Adaptive predictor control for stabilizing pressure in a managed pressure drilling system under time-delay

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\textbf{A B S T R A C T}

In this paper, we address adaptive predictor feedback design for a simplified ODE drilling system in the presence of unknown parameter, disturbance and time-delay. The main objective is to stabilize the bottomhole pressure at a critical depth at a desired set-point directly. The time-delay in the transmission line of the drilling systems is considered. The stabilization of the dynamic system and the asymptotic tracking are demonstrated by the proposed predictor control, where the adaptation employs Lyapunov update law design with normalization. The proposed method is evaluated using a high fidelity drilling simulator and cases from a North Sea drilling operation are simulated. The results show that the proposed predictor controller is effective to stabilize the bottom hole pressure within the desired bound and compensate the effects of the delay in MPD. The simulation also shows that the proposed method provides good tracking, good disturbance rejection, and good compensation of time-delay.

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1. Introduction

1.1. Managed pressure drilling

Managed pressure drilling (MPD) shown in Fig. 1 is defined as an adaptive drilling process used to more precisely control the annular pressure profile throughout the wellbore given by the International Association of Drilling Contractors (IADC) in [1]. The objectives of MPD are to ascertain the bottomhole pressure (BHP) environment limits and to manage the annular hydraulic pressure profile accordingly. This is typically achieved through a closed, pressurized fluid system in which flow rate, mud density, and back pressure on the fluid returns (choke manifold) are used to set and control the BHP under both static and dynamic conditions. MPD concepts come in many variants, such as Pressurized Mud Cap Drilling, Constant Bottomhole Pressure Control, Reverse Circulation, Dual-Gradient Drilling, etc.

1.2. Pressure control

A stable wellbore promotes efficient drilling and personnel safety. A destabilized wellbore can reduce or eliminate production [2]. Too low mud pressure can lead to a kick or wellbore collapse and too high mud pressure can create wellbore fracturing and losses as shown in Fig. 2. Preventing these costly stability problems requires accurate pressure control. The main objective is to precisely control BHP throughout the wellbore continuously while drilling, i.e. to maintain the pressure in the wellbore above the pore or collapse pressure and below the fracture or sticking pressure. Usually, this amounts to stabilizing the BHP at a critical depth at a desired set-point directly, i.e. either at a particular depth where the pressure margins are small, or at the drill bit where conditions are the most uncertain.

Constant BHP control is a challenging task during well drilling, due to the complex dynamics of the multiphase flow potentially consisting of drilling mud, oil, gas and cuttings. By allowing manipulation of the topside choke and pumps, MPD provides a means of quickly affecting pressure to counteract disturbances, and several control schemes are found in the literature. State-of-the-art solutions typically employ

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conventional Proportional Integral (PI) control applied to the choke in drilling process in [3–6]. There are significant drawbacks using PI control. One is that the control system based on conventional PI control will react slowly to pressure changes and flowrate changes, which results from movements of the drill string and from the draw-down test or pipe connection. Another drawback is the uncertainty in the pressured mud system, due to uncertainties in the friction and mud compressibility parameters in both the drill string and annulus.

A lot of effort has been put into developing advanced complicated models that capture all aspects of the drilling fluid hydraulics. However, a main drawback is the resulting complexity of these models, which require expert knowledge to set up and calibrate, making it a high-end solution. The complexity is also increased by the fact that many of the parameters in such models are uncertain and possibly slowly changing, which implies that they would need to be tuned when operating conditions change. In order to reduce the complexity, a simplified low order model for control and estimation of the BHP is used in [7–12], where ordinary differential equations (ODE) are used to capture the dominant phenomena of the drilling system. Based on this low order ODE model, several control schemes are found in the literature, such as [9,10,13–17]. Model based control was also presented for directional drilling systems in [18,19]. Research on BHP estimation in MPD has been conducted in our articles [9,10,13,15,20], where adaptive observers are used. The model predictive control has been studied in [17] for in DGD operation. In [14–16,21], an automatic switch control scheme is developed for kick attenuation and pressure regulation and when there is an influx in MPD. Kick detection and control has also been studied in [22–24]. However the time delay problem is not considered in these papers.
1.3. Time delay

