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Global attitude/position regulation for underwater vehicles

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In this paper a nonlinear controller is designed for a 6 DOF model of an unmanned underwater vehicle (UUV) which includes both the kinematics and the dynamics. It is shown how the use of a Lyapunov function, consisting of a quadratic term in the velocity (both linear and angular), a quadratic term in the position and a logarithmic term in the attitude leads to a design of a control law that achieves global asymptotic stabilization to an arbitrary set point in position/attitude. The control law is made linearly bounded by avoiding cancellation of some of the quadratic nonlinearities in the model. No information about the inertia matrix, the damping, and the Coriolis/centripetal parameters is used in the controller, endowing it with a certain amount of parametric robustness. The control law is given in terms of the Modified Rodrigues parameters. An extensive simulation study shows that the proposed control law achieves excellent tracking for slowly changing trajectories, even though it is designed only for set point regulation. The nonlinear controller dramatically outperforms a liner controller.

1. Introduction

Unmanned underwater vehicles (UUVs) is an area of growing interest due to their ability to operate at depths and in areas that are inaccessible to other types of vessels. The scope of their potential application varies from scientific research of ocean depths, surveillance and inspection of commercial undersea facilities and installations, to various military purposes. Due to the need to use nonlinear 6 DOF modelling (because of significant coupling between the rotational and translational motion), UUVs are a considerably more challenging problem for control design than surface ships.

Leonard's pioneering work on stability analysis and control of UUVs, using tools from geometric mechanics (Leonard 1995a–c, 1996a,b, and 1997a,b, Leonard and Krishnaprasad 1994a,b, Leonard and Marsden 1997), has solved problems such as control after actuator failure and stabilization of steady motion. Fossen and coworkers (Fjellstad and Fossen 1994a–c, Fossen and Fjellstad 1993, 1995, Schjolberg and Fossen 1994) developed control schemes based on feedback linearization which allow (local) trajectory tracking and can accommodate some actuator dynamics. Other notable work on control of UUVs includes that by Pettersen and Egeland (1996), Cristi *et al.* (1990), Healey and Lienard (1993), and Juul *et al.* (1994).

In this paper the problem of set point regulation for a general nonlinear 6 DOF model of a UUV with kinematics represented by Modified Rodrigues parameters is addressed. A form of a Lyapunov function with a logarithmic term in the kinematic variable proposed by Tsiotras (1996) for a 3 DOF spacecraft problem is employed. A similar idea was independently arrived at and used by Fjellstad and Fossen (1994a) for stabilization to the origin in a 6 DOF model with classical Cayley–Rodrigues parameters. This paper extends the result of Fjellstad and Fossen (1994a) to an arbitrary constant set of points and shows by simulation that the same method achieves good approximate tracking for slow time-varying trajectories.

The control law designed in this paper does not require any information about the inertia matrix of the UUV and is therefore *robust* with respect to uncertainties of system parameters. It is also shown that the control law is *linearly bounded* in terms of the regulation error, which guarantees that the control effort will remain reasonable even for large deviations from the desired position and attitude.

The paper is organized as follows. Section 2 introduces attitude representation in Modified Rodrigues parameters and 6 DOF equations of motion of the UUV. A controller based on Lyapunov theory is

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designed in section 3, while the simulation study is presented in section 4.

2. Model of UUVs

2.1. *Attitude representation using Modified Rodrigues parameters*

Let θ denote the principal angle and let λ denote the principal axis associated with Euler's Theorem. The Euler parameters (unit quaternions) $q = [\eta, \epsilon_1, \epsilon_2, \epsilon_3]^{\mathrm{T}}$ are defined as

$$\eta_q = \cos\left(\frac{\theta}{2}\right), \quad \epsilon_i = \lambda_i \sin\left(\frac{\theta}{2}\right), \quad i = 1, 2, 3 \qquad (2.1)$$

and they satisfy the constraint

$$\eta_q^2 + \epsilon^{\mathrm{T}}\epsilon = \eta_q^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 = 1. \qquad (2.2)$$

The attitude representation in unit quaternions is a four-dimensional parametrization and therefore it is non-minimal. Introducing a new set of coordinates (Cayley–Rodrigues parameters) defined as the ratio of unit quaternions

$$\rho_i = \frac{\epsilon_i}{\eta_q}, \qquad i = 1, 2, 3 \qquad (2.3)$$

the constraint (2.2) is eliminated. The vector $\rho = [\rho_1, \rho_2, \rho_3]^{\mathrm{T}}$ is related to the principal vector λ and principal angle θ as

$$\rho = \lambda \tan\left(\frac{\theta}{2}\right). \qquad (2.4)$$

The introduction of minimal three-parameter descriptions is mainly motivated by their potential advantages in stabilization and control related problems. Un-fortunately, all three parameter representations contain singularity points. One can see from equation (2.4) that this representation presents a singularity at $\theta = \pi$, i.e. the classical Cayley–Rodrigues parameters cannot be used for describing eigenaxis rotations of more than 180°. If instead of (2.3) a renormalization defined as

