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Brief Paper

Robustness of Adaptive Nonlinear Control to Bounded Uncertainties*

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Key Words—Adaptive control; robust control; nonlinear systems; disturbance rejection; recursive approaches.

Abstract—Two robust adaptive control methods are outlined for a class of nonlinear systems. The first method is based on the tuning function design of Krstić *et al.* (1992), and the second method is based on the modular design of Krstić and Kokotović (1995). Both methods guarantee robustness with respect to bounded uncertainties and exogenous disturbances, and L_∞/L_2 estimates are given on the effects of these uncertainties/disturbances on the tracking error.

1. *Introduction*

A state-space model of a plant is likely to contain uncertain nonlinearities. Depending on how such uncertainties are characterized, different nonlinear design techniques may apply. Robust control techniques, including sliding mode control, account for arbitrarily fast time variations of the uncertainty (such as those arising from exogenous disturbances), but usually high gain is required to guarantee a small tracking error. In contrast, adaptive control techniques only account for constant parametric uncertainties, but they guarantee convergence of the tracking error to zero without high gain The combination of tools from both robust and adaptive control is likely to produce designs better than those produced by either method alone. To illustrate this point, suppose the model of our plant contains an uncertain nonlinearity $f(x, t)$, where x is the state vector. We first split this nonlinearity into a known nominal part $f_0(x)$ plus an uncertain part $\Delta f(x, t)$ as follows:

$$f(x,t) = f_0(x) + \Delta f(x,t). \tag{1}$$

Robust control techniques would use high gain to reduce the effect of $\Delta f(x,t)$ on the error signals. A less conservative approach would be to further split $\Delta f(x\,t)$ into a parametric part $p(x)\theta$ plus a time-varying part $\delta(x,t)$ as follows:

$$f(x,t) = f_0(x) + p(x)\theta + \delta(x,t), \tag{2}$$

where θ is an unknown constant parameter vector. If adaptive control techniques can be used to account for θ, then the robust control techniques need only counteract the uncertainty $\delta(x,t)$ which is likely to be much smaller than the original uncertainty $\Delta f(x,t)$. Thus, the performance of a robust controller might be greatly improved if it includes some adaptation. Likewise, adaptive controllers should benefit from robustification.

Such considerations motivate the development of robust versions of nonlinear adaptive control designs, and results in this direction are beginning to appear (Polycarpou and Ioannou, 1993; Yao and Tomizuka, 1995; Pan and Başar, 1995; Freeman and Kokotović, 1995). In this paper, we outline two design methods which combine tools from nonlinear robust control (Marino and Tomei, 1993; Qu, 1993; Freeman and Kokotović, 1993) and nonlinear adaptive control (Krstić *et al.*, 1995, and the references therein). The first method is based on the tuning function design of Krstić *et al.* (1992), and the second method is based on the modular design of Krstić and Kokotović (1995).

2. *Class of systems*

We consider single-input–single-output nonlinear systems which can be transformed into the following *strict feedback form*:

$$\begin{aligned}
\dot{x}_1 &= x_2 + p_1(x_1)\theta + q_1(x_1)w,\\
\dot{x}_2 &= x_3 + p_2(x_1,x_2)\theta + q_2(x_1,x_2)w,\\
&\ \vdots\\
x_n &= u + p_n(x_1,\,\dots,\,x_n)\theta + q_n(x_1,\,\dots,\,x_n)w,\\
y &= x_1,
\end{aligned} \tag{3}$$

where $x = [x_1 \cdots x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the variable to be tracked, $\theta \in \mathbb{R}^p$ is a unknown constant parameter vector, $w \in \mathbb{R}^q$ is an unmeasured disturbance input, and the functions p_i and q_i are smooth. The main structural condition is that the functions p_i and q_i depend only on the first i state variables.