Recent experience indicates that in order to optimize the drilling operation, not just the mechanics or software, the entire drilling system needs to be designed from a control system point of view. Automatic drilling operations in MPD systems require investigation of the system ability to operate during various incidents, especially for long well with oil-based mud. Long wells with narrow pressure margins are complicated with respect to maintaining accurate pressure control during drilling. The high compressibility of the drilling mud that is typically used in such wells result in large pressure transients that need to be handled by MPD system. Also, the time delay in the BHP measurement and in the transmission line arises in such wells and deteriorates the performance of closed-loop systems, such as to cause fluctuations of the BHP and make drilling system lose stability. Adaptive control of systems with input time delay has been considered in [25–33].

Therefore there is significant potential to improve existing control algorithms to compensate the pressure fluctuations due to the transmission delay and disturbance in a drilling system. Adaptive predictor feedback design is an effective way to control of systems with time-delay and unknown parameters, such as in [26–28]. By using predictor feedback control with adaptation of uncertain parameters rather than integral action in the controller, one typically enable faster reaction to changes in set-point and disturbance and compensate the effects of time delay.

The control objective is to maintain BHP to track at the desired set-point pressure in MPD, in presence of several difficulties the drilling system may encounter: (i) unknown time delay; (ii) uncertain parameter; (iii) unknown disturbance. Each of these situations represents a different challenge in drilling operation and calls for the introduction of some sort of adaptation. This paper addresses the three presented problems in drilling system by using the predictor feedback control. Through the backstepping transformation in PDE representation, the stabilization of the dynamic system and the asymptotic pressure regulation are demonstrated by the proposed predictor control, where the adaptation employs Lyapunov update law design with normalization.

This paper is organized as follows. In Section 2, an ODE model is presented that captures the dominant phenomena of the drilling system and forms the basis for control design. In Section 3, the design of a predictor controller is presented for systems with known parameters and known disturbance. In Section 4, the illustration of the design of an adaptive predictor control scheme is presented and the convergence properties of the closed-loop system is shown. The simulation results are obtained with a high fidelity drilling simulator in Section 5. Finally, we draw some conclusions in Section 6.

2. Model

A simplified schematic drawing of a MPD system is shown in Fig. 3, which was presented in [34]. During well drilling, a drilling fluid is pumped into the drill string topside and through the drill bit at the bottomhole of the well. The mud then transports cuttings in the annulus side of the well (i.e. in the well bore outside the drill string) up to the drill rig, where a choke valve and a backpressure pump are used to control the annular pressure. A more elaborate description of the drilling process is given in [11]. A mass-balance model is used to relate the bottomhole pressure and the flowrates through the mud pump, backpressure pump and choke, described as

$$\frac{d}{dt}(\rho_A V_A) = \rho_{pump} q_{pump}(t) + \rho_{bpp} q_{bpp}(t) - \rho_{ch} q_{ch}(t).$$

where $\rho_A$, $\rho_{pump}$, $\rho_{bpp}$ and $\rho_{ch}$ are the average mud densities (kg/m$^3$) in the annulus, the mud pump, the backpressure pump and the choke, respectively, $q_{pump}$, $q_{bpp}$ and $q_{ch}$ are the flows rates (m$^3$/sec) through the mud pump, the backpressure pump and the choke, respectively. $V_A$ is the volume (m$^3$) in the annulus. The drill string volume is assumed incompressible. Introducing a factor $\beta_A = 1/(\rho_A V_A)$ and neglecting the dependence on the temperature and density changes, the BHP at drill bit $p_{hit}$ (bar) can be described as the following ODE.

$$p_{hit} = \frac{\beta_A}{V_A} (q_{pump}(t-D) + q_{bpp}(t-D) - q_{ch}(t-D) + d).$$
where \(d\) is the disturbance due to the slow variations in the annulus and uncertainties in the drill bit, and \(D\) is the delay for transportation of BHP measurement from downhole to top side and in the transmission line of MPD. The main variable of interest is \(p_{\text{bit}}\) and the main objective is to control the bottomhole pressure at the desired set-point during drilling operation.