$$\sigma_i = \frac{\epsilon_i}{1 + \eta_q}, \qquad (i = 1, 2, 3) \qquad (2.5)$$

is introduced, one can easily show that the Modified Rodrigues parameter vector $\sigma = [\sigma_1, \sigma_2, \sigma_3]^{\mathrm{T}}$ is related to the principal vector and principal angle as

$$\sigma = \lambda \tan\left(\frac{\theta}{4}\right). \qquad (2.6)$$

It is obvious from equation (2.6) that the Modified Rodrigues parameters are superior to any other three-parameter representation. First, all eigenaxis rotations in the range $0 \le \theta < 360°$ are well defined. Second, unlike other three-parameter representations (Eulerian angles or Cayley–Rodrigues parameters) which eliminate an infinite number of possible orientation configurations due to singularity, this parametrization eliminates only one attitude configuration being singular (namely, $\theta = 360°$ implies $\eta_q = -1$, and (2.2) implies $\epsilon = 0_{3\times 1}$).

2.2. *Kinematic equations of motion*

The work on the UUVs starts with the representation of the kinematics in the Modified Rodrigues parameters. The kinematic model describes the geometric relationship between earth-fixed and body-fixed reference frames. The motion in body-fixed and earth-fixed reference frames is related through a transformation matrix $J(\eta)$ as

$$\eta = J(\eta)\upsilon = \begin{bmatrix} J_1(\eta) & 0_{3\times 3} \\ 0_{3\times 3} & J_2(\eta) \end{bmatrix} \upsilon. \qquad (2.7)$$

The vectors η and υ are defined as

$$\eta = \begin{bmatrix} x \\ \sigma \end{bmatrix}, \qquad \upsilon = \begin{bmatrix} \nu \\ \omega \end{bmatrix}, \qquad (2.8)$$

where $x = [x_1, x_2, x_3]^{\mathrm{T}}$ represents the position vector in the inertial reference frame, $\sigma = [\sigma_1, \sigma_2, \sigma_3]^{\mathrm{T}}$ the vector of Modified Rodrigues parameters representing the attitude, ν and ω are the linear and angular velocities in the body-fixed reference frame. The matrices $J_1(\eta)$ and $J_2(\eta)$ are defined as

$$J_1(\sigma) = I + \frac{8}{(1 + |\sigma|^2)^2} S(\sigma)\left(S(\sigma) - \frac{1 - |\sigma|^2}{2} I\right) \qquad (2.9)$$

$$J_2(\sigma) = \frac{1}{2}\left[I + \sigma\sigma^{\mathrm{T}} - S(\sigma) - \frac{1 + \sigma^{\mathrm{T}}\sigma}{2} I\right] \qquad (2.10)$$

where the matrix $S(\cdot)$ is defined as

$$S(\sigma) = \begin{bmatrix} 0 & \sigma_3 & -\sigma_2 \\ -\sigma_3 & 0 & \sigma_1 \\ \sigma_2 & -\sigma_1 & 0 \end{bmatrix}. \qquad (2.11)$$

From the definition it is obvious that the matrix S is skew-symmetric with the property

$$S(a)b = -a \times b = b \times a = -S(b)a. \qquad (2.12)$$

The matrices $J_1(\sigma)$ and $J_2()$ are invertible with the properties

$$J_1(\sigma)^{-1} = J_1(\sigma)^{\mathrm{T}}$$

$$J_2(\sigma)^{-1} = \left(\frac{4}{1 + |\sigma|^2}\right)^2 J_2(\sigma)^{\mathrm{T}}. \qquad (2.13)$$

Finally, the system consisting of equations (2.9) and (2.10) can be written as

$$\dot{x} = \left[I + \frac{8}{(1+|\sigma|^2)^2} S(\sigma)\left(S(\sigma) - \frac{1-|\sigma|^2}{2}\right) I\right] v \tag{2.14}$$

$$\dot{\sigma} = \frac{1}{2}\left[I + \sigma\sigma^{\mathrm{T}} - S(\sigma) - \frac{1+\sigma^{\mathrm{T}}\sigma}{2} I\right]\omega. \tag{2.15}$$