The disturbance input w need not be purely exogenous; we allow it to be any (sufficiently regular) uniformly bounded nonlinear function $w = w(x, u, t)$, and we do not require knowledge of its bound. This allows us to consider systems outside the class of parametric strict feedback systems identified by Kanellakopoulos *et al.* (1991). For example, consider the third-order system

$$\begin{aligned}
\dot{x}_1 &= x_2 + \alpha x_1^3 + \beta x_1^2\cos(x_3u) + n(t),\\
\dot{x}_2 &= x_3\\
\dot{x}_3 &= u,\\
y &= x_1,
\end{aligned} \tag{4}$$

where α and β are unknown constant parameters and $n(t)$ is an unmeasured bounded exogenous disturbance. If we were to make the natural choice of $\theta = [\alpha\ \beta]^{\mathrm{T}}$ and $w = n(t)$, than this system would not be in the form (3) because the function p_1 would depend on x_3 and u. However, if we chose $\theta = \alpha$ and $w = [\beta\cos(x_3u)\ n(t)]^{\mathrm{T}}$ with $p_1(x_1) = x_1^3$ and $q_1(x_1) = [x_1^2\ 1]$, then we do obtain a system in the form (3). This choice is possible because even though the variables x_3 and u enter the equation for $\dot{x}_1$ and thus appear too early, they do so through

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the bounded cosine nonlinearity. Note that we also have the option of treating the unknown constant α as part of the disturbance vector w, but this would lead to a more conservative design with a less desirable trade-off between control effort and tracking error.

We can also allow the uncertainty w to include uniformly bounded functions of outputs of unmodeled dynamics. For example, the nonlinearity $\cos(x_3u)$ in equation (4) could be replaced by $\cos(\xi^T\xi)$, where ξ is governed by dynamics of the form

$$\dot{\xi} = \Xi(\xi, x, u, t) \tag{5}$$

and it is known that ξ is bounded whenever x and u are bounded.

3. *Control objective and main reuslts*

Our control objective is to force the output $y(t)$ to track a given bounded reference signal $y_r(t)$ having bounded derivatives. We will consider dynamic state feedback controllers of the form

$$u = \mu(x, \hat{\theta}, \eta, y_r, \dot{y}_r, \ldots, y_r^{(n)}), \tag{6}$$

$$\dot{\hat{\theta}} = T(x, \hat{\theta}, \eta, y_r, \dot{y}_r, \ldots, y_r^{(n)}), \tag{7}$$

$$\dot{\eta} = H(x, \hat{\theta}, \eta, y_r, \dot{y}_r, \ldots, y_r^{(n)}), \tag{8}$$

where μ and H are smooth functions, T is a locally Lipschitz continuous function, and $\hat{\theta} \in \mathbb{R}^p$ is of the same dimension as the unknown constant parameter θ. The design of the controller will be based on knowledge of a compact convex set $\Theta \subset \mathbb{R}^p$ in which the true parameter θ is known to lie.

We require the closed-loop system to have the following properties: for every initial condition $x(0)$ of the plant, every admissible† pair of initial conditions $\hat{\theta}(0)$, $\eta(0)$ of the controller, every bounded disturbance $w \in L_\infty$, every bounded reference signal $y_r(t)$ having bounded derivatives, and every constant parameter $\theta \in \Theta$,

P1. *Global boundedness*: the states $x(t)$, $\hat{\theta}(t)$, and $\eta(t)$ exist and are bonded for all $t \geq 0$.

P2. *Asymptotic tracking*: if $w \equiv 0$, then $y(t) \to y_r(t)$ and $\hat{\theta}(t) \to \Theta$ as $t \to \infty$.

P3. *Finite L_∞-gain*: the tracking error $y(t) - y_r(t)$ satisfies

$$\|y - y_r\|_\infty \leq \lambda\|w\|_\infty + \lambda_0, \tag{9}$$

where λ is constant and λ_0 depends only on initial conditions, and.

P4. *Finite L_2-gain*: if $w \in L_2$, *then* $(y - y_r) \in L_2$ and

$$\|y - y_r\|_2 \leq \chi\|w\|_2 + \chi_0, \tag{10}$$

where χ is constant and χ_0 depends only on initial conditions.