**Assumption 1.** The compressibility factor \(\beta_a\) is an unknown non-zero constant satisfying \(\beta_{\min} \leq \beta_a \leq \beta_{\max}\), where \(\beta_{\min}\) and \(\beta_{\max}\) are known positive constants.

### 3. Control strategy with delay estimation

Now a control input \(q(t)\) is introduced, which includes three physical actuation devices: the choke valve, the backpressure pump and the mud pump. The control input \(q(t)\) is defined as,

\[
q(t) = q_{\text{pump}}(t) + q_{\text{bhp}}(t) - q_{\text{choke}}(t),
\]

where the mud pump and the backpressure pump are controlled manually in MPD.

#### 3.1. Without input delay

When the delay \(D = 0\) and the disturbance \(d\) is known, a predictor control law for system (2) is given as

\[
q(t) = -k(p_{\text{bit}}(t) - p_{\text{set}}) - d,
\]

where the gain \(k\) is a positive constant and \(p_{\text{set}}\) (bar) is the desired set-point for bottomhole pressure. Clearly, the controller (4) in the closed-loop with system (2) without delay achieves exponential set-point regulation, that is

\[
\lim_{t \to \infty} p_{\text{bit}}(t) = p_{\text{set}}.
\]

#### 3.2. Known system with known input delay

When the delay \(D \neq 0\), the parameter \(\beta_a\) and the disturbance \(d\) are known, a stabilizing predictor control law for system (2) is given as

\[
q(t) = -k \left( p_{\text{bit}}(t) - p_{\text{set}} \right) + \frac{\beta_a}{V_a} \int_{t-D}^{t} q_1(s)ds - d,
\]

where \(q_1(t) = q(t) + d\). It achieves exponential stability and compensates the effects of the input delay \(D\). The structure of the compensator (6) involves a feedback distributed control law as in [35,36].

We now represent the system (2) with a transport PDE form as

\[
p_{\text{bit}} = \frac{\beta_a}{V_a} u(0, t),
\]

\[
u_t(x, t) = u(x, t), \quad \text{for all } x \in [0, D]
\]

\[
u(D, t) = q(t).
\]

Note that with this representation, it is given

\[
u(x, t) = q_1(t + x - D), \quad \text{for all } x \in [0, D].
\]

The backstepping transformation in the PDE is given as

\[
w(x, t) = u(x, t) + k p(x, t).
\]

The inverse backstepping transformation in PDE notation is given by

\[
u(x, t) = w(x, t) - k e^{-\frac{\beta_a}{V_a} x} e(t) - k \frac{\beta_a}{V_a} \int_0^x e^{-\frac{\beta_a}{V_a} (x-y)} w(y, t)dy,
\]

where \(e(t) = p_{\text{bit}}(t) - p_{\text{set}}\). Then the system (2) can be transformed to the following target system as

\[
\dot{e} = -k \frac{\beta_a}{V_a} e(t) + \frac{\beta_a}{V_a} w(0, t)
\]

\[
w_0(x, t) = w_0(x, t), \quad \text{for all } x \in [0, D]
\]

\[
w(D, t) \equiv 0.
\]