2.3. *Dynamic equations of motion*

The dynamic equations (Newton's equations of motion for a rigid body with respect to the body-fixed reference frame) are given by:

$$m(\dot{v} + \omega \times v + \dot{\omega} \times r_g + \omega \times (\omega \times r_g)] = f \tag{2.16}$$

$$J\dot{\omega} + \omega \times (J\omega) + mr_g \times (\dot{v} + \omega \times v) = \tau, \tag{2.17}$$

where $r_g = [x_g, y_g, z_g]^{\mathrm{T}}$ is the centre of gravity, m is the mass of the UUV, $J = J^{\mathrm{T}} > 0$ is the inertia matrix of UUV, and f and τ are vectors of external forces and moments (including control and hydrodynamic forces/moments). Upon the inclusion of hydrodynamic forces/moments, it was shown in Fossen (1994) that equations (2.16) and (2.17) give

$$M\dot{\upsilon} + C(\upsilon)\upsilon + D(\upsilon)\upsilon + g(\sigma) = u \tag{2.18}$$

where u is a vectr of actuator control forces and moments, M is a constant positive definite and symmetric inertia matrix (which includes the added inertia), $C(\upsilon)$ is a skew-symmetric matrix linear in υ containing the Coriolis and centripetal terms, $D(\upsilon)$ is a positive definite damping matrix containing drag and lift terms (and possibly skin friction and viscous damping), and $g(\sigma)$ is the vector of restoring (gravitational and buoyant) forces/moments. The wave-induced forces/moments are assumed to be negligible, since UUVs operate below the wave affected zone (the operating depth is significantly greater than 20 m). The variations of water density are also considered to be negligible.

2.4. *Restoring forces/moments*

Let m be the mass of UUV (including water in free floating spaces), ∇ the volume of fluid displaced by the vehicle, g_0 the acceleration of gravity (positive downwards) and ρ_f the fluid density. Then, the submerged weight of the body and buoyancy force in the earth-fixed frame are respectively $W = mg_0$ and $B = \rho_f g_0 \nabla$. Assuming that UUV is neutrally buoyant $(W = B)$ and that the distance between the centre of gravity CG and centre of buoyancy CB in the body-fixed frame is $\overline{BG} = [\overline{BG_x}, \overline{BG_y}, \overline{BG_z}) = [0, 0, \overline{BG_z}]$, the vector of gravitational and buoyant forces/moments becomes

$$g(\sigma) = 2\overline{BG_z} W \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{4}{(1+|\sigma|^2)^2}\sigma_2\sigma_3 + \dfrac{2(1-|\sigma|^2)}{(1+|\sigma|^2)^2}\sigma_1 \\ \dfrac{2(1-|\sigma|^2)}{(1+|\sigma|^2)^2}\sigma_2 - \dfrac{4}{(1+|\sigma|^2)^2}\sigma_1\sigma_3 \\ 0 \end{bmatrix}. \tag{2.19}$$

3. Control of UUVs

In order to achieve stabilization of the system (2.14), (2.15), (2.18) at the point $(x, \sigma, \upsilon) = (x_d, \sigma_d, 0)$, regulation error variables $\tilde{x}$ and $\tilde{\sigma}$ are introduced as

$$\tilde{x} = x - x_d \tag{3.1}$$

$$\tilde{\sigma} = \sigma - \sigma_d. \tag{3.2}$$

Taking

$$V = \frac{1}{2}\upsilon^{\mathrm{T}} M\upsilon + \frac{k_1}{2}\tilde{x}^{\mathrm{T}}\tilde{x} + k_2 \ln(1 + \tilde{\sigma}^{\mathrm{T}}\tilde{\sigma}) \tag{3.3}$$

as a Lyapunov function candidate, one finds its derivative to be

$$\begin{aligned}\dot{V} &= -\upsilon^{\mathrm{T}} C(\upsilon)\upsilon - \upsilon^{\mathrm{T}} D(\upsilon)\upsilon + \upsilon^{\mathrm{T}}(u - g(\sigma)) \\ &\quad + \frac{k_2\tilde{\sigma}^{\mathrm{T}}}{1+|\tilde{\sigma}|^2}\left[I + \sigma\sigma^{\mathrm{T}} - S(\sigma) - \frac{1+\sigma^{\mathrm{T}}\sigma}{2} I\right]\omega \\ &\quad + k_1\tilde{x}^{\mathrm{T}}\left[I + \frac{8}{(1+|\sigma|^2)^2} S(\sigma)\left(S(\sigma) - \frac{1-|\sigma|^2}{2} I\right)\right] v.\end{aligned} \tag{3.4}$$

Since $C(\upsilon)$ is skew-symmetric and $D(\upsilon)$ is positive definite one gets

$$\begin{aligned}\dot{V} &\le \upsilon^{\mathrm{T}}(u - g(\sigma)) \\ &\quad + \frac{k_2\omega^{\mathrm{T}}}{1+|\tilde{\sigma}|^2}\left[I + \sigma\sigma^{\mathrm{T}} - S(\sigma) - \frac{1+\sigma^{\mathrm{T}}\sigma}{2} I\right]^{\mathrm{T}}\tilde{\sigma} \\ &\quad + k_1 v^{\mathrm{T}}\left[I + \frac{8}{(1+|\sigma|^2)^2} S(\sigma)\left(S(\sigma) - \frac{1-|\sigma|^2}{2} I\right)\right]^{\mathrm{T}}\tilde{x} \\ &= \upsilon^{\mathrm{T}}(u - g) + \upsilon^{\mathrm{T}} \\ &\quad \times \begin{bmatrix} k_1\left[I + \dfrac{8}{(1+|\sigma|^2)^2} S(\sigma)\left(S(\sigma) + \dfrac{1-|\sigma|^2}{2} I\right)\right]\tilde{x} \\ k_2\left[I + \sigma\sigma^{\mathrm{T}} + S(\sigma) - \dfrac{1+\sigma^{\mathrm{T}}\sigma}{2} I\right]\tilde{\sigma} \end{bmatrix}.\end{aligned} \tag{3.5}$$