Note that in the asymptotic tracking property P2, we do not require the state $\hat{\theta}$ to converge to the true value of the parameter θ in the absence of disturbances; we only require that it converge to the set Θ. Also, the finite-gain properties P3 and P4 are with respect to the tracking error only; we do not require a finite gain between the disturbances and the other signals in the system. Finally, χ_0 will be zero for some initial condition, but λ_0 will have a minimum value of $\lambda^* > 0$ for all initial conditions. This minimum value will be proportional to the constant M given by

$$M := \max_{\hat{\theta} \in \Theta} |\theta - \hat{\theta}|. \tag{11}$$

Note that M is bounded from above by the diameter of the set Θ in which the parameter θ is known to lie.

Theorem 1. There exist functions μ, T, and H such that controller (6)–(8) meets the design objective P1–P4 for system (3). Furthermore, the constants λ, λ^*, and χ can be reduced arbitrarily by using high gain in the controller.

†In the modular design, the controller initial conditions must lie in certain sets.

In the next sections, we outline two different proofs of this theorem which correspond to two different controller designs, the tuning function design and the modular design.

4. *Tuning function design*

The tuning function adaptive control design of Krstić *et al.* (1992) is a Lyapunov design in which a Lyapunov function for the complete closed-loop system is constructed in a recursive fashion. Controller (6)–(7), which in this design does not include the η-dynamics in equation (8), is chosen to make the derivative of this Lyapunov function negative along closed-loop trajectories. When our robust version of this design is complete, we obtain a Lyapunov function of the form

$$V(x, \hat{\theta}, t) = |z|^2 + \frac{1}{\gamma}|\theta - \hat{\theta}|^2, \tag{12}$$

where $\gamma > 0$ is the adaptation gain and $z \in \mathbb{R}^n$ is a transformed state variable whose first component is the tracking error, that is, $z_1(t) := y(t) - y_r(t)$. The derivative of this Lyapunov function along closed-loop trajectories satisfies

$$\dot{V} \leq -c|z|^2 - 2(\theta - \hat{\theta})^T L(\hat{\theta}) + \frac{1}{4d}|w|^2, \tag{13}$$

where $c > 0$ and $d > 0$ are design parameters and the function $L(\hat{\theta})$ appears in the update law T in equation (7) as a type of σ-modification (Ioannou and Sun, 1995). The construction of the variable z, the control law μ, and the updata law T follows the recursive procedure given in Krstić *et al.* (1992) with two modifications. First, nonlinear damping terms are added to the control law at each recursion step to dominate the effects of the disturbance w. Second, the first tuning function includes an extra term of the form $\gamma L(\hat{\theta})$ so that the update law (7) becomes

$$\dot{\hat{\theta}} = \gamma L(\hat{\theta}) + \text{modified tuning functions}. \tag{14}$$

The derivatives of L will appear in the definitions of z, μ, and the modified tuning functions. We will not give the construction here; it is a straightforward extension of the procedure outlined in Krstić *et al.* (1992).

The function L is chosen to have the following properties:

L1. $L(\hat{\theta}) = 0 \quad$ for all $\hat{\theta} \in \Theta$,

L2. $-2(\theta - \hat{\theta})^T L(\hat{\theta}) < 0 \quad$ for all $\hat{\theta} \notin \Theta$

and

L3. $-2(\theta - \hat{\theta})^T L(\hat{\theta}) \leq -\frac{c}{\gamma}|\theta - \hat{\theta}|^2 + \frac{2c}{\gamma}M^2 \quad \forall \hat{\theta} \in R^p$.

where M is defined in equation (11). One can show that a C^∞ function L with these properties always exists provided Θ is compact and convex with $\theta \in \Theta$. The smoothness of L is crucial because its derivatives (of up to order $n - 2$) will appear in the functions μ and T which define controller (6) and (7). Such smoothness also means that the update law T in equation (7) can be chosen to be smooth rather than merely locally Lipschitz in this design. From equation (13) and L3 we obtain

$$\dot{V} \leq -cV + \frac{2c}{\gamma}M^2 + \frac{1}{4d}|w|^2 \tag{15}$$

form which the boundedness property P1 follows. The other desired closed-loop properties P2–P4 can be derived from equations (13), (15), and L1–L3. The robustness estimates λ, λ_0, χ, and χ_0 in P3 and P4 are given by

