 rad/s]T, b(0) = [10 kN, 10 kN, 100 kNm]T, 𝜎(0) = 10 and \( \dot{d}_1(0) = \dot{d}_2(0) = \dot{d}_3(0) = \theta_{20x1} \). In addition, the measurement noises are taken as the zero-mean Gaussian white noises whose standard deviations are 0.05 m in north position \( x \), 0.05 m in east position \( y \), and \( (1^\circ/180^\circ) \times \pi \) rad in heading \( \psi \), respectively.

To filter out the first-order wave-induced motions and the measurement noises of the ship position and heading, the low-pass filter (34) and the notch filter (35) are chosen as

\[
h_{LP}(s) = \frac{1}{1 + 1.2s} \tag{34}
\]

\[
h_{NO}(s) = \frac{s^2 + 0.17s + 0.69}{s^2 + 1.66s + 0.69}. \tag{35}
\]

The simulation results on Example 1 are depicted with solid lines in Figures 1B to 1E. To quantitatively evaluate the control performance of the developed DP control law, the DP control performance indices are summarized in Table 1, where the settling time \( t_s \) is the required time for all the ship north position \( x \), east position \( y \), and heading \( \psi \) to arrive at and remain within the specified error bands 3 m, 3 m, and \( (1^\circ/180^\circ) \times \pi \) rad, respectively. It is observed from Figure 1B and Table 1 that the ship position \((x, y)\) and heading \( \psi \) converge to the desired value \( \eta_d = [0 \text{ m}, 0 \text{ m}, 0 \text{ rad}]^T \) in around 90 seconds. Figure 1C shows the boundedness of the ship surge velocity \( u \), sway velocity \( v \), and yaw rate \( r \). Figure 1D demonstrates the boundedness of the two-norms \( ||\dot{\theta}_1||, ||\dot{\theta}_2||, ||\dot{\theta}_3|| \). Figure 1E shows that the control inputs \( r_1, r_2, r_3 \) are bounded and reasonable.

**Example 2.** The ship example is a semisubmersible vessel of 115.0 m in length, whose data is included in MSS.

In the simulations, change the mean wave direction to \((35^\circ/180^\circ) \times \pi \) rad, the peak frequency to 0.92 rad/s, and the significant wave height to 0.55 m, respectively, corresponding to the sea state 3. The current direction, current speed, mean wind angle, and wind speed are the same as those of Example 1. Then, the corresponding LF disturbance curves are shown in Figure 2A. In addition, the ship desired position vector, the remaining design parameters, and the initial conditions are identical to the counterparts of Example 1. This means that the DP robust adaptive regulating control law is kept unchanged for Example 1 and Example 2.

The simulation results on Example 2 are depicted with solid lines in Figures 2B to 2E and the DP control performance indices are listed in Table 1. As seen from Figures 1B to 1E, 2B to 2E, and Table 1, our developed DP robust adaptive control law exhibits satisfactory control performance for both Example 1 and Example 2 under the different environmental conditions, which demonstrates that our developed DP robust adaptive control law has the robustness against environmental disturbances and the adaptability to the model parameter changes.

### 4.2 Comparisons with existing DP control law

In this section, we compare our developed DP control law with the DP robust adaptive neural control law (36) with the parameter adaptive laws (37) (see the work of Du et al\textsuperscript{26})

\[
\begin{align*}
\tau_{com} &= -J^T(\psi)z_1 - K_2z_2 + \mathbf{\hat{\theta}}_{com}^T \hat{\beta}(\mathbf{z}) \tag{36} \\
\dot{\mathbf{\hat{\theta}}}_{com} &= -\Gamma_i[\hat{\beta}(\mathbf{z})z_2 + \sigma \mathbf{\hat{\theta}}_{com}], \quad i = 1, 2, 3. \tag{37}
\end{align*}
\]

In the simulations, the design parameter values in (40)-(41) are taken as the same as those of the counterparts of our developed DP control law. The simulations are then carried out on Example 1 and Example 2 respectively, in Section 4.1.

The simulation results on Example 1 and Example 2 are depicted with dashed lines in Figures 1B to 1E and 2B to 2E, respectively. The DP control performance indices are summarized in Table 1. It is observed from Figures 1D and 2D that

<table>
<thead>
<tr>
<th>Performance Indices</th>
<th>Control Law ( \tau )</th>
<th>Control Law ( \tau_{com} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Example 1</td>
<td>Example 2</td>
</tr>
<tr>
<td>Settling time</td>
<td>90 s</td>
<td>507 s</td>
</tr>
<tr>
<td>Mean deviation in north</td>
<td>0.5790 m</td>
<td>2.0685 m</td>
</tr>
<tr>
<td>Mean deviation in east</td>
<td>0.2766 m</td>
<td>0.6840 m</td>
</tr>
<tr>
<td>Mean deviation in yaw</td>
<td>0.2086°</td>
<td>0.8410°</td>
</tr>
</tbody>
</table>
FIGURE 2 Simulation results on Example 2 under $\tau$ and $\tau_{com}$. A, Low-frequency disturbances $d_1$, $d_2$, and $d_3$ acting on the ship; B, Ship position $(x, y)$ and heading $\psi$; C, Ship surge velocity $u$, sway velocity $v$, and yaw rate $r$; D, Two-norms $||\hat{\theta}_1||$, $||\hat{\theta}_2||$, $||\hat{\theta}_3||$, and $||\hat{\theta}_{com1}||$, $||\hat{\theta}_{com2}||$, $||\hat{\theta}_{com3}||$; E, Control inputs $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_{com1}$, $\tau_{com2}$, $\tau_{com3}$. [Colour figure can be viewed at wileyonlinelibrary.com]
the two-norms $\|\hat{\theta}_{\text{com}1}\|$, $\|\hat{\theta}_{\text{com}2}\|$, and $\|\hat{\theta}_{\text{com}3}\|$ of parameter estimates drift, which is caused by the first-order waves, the measurement noises, etc. It is clearly seen from Figures 1B, 2B, and Table 1 that the control performance of our developed DP control law is superior to $r_{\text{com}}$.

Remark 3. In the simulations, we firstly choose the design parameters $K_1$ and $K_2$ satisfying $\lambda_{\text{min}}(K_1) > 0$ and $\lambda_{\text{min}}(K_2) > 0$ through the trial and error to ensure that the system is stable. Furthermore, we properly regulate the other design parameters satisfying $\lambda_{\text{min}}(G_i) > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$, $b_i > 0$, $\sigma_i > 0$, and $\rho > 0$ to get the satisfactory control performance. The chosen design parameters could affect the results of simulations. An amount of simulations done in many scenarios show that the larger the parameters $K_1$, $K_2$, $G_i$, and $\gamma_1$ are, the better the control accuracy is, while the larger the control signal $r$ is.

5 CONCLUSIONS

We have developed a robust adaptive regulating control law for the DP of ships with unknown dynamics and unknown disturbances incorporating the RBF NNs, the dead zone adaptive technique, and a robust control term into the vectorial backstepping method. The dead zone has been introduced in the NN weight adaptive laws to prevent the parameter drift and burst. The ship position and heading are maintained at the desired values with arbitrarily small errors, while all signals in the DP closed-loop control system are guaranteed to be uniformly ultimately bounded. The high-fidelity simulation results on two supply ships and the simulation comparisons with an existing DP control law have validated the developed DP control law.

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