Denoting

$$u = \begin{bmatrix} u_v \\ u_\omega \end{bmatrix}, \qquad g = \begin{bmatrix} g_v \\ g_\omega \end{bmatrix}, \tag{3.6}$$

and using the facts that $\sigma = \sigma_d + \tilde{\sigma}$ and $S(\tilde{\sigma}) \times \tilde{\sigma} = \tilde{\sigma} \times \tilde{\sigma} = 0$ one gets

$$\dot{V} \le \upsilon^{\mathrm{T}} \begin{bmatrix} u_v - g_v + k_1 \tilde{x} + k_1 \left[\dfrac{8}{(1+|\sigma|^2)^2} S(\sigma) \times \left(S(\sigma) + \dfrac{1-|\sigma|^2}{2} I \right) \right] \tilde{x} \\ u_\omega - g_\omega + k_2 \tilde{\sigma} + \dfrac{k_2}{1+|\tilde{\sigma}|^2} \times \left[\sigma_d \sigma_d^{\mathrm{T}} + \sigma_d \tilde{\sigma}^{\mathrm{T}} + \tilde{\sigma} \sigma_d^{\mathrm{T}} + S(\sigma_d) - \dfrac{1+|\sigma|^2}{2} I \right] \tilde{\sigma} \end{bmatrix}. \tag{3.7}$$

Choosing

$$u_v = g_v(\sigma) - k_1 \tilde{x} - c_1 v - k_1 \frac{8}{(1+|\sigma|^2)^2} \times S(\sigma) \left(S(\sigma) + \frac{1-|\sigma|^2}{2} I \right) \tilde{x} \tag{3.8}$$

$$u_\omega = g_\omega(\sigma) - k_2 \tilde{\sigma} - c_2 \omega - \frac{k_2}{1+|\tilde{\sigma}|^2} \times \left[\sigma_d \sigma_d^{\mathrm{T}} + \sigma_d \tilde{\sigma}^{\mathrm{T}} + \tilde{\sigma} \sigma_d^{\mathrm{T}} + S(\sigma_d) - \frac{1+|\sigma|^2}{2} I \right] \tilde{\sigma} \tag{3.9}$$

yields

$$\dot{V} \le -c_1 |v|^2 - c_2 |\omega|^2. \tag{3.10}$$

By LaSalle's invariance theorem (Khalil 1996), the solutions converge to the largest invariant set inside the set $v = \omega = 0$, that is, the set where $u = g$:

$$\begin{aligned} 0 &= -k_1 \tilde{x} - k_1 \left[\frac{8}{(1+|\sigma|^2)^2} S(\sigma) \left(S(\sigma) + \frac{1-|\sigma|^2}{2} I \right) \right] \tilde{x} \\ 0 &= -k_2 \tilde{\sigma} - \frac{k_2}{1+|\tilde{\sigma}|^2} \times \left[\sigma_d \sigma_d^{\mathrm{T}} + \sigma_d \tilde{\sigma}^{\mathrm{T}} + \tilde{\sigma} \sigma_d^{\mathrm{T}} + S(\sigma_d) - \frac{1+|\sigma|^2}{2} I \right] \tilde{\sigma}, \end{aligned} \tag{3.11}$$

which is equivalent to

$$\begin{aligned} J_1(\eta)^{\mathrm{T}} \tilde{x} &= 0 \\ J_2(\eta)^{\mathrm{T}} \tilde{\sigma} &= 0. \end{aligned} \tag{3.12}$$

Since both $J_1(\eta)$ and $J_2(\eta)$ are invertible, the equilibrium $\tilde{x} = \tilde{\sigma} = v = \omega = 0$ is globally asymptotically stable.

Theorem 3.1: *The system (2.7), (2.18), (3.8), (3.9) is globally asymptotically stable at $\eta = \upsilon = 0$.*

The control law (3.8), (3.9) does not require information about the parameters of the inertia matrix, damping matrix, and the Coriolis/centripetal matrix, and is thus robust to parametric uncertainties in the UUV model.