$$\lambda = \frac{1}{2\sqrt{cd}}, \tag{16}$$

$$\lambda = \frac{1}{\sqrt{\gamma}}|\tilde{\theta}(0)| + \sqrt{\frac{2}{\gamma}}M + |z(0)|, \tag{17}$$

$$\chi = \frac{1}{2\sqrt{cd}}, \tag{18}$$

$$\chi_0 = \frac{1}{\sqrt{c\gamma}}|\tilde{\theta}(0)| + \frac{1}{\sqrt{c}}|z(0)|. \tag{19}$$

5. *Modular design*

The modular design is less complex than the tuning function design. While in the tuning function design the controller and the identifier are interlaced and their construction is guided by a single Lyapunov function, in the modular design the controller and the identifier are independent. The independence is achieved using a strong controller whose nonlinear damping terms assure the boundedness of the plant state whenever the identifier guarantees certain boundedness properties and the disturbance w is bounded.

The parameter estimate $\hat{\theta}$ is kept inside Θ using parameter projection. This is in contrast to the tuning function design where it was not possible to employ projection because it is not differentiable (projection is only locally Lipschitz, see Krstić *et al.*, 1995; Lemma E.1).

5.1. *ISS-controller*. The controller of Krstić and Kokotović (1995), augmented by nonlinear damping terms counteracting the disturbance w, results in the closed-loop system

$$\dot{z} = A_z(z,\hat{\theta},t)z + P(z,\hat{\theta},t)^{\mathrm{T}}\tilde{\theta} + R(z,\hat{\theta},t)^{\mathrm{T}}\dot{\hat{\theta}} + Q(z,\hat{\theta},t)^{\mathrm{T}}w, \quad (20)$$

where $z \in \mathbb{R}^n$ is a transformed state variable whose first component is the tracking error, and A_z, P, Q, and R are matrix-valued functions of z, $\hat{\theta}$, and t:

$$A_z = \begin{bmatrix} -c_1 - s_1 & 1 & 0 & \cdots & 0 \\ -1 & -c_2 - s_2 & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 & -c_n - s_n \end{bmatrix} \quad P^{\mathrm{T}} = \begin{bmatrix} \bar{p}_1^{\mathrm{T}} \\ \bar{p}_2^{\mathrm{T}} \\ \vdots \\ \bar{p}_n^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{n\times p}$$

$$Q^{\mathrm{T}} = \begin{bmatrix} \bar{q}_1^{\mathrm{T}} \\ \bar{q}_2^{\mathrm{T}} \\ \vdots \\ \bar{q}_n^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{n\times q} \qquad R^{\mathrm{T}} = \begin{bmatrix} \bar{r}_1^{\mathrm{T}} \\ \bar{r}_2^{\mathrm{T}} \\ \vdots \\ \bar{r}_n^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{n\times p} \quad (21)$$

and the nonlinear damping functions s_i are given by

$$s_i = \kappa_i|\bar{p}_i|^2 + d_i|\bar{q}_i|^2 + g_i|\bar{r}_i|^2. \quad (22)$$

For system (20) we can show that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}|z|^2\right) \le -c|z|^2 + \frac{1}{4}\left(\frac{1}{\kappa}|\tilde{\theta}|^2 + \frac{1}{g}|\dot{\hat{\theta}}|^2 + \frac{1}{d}|w|^2\right), \quad (23)$$

where

$$c = \min\{c_1, \ldots, c_n\}, \quad \kappa_m = \min\{\kappa_1, \ldots, \kappa_n\}$$

$$d = \left(\sum_{i=1}^{n}\frac{1}{d_i}\right)^{-1}, \quad \kappa = \left(\sum_{i=1}^{n}\frac{1}{\kappa_i}\right)^{-1}, \quad g = \left(\sum_{i=1}^{n}\frac{1}{g_i}\right)^{-1}.$$

It follows that z will be bounded provided $\tilde{\theta}$, $\dot{\hat{\theta}}$, and w are bounded. However, it is not necessary that these inputs be bounded. It is enough if they are square integrable, and even if they are only "mean-square" bounded (see Corollary 3.3.3 in Ioannou and Sun, 1995).

