On the applicability of PID control to nonlinear second-order systems

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Proportional-integral-derivative (PID) controllers, which react to the regulated signal's present, past and predicted future behavior, are popular, for two reasons. First, they are model-free and require minimal background and preliminary effort from the user. Second, they possess the capability—albeit limited—of shaping the system’s both transient and asymptotic performance.

The great popularity of PID controllers goes hand in hand with their widespread misuse. With only one degree of freedom to influence asymptotic performance (the I part), PID controllers are capable of only the rejection of constant disturbances and tracking of constant commands. With only two degrees of freedom for shaping the system’s transients (the PD part), PID controllers are sufficiently general only for systems that have up to two states, namely second-order systems.

Zhao and Guo [1] from the Chinese Academy of Sciences provide a highly valuable paper for both the control practitioner and for the theorist, by developing a detailed analysis of applying PID control to general second-order systems. They note that fully actuated mechanical systems, modeled by Newton’s second law, are second-order and, therefore, the rigorous focus of their work comes with little sacrifice in terms of relevance.

In Theorem 1, the authors prove that, if the proportional gain is larger than an ‘anti-stiffness’ function of the plant, then the integral gain can be chosen sufficiently small, but positive, so that global output regulation is achieved. The general result of Theorem 1 is both specialized in Proposition 1 and Corollary 1 and further generalized in Theorem 2 but in the absence of the I-term for plants for which the setpoint is an equilibrium.

The results are strong and their proofs are not elementary. The most general results are proved using Lyapunov-like techniques, including those by LaSalle and Yoshizawa, as well as by eigenvalue-based techniques. For the case in which the I-action is not needed, a theorem based on the Markus-Yanabe Jacobian-based conjecture is employed in the proof.

The authors provide conditions on the PID gains such that global regulation is guaranteed independently of the output setpoint. Such results are highly valuable for the practitioner by giving large ranges of ‘safe’ controller parameters. However, as the authors indicate in the introduction, from the early days of PID control, there has, additionally, been interest in providing recipes to the user for not only safe choices, but the best choices of the three parameters. The Ziegler-Nichols procedure is the best-known recipe, but also known to exhibit interior transients while asymptotically rejecting constant disturbances.

Most of the nine methods listed by the authors are developed based on the assumption of linearity of the plant.

The only method known to us in which the parameter optimization is conducted regardless of the plant’s linearity or nonlinearity, or event the plant’s dimension, is the extremum-seeking (ES) method in [2]. This method views the system’s response as a map from the three gains into a functional of the system’s response over a time interval of interest. For linear systems, ES matches or beats the best performance attainable by linearity-based methods. An alternative to deterministic ES [2] is the stochastic ES method in [3]. Hence, the papers [2,3] can be viewed as companion references for the practical user of the results in the highlighted paper. However, the user should be aware that, for nonlinear plants, optimal parameter choices will be dependent on the plant’s initial condition and the setpoint value.

REFERENCES