Noting that $|a \times b| \le |a||b|$, one can observe that $u - g(\sigma)$ is linearly bounded (in $\tilde{x}, \tilde{\sigma}, v, \omega$) because

$$\left| \left[\frac{8}{(1+|\sigma|^2)^2} S(\sigma) \left(S(\sigma) + \frac{1-|\sigma|^2}{2} I \right) \right] \tilde{x} \right| \tag{3.13}$$

$$= \left| \frac{8}{(1+|\sigma|^2)^2} \left(\tilde{x} \times \sigma \times \sigma + \frac{1-|\sigma|^2}{2} (\tilde{x} \times \sigma) \right) \right| \le 12 |\tilde{x}| \tag{3.14}$$

and

$$\left| \frac{1}{1+|\tilde{\sigma}|^2} \left[\sigma_d \sigma_d^{\mathrm{T}} + \sigma_d \tilde{\sigma}^{\mathrm{T}} + \tilde{\sigma} \sigma_d^{\mathrm{T}} + S(\sigma_d) - \frac{1+|\sigma|^2}{2} I \right] \tilde{\sigma} \right| \tag{3.15}$$

$$= \left| \frac{1}{1+|\tilde{\sigma}|^2} \left[\sigma_d \sigma_d^{\mathrm{T}} \tilde{\sigma} + \sigma_d \tilde{\sigma}^{\mathrm{T}} \tilde{\sigma} + \tilde{\sigma} \sigma_d^{\mathrm{T}} \tilde{\sigma} + \tilde{\sigma} \times \sigma_d - \frac{1 + (\sigma_d + \tilde{\sigma})^{\mathrm{T}} (\sigma_d + \tilde{\sigma})}{2} \tilde{\sigma} \right] \right| \tag{3.16}$$

$$\le \frac{7}{2} |\sigma_d| + \frac{3}{4} |\sigma_d|^2 + \frac{1}{2} |\tilde{\sigma}|. \tag{3.17}$$

This is a remarkable property for a system with nonlinearities of quadratic growth and indicates that, in the control design, cancellation of nonlinearities that are not of destabilizing nature is avoided. As a result, wasting control effort is avoided and robustness is improved.

4. Simulation study

The controller designed in section 3 was simulated for UUV with the following set of parameters (Fjellstad and Fossen 1994a):

Inertia matrix

$$\begin{aligned} M &= \mathrm{diag}\{m_{11}, m_{22}, m_{33}, m_{44}, m_{55}, m_{66}\} \\ &= \mathrm{diag}\{215, 265, 265, 40, 80, 80\}. \end{aligned} \tag{4.1}$$

Damping matrix

$$D(\upsilon) = \mathrm{diag}\{70, 100, 100, 30, 50, 50\} + \mathrm{diag}\{100|v_1|, 200|v_2|, 200|v_3|, 50|\omega_1|, 100|\omega_2|, 100|\omega_3|\}, \quad (4.2)$$

Coriolis and centripetal terms matrix

$$C(\upsilon) = \begin{bmatrix} 0 & 0 & 0 & 0 & m_{33}v_3 & -m_{22}v_2 \\ 0 & 0 & 0 & -m_{33}v_3 & 0 & m_{11}v_1 \\ 0 & 0 & 0 & m_{22}v_2 & -m_{11}v_1 & 0 \\ 0 & m_{33}v_3 & -m_{22}v_2 & 0 & m_{66}\omega_3 & -m_{55}\omega_2 \\ -m_{33}v_3 & 0 & m_{11}v_1 & -m_{66}\omega_3 & 0 & m_{44}\omega_1 \\ m_{22}v_2 & -m_{11}v_1 & 0 & m_{55}\omega_2 & -m_{44}\omega_1 & 0 \end{bmatrix}. \quad (4.3)$$

The UUV has mass of $m = 185\,\mathrm{kg}$ and is assumed to be neutrally buoyant. It is assumed that the centre of buoyancy coincides with the centre of gravity and that environmental disturbances are negligible.

When plotting three component vectors on a single figure, components one, two, and three are represented with dotted, dash–dot, and dashed lines respectively. For representing the attitude in quaternions $q = [\eta, \epsilon^{\mathrm{T}}]^{\mathrm{T}}$, η will be plotted as a solid line and the components of the vector ϵ will conform to the vector representation rule.

Figure 1 shows the time evolution of position x, attitude q, and actuator control forces and moments and u_v and u_ω. The initial values were chosen to be $\eta(0) = [10, 10, 10, 1, 1, 1]^{\mathrm{T}}$ for the position/attitude vector in the earth-fixed reference frame, $\upsilon(0) = 0_{6\times 1}$ for the velocity vector in the body-fixed reference frame, and the regulation set-point was chosen as $\eta_{\mathrm{d}} = [0, 0, 0, 0, 0, 0]^{\mathrm{T}}$. Control parameters were chosen to be $k_1 = 50$, $k_2 = 50$, $c_1 = 10$, $c_2 = 10$. It should be noted that $\sigma(0) = [1, 1, 1]^{\mathrm{T}}$ corresponds to $q(0) = [-0.5, 0.5, 0.5, 0.5]^{\mathrm{T}}$, i.e. to the initial eigenaxis rotation of 240°.