5.2. *Scheme with passive identifier*. We employ the observer

$$\dot{\hat{z}} = A_z(z,\hat{\theta},t)\hat{z} + R(z,\hat{\theta},t)^{\mathrm{T}}\dot{\hat{\theta}}. \quad (24)$$

The observer error

$$\varepsilon = z - \hat{z} \quad (25)$$

is governed by an equation driven by $\tilde{\theta}$ and w:

$$\dot{\varepsilon} = A_z(z,\hat{\theta},t)\varepsilon + P(z,\hat{\theta},t)^{\mathrm{T}}\tilde{\theta} + Q(z,\hat{\theta},t)^{\mathrm{T}}w. \quad (26)$$

The update law for $\hat{\theta}$,

$$\dot{\hat{\theta}} = \operatorname*{Proj}_{\Theta}\{\Gamma P(z,\hat{\theta},t)\varepsilon\}, \quad \Gamma = \Gamma^{\mathrm{T}} > 0 \quad (27)$$

employs parameter projection to keep $\hat{\theta}$ inside Θ. For the properties of the projection operator, the reader is referred to Appendix E of Krstić *et al.* (1995). For simplicity, the subsequent analysis assumes $\Gamma = \gamma I$.

Following the proof of Lemma 5.10 in Krstić *et al.* (1995), the Lyapunov-like function

$$V = \frac{1}{2\gamma}|\tilde{\theta}|^2 + \frac{1}{2}|\varepsilon|^2 \quad (28)$$

can be shown to have the derivative satisfying

$$\dot{V} \le -c|\varepsilon|^2 - \frac{\kappa_m}{\gamma^2 n}|\dot{\hat{\theta}}|^2 + \frac{1}{4d}|w|^2. \quad (29)$$

Since w is bounded, equation (29) can be used to show that $\dot{\hat{\theta}}$ is mean-square bounded. Along with boundedness of $\tilde{\theta}$ (which is guaranteed by projection) and w, with Ioannou and Sun (1995; Corollary 3.3.3) applied to equation (23) we prove that z is bounded.

The L_2 and L_∞ performance computations are carried out using equation (29) and the following two inequalities:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu|\hat{z}|^2 + |\varepsilon|^2 + |\tilde{\theta}|^2_{\Gamma^{-1}}}{2}\right) \le -\mu c|\hat{z}|^2 - c|\varepsilon|^2 + \frac{1}{4d}|w|^2, \quad (30)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\nu|z|^2 + |\varepsilon|^2}{2}\right) \le -c(\nu|z|^2 + |\varepsilon|^2) + \frac{\nu+2}{4\kappa}|\tilde{\theta}|^2 + \frac{\nu+1}{4d}|w|^2, \quad (31)$$

where $\mu = 2g\kappa_m/n\bar{\gamma}^2$ and $\nu = g\kappa_m/n\gamma^2$. Inequalities (30) and (31) can be obtained by modifying proofs of Theorems 5.11 and 5.19, respectively, in Krstić *et al.* (1995). The robustness estimates λ, λ_0, χ, and χ_0 in P3 and P4 are given by

$$\lambda = \frac{1}{2\sqrt{cd}}\left(1 + \frac{n\gamma^2}{g\kappa_m}\right)^{1/2}, \quad (32)$$

$$\lambda_0 = \frac{1}{2\sqrt{c\kappa}}\left(1 + \frac{2n\gamma^2}{g\kappa_m}\right)^{1/2} M + |z(0)|, \quad (33)$$

$$\chi = \frac{1}{\sqrt{2cd}}\left(1 + \frac{n\gamma^2}{2g\kappa_m}\right)^{1/2}, \quad (34)$$

$$\chi_0 = \frac{1}{\sqrt{c\gamma}}\left(1 + \frac{n\gamma^2}{2g\kappa_m}\right)^{1/2}|\tilde{\theta}(0)| + \frac{1}{\sqrt{c}}|z(0)|. \quad (35)$$