In order to understand the importance of the nonlinear part of the control law, previous simulation is repeated with all the initial conditions and parameter values the same, but with a linear control law (higher order terms in $\tilde{x}$ and $\tilde{\sigma}$ in the original control law are neglected giving the 'proportional feedback'

$$u_v = -k_1 x - c_1 v, \qquad u_\omega = -\frac{k_2}{2}\sigma - c_2\omega$$

for this particular simulation). Results are shown in figure 2. The system will eventually reach the desired values for attitude/position using the linear control law as expected (it was proven in section 3 that a linear bound can be found for the nonlinear portion of the control law), but will experience an extremely long period of oscillatory behaviour with large overshoot in the desired position and waste of control effort.

Next, the proposed design is applied for set point regulation to try to track a time-varying reference signal. Figures 3 and 4 show the time evolution of position tracking error $\tilde{x}$, attitude q, actuator control forces and moments u_v and u_ω, and UUV's trajectory in the $x_1x_2x_3$-space for tracking of the desired trajectory $x_{\mathrm{d}}(t) = [10\sin(0.01t),\ 10\cos(0.01t), 0]^{\mathrm{T}}$. Evidently, the

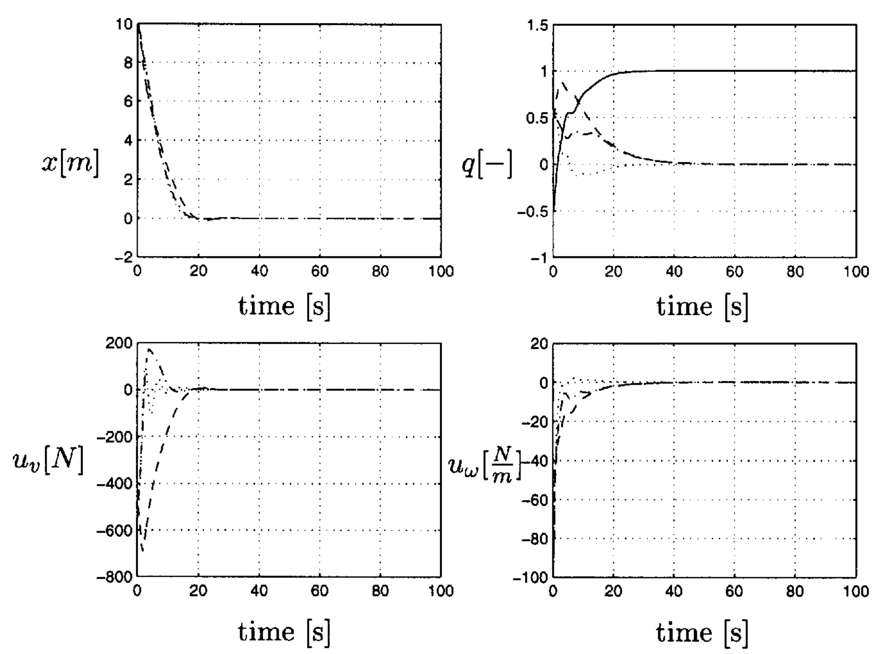

Figure 1. Position/attitude control for $q(0) = [-0.5, 0.5, 0.5, 0.5]^{\mathrm{T}}$.

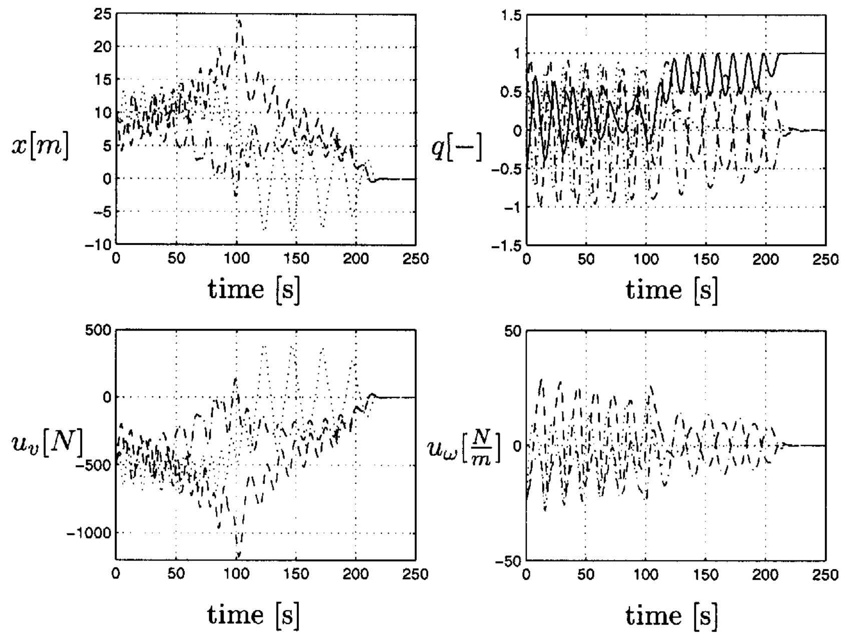

Figure 2. Position/attitude control for $q(0) = [-0.5, 0.5, 0.5, 0.5]^{\mathrm{T}}$ using linear control law.