5.3. *Scheme with swapping identifier*. We employ the following filters and the estimation error:

$$\dot{\Omega}^{\mathrm{T}} = A_z(z,\hat{\theta},t)\Omega^{\mathrm{T}} + P(z,\hat{\theta},t)^{\mathrm{T}}, \quad (36)$$

$$\dot{\Omega}_0 = A_z(z,\hat{\theta},t)\Omega_0 + P(z,\hat{\theta},t)^{\mathrm{T}}\hat{\theta} - R(z,\hat{\theta},t)^{\mathrm{T}}\dot{\hat{\theta}}, \quad (37)$$

$$\varepsilon = z + \Omega_0 - \Omega^{\mathrm{T}}\hat{\theta}. \quad (38)$$

The estimation error satisfies

$$\varepsilon = \Omega^{\mathrm{T}}\tilde{\theta} + \tilde{\varepsilon}, \quad (39)$$

where $\tilde{\varepsilon}$ is the filtered disturbance:

$$\dot{\tilde{\varepsilon}} = A_z(z,\hat{\theta},t)\tilde{\varepsilon} + Q(z,\hat{\theta},t)^{\mathrm{T}}w. \quad (40)$$

The update law for $\hat{\theta}$ is either the gradient:

$$\dot{\hat{\theta}} = \operatorname*{Proj}_{\Theta}\{\Gamma\Omega\varepsilon\}, \quad \Gamma = \Gamma^{\mathrm{T}} > 0 \quad (41)$$

or the least squares

$$\dot{\hat{\theta}} = \operatorname*{Proj}_{\Theta}\{\Gamma\Omega\varepsilon\}, \qquad \dot{\Gamma} = -\Gamma\Omega\Omega^{\mathrm{T}}\Gamma, \quad \Gamma(0) = \Gamma(0)^{\mathrm{T}} > 0. \quad (42)$$

In our analysis here, we will consider only the gradient update law. All the results can be extended to the case of least-squares update law.

To establish boundedness of all signals, we note that w is bounded, $\tilde{\theta}$ is bounded by projection, so in order to use inequality (23), we only need to show that $\hat{\theta}$ is bounded. This is straightforward due to nonlinear damping terms embedded in the matrix A_z present in equations (36) and (40). These terms guarantee boundedness of Ω and $\tilde{\varepsilon}$, so we show that $\hat{\theta}$ is bounded. Hence, z is bounded, which, in turn, implies that Ω_0 is bounded.

An L_∞ performance bound is obtained using equation (23). An L_2 performance bound is obtained by extending the proof of Theorem 6.11 in Krstić *et al.* (1995) to the case with a disturbance w. The robustness estimates λ, λ_0, χ, and χ_0 in P3–P4 are given by

$$\lambda = \frac{1}{2\sqrt{cd}}\left(1 + \frac{\gamma^2}{8c^2 g\kappa}\right)^{1/2}, \tag{43}$$

$$\lambda_0 = \frac{1}{2\sqrt{c\kappa}}\left(1 + \frac{\gamma^2}{8c^2 gk}\right)^{1/2} M + |z(0)|, \tag{44}$$

$$\chi = \frac{1}{2\sqrt{cd}}\left[1 + \frac{\gamma}{2c\sqrt{2\kappa}}\left(\frac{1}{g} + \frac{1}{2c^2\kappa}\right)^{1/2}\right], \tag{45}$$

$$\chi_0 = \sqrt{\frac{\gamma}{4c^2\kappa}}\left(\frac{1}{g} + \frac{1}{2c^2\kappa}\right)^{1/2} |\tilde{\theta}(0)| + \frac{2}{\sqrt{\gamma}}|\tilde{\theta}(0)| + \frac{1}{\sqrt{c}}|z(0)|. \tag{46}$$

6. *Conclusion*

We have shown how two recently developed nonlinear adaptive control designs can be made robust with respect to structured state-space uncertainty and exogenous disturbances. These modified designs are likely to provide improvements over either strictly robust or strictly adaptive designs. Still open are important questions about control effort, performance, and robustness with respect to unmodeled dynamics and measurement error.

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