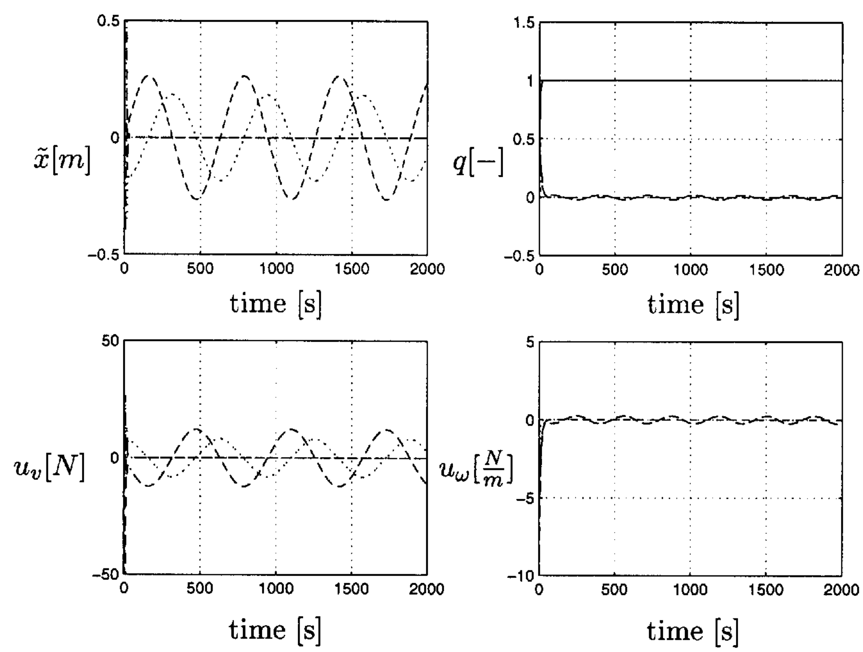

Figure 3. Position tracking for $x_d(t) = [10\sin(0.01t), 10\cos(0.01t), 0]^{\mathrm{T}}$ (the time evolution of position tracking error $\tilde{x}$, attitude q, and actuator control forces and moments u_v and u_ω).

control design achieves excellent tracking (the error is less than 2.5% of the desired amplitude) after less than 50 s, with small control effort. All initial conditions and control parameters are the same as in the previous two cases except for the initial condition for attitude which is taken as $\sigma(0) = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]^{\mathrm{T}}$ (this corresponds to $q(0) = [0.5, 0.5, 0.5, 0.5]^{\mathrm{T}}$).

Good tracking can be achieved (under 7.5% error) for a significantly higher frequency of desired trajectory $(x_{\mathrm{d}}(t) = [10\sin(0.1t), 10\cos(0.1t), 0]^{\mathrm{T}})$ with higher control gains $k_1 = 500$, $k_2 = 500$, $c_1 = 50$, $c_2 = 50$, for all the initial conditions the same as in the previous simulation (figures 5 and 6). Owing to the highly nonlinear nature of the system, the control effort is going to be much larger compared with the previous case.

5. Conclusions

A controller based on Lyapunov theory which achieves global attitude/position set point regulation for a 6 DOF UUV model with kinematics represented

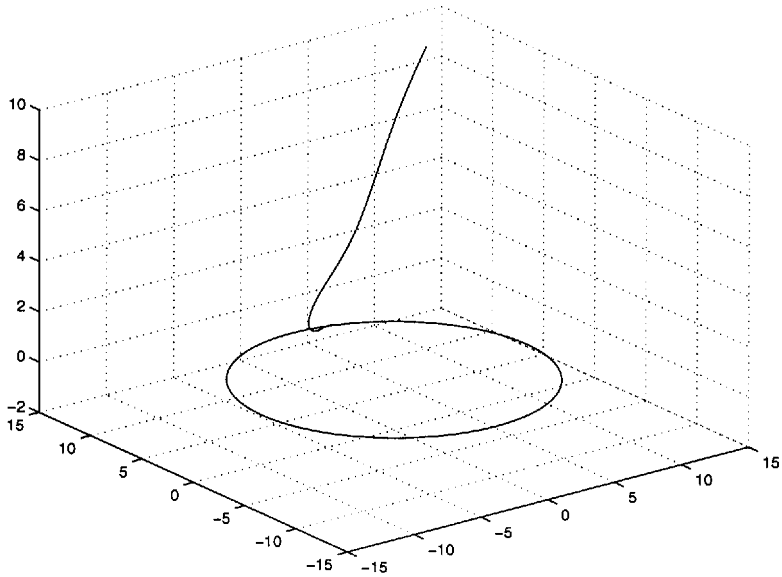

Figure 4. The trajectory in the $x_1x_2x_3$-space ($x_d(t)=[10\sin(0.01t), 10\cos(0.01t), 0]^T$).

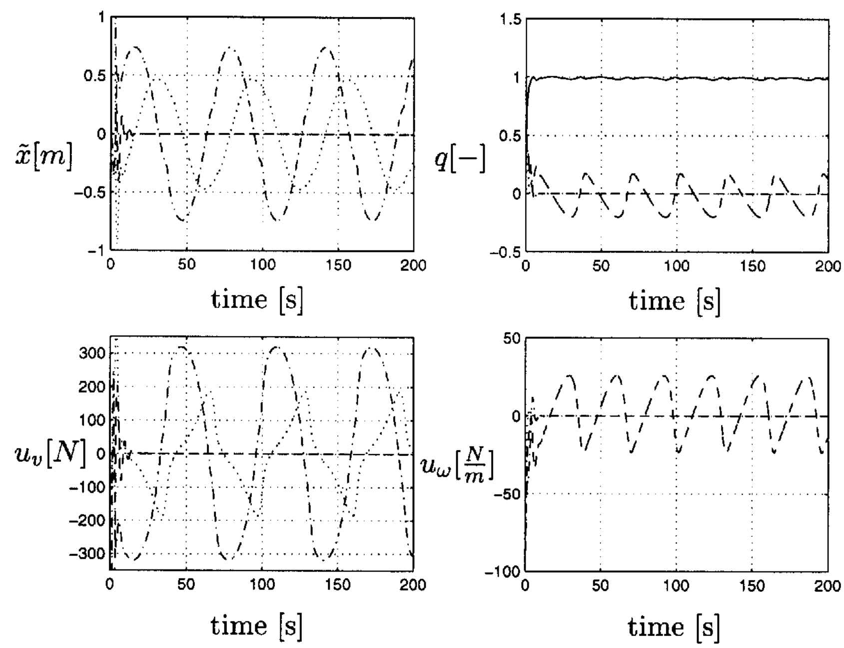

Figure 5. Position tracking for $x_d(t)=[10\sin(0.1t), 10\cos(0.1t), 0]^T$ (the time evolution of position tracking error $\tilde{x}$, attitude q, and actuator control forces and moments u_v and u_ω).

in Modified Rodrigues parameters has been derived. The controller does not require any information about UUV model parameters except those related to restoring forces and is therefore robust with respect to parametric uncertainties. The control law avoids cancellation of nonlinearities and is shown to be linearly bounded.

The simulation study indicates that the overall system performance is excellent. The controller, although designed for constant set point regulation, achieves a small tracking error even for slowly time-varying desired trajectories.

The control design in this paper achieves stabilization but is not optimal. It will be of interest to extend the inverse optimal design for the 3 DOF spacecraft model in Krstić and Tsiotras (1998) to the 6 DOF UUV model. Inverse optimality is achieved using the backstepping method (Krstić *et al.* 1995) and guarantees infinite gain margin and fuel efficiency for the UUV.

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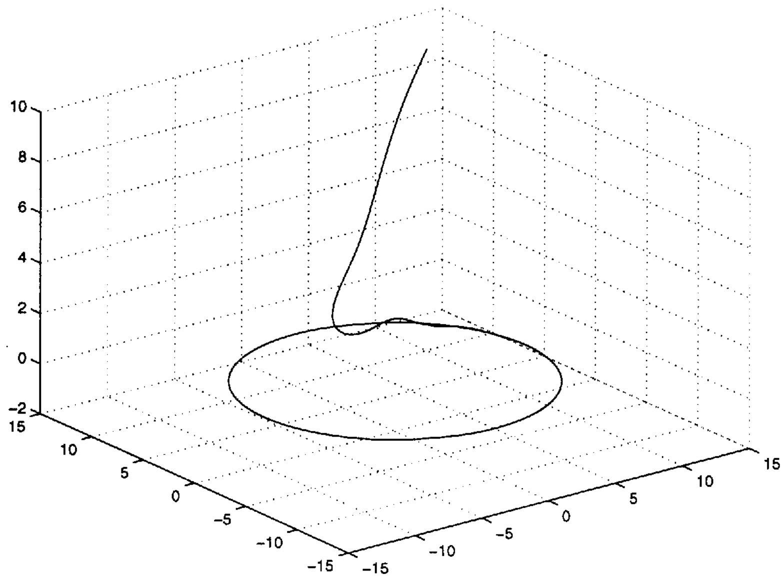

Figure 6. The trajectory in the $x_1x_2x_3$-space ($x_d(t) = [10\sin(0.1t), 10\cos(0.1t), 0]^T$).

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