**Theorem 1.** The controller (6) in the closed-loop system of the plant (2) in presence of input delay \(D\) achieves exponential set-point regulation, that is

\[
\lim_{t \to \infty} p_{\text{bit}}(t) = p_{\text{set}}.
\]

and there exist positive constants \(\lambda\) and \(\sigma\) such that the following holds:

\[
|e(t)| + ||u(t)|| \leq \sigma(|e(0)| + ||u(0)||) e^{-\lambda t}
\]

for all \(t \geq 0\) with \(||u(t)||^2 = \int_0^D u(x, t)^2 dx\).
Proof. In order to use the Lyapunov technique, a PDE representation of the predictor state $p_{\text{bar}}(t+x)(\text{bar})$ is introduced as in [26].

$$p(x, t) = e(t) + \frac{\beta_s}{V_a} \int_0^x u(y, t) \, dy, \quad \text{for all } x \in [0, D].$$

(18)

A Lyapunov functional is defined as the following

$$V(t) = e(t)^2 + \epsilon \int_0^D e^\epsilon w(x, t)^2 \, dx,$$

(19)

where $\epsilon \geq \frac{\beta_s}{V_a}$. Using integration by parts and Young’s inequality, the derivative of (19) along (13)–(15) is given as

$$\dot{V}(t) = -2k \frac{\beta_s}{V_a} e(t)^2 + 2 \frac{\beta_s}{V_a} e(t)w(0, t) + 2 \epsilon \int_0^D e^\epsilon w(x, t)w_x(x, t) \, dx = -2k \frac{\beta_s}{V_a} e(t)^2 + 2 \frac{\beta_s}{V_a} e(t)w(0, t) - \epsilon w(0, t)^2$$

$$- \epsilon \int_0^D e^\epsilon w(x, t)^2 \, dx \leq -k \frac{\beta_s}{V_a} e(t)^2 - \epsilon \int_0^D e^\epsilon w(x, t)^2 \, dx \leq -\mu V(t),$$

(20)

where $\mu = -\min \left\{ k \frac{\beta_s}{V_a}, 1 \right\}$. Thus the exponential set-point regulation is obtained by applying the LaSalle–Yoshizawa theorem to (20), such as

$$V(t) \leq e^{-\mu t}V(0), \quad \text{for all } t \geq 0$$

(21)

Now we derive the bound stated in Theorem 1. It is observed from relation (19) that

$$\mu_1 |e(t)|^2 + \int_0^D w(x, t)^2 \, dx \leq V(t) \leq \mu_2 |e(t)|^2 + \int_0^D w(x, t)^2 \, dx,$$

(22)

where

$$\mu_1 = \min \left\{ 1, \frac{\beta_s}{V_a k} \right\}$$

(23)

$$\mu_2 = \max \left\{ 1, \frac{\beta_s}{V_a k^\epsilon} \right\}$$

(24)

Therefore

$$|e(t)|^2 + \int_0^D w(x, t)^2 \, dx \leq \frac{\mu_2}{\mu_1} |e(0)|^2 + \int_0^D w(x, 0)^2 \, dx e^{-\mu t},$$

(25)

Since we have

$$|e(t)|^2 + \int_0^D w(x, t)^2 \, dx \leq v_1 |e(t)|^2 + \int_0^D u(x, t)^2 \, dx,$$

(26)

$$|e(t)|^2 + \int_0^D u(x, t)^2 \, dx \leq v_2 |e(t)|^2 + \int_0^D w(x, t)^2 \, dx,$$

(27)

where

$$v_1 = \max \left\{ 2 + 4k^2 D^2 \frac{\beta_s}{V_a}, 1 + 2k^2 D \right\}$$

(28)

$$v_2 = \max \left\{ 2 + 4k^2 D^2 \frac{\beta_s}{V_a} e^{2(\frac{\beta_s}{V_a} k^\epsilon)}, 1 + 2k^2 D e^{2(\frac{\beta_s}{V_a} k^\epsilon)} \right\}$$

(29)

Combining (25)–(27), we get (17) with

$$\sigma = \sqrt{\frac{2\mu_2 v_1 v_2}{\mu_1}}$$

(30)

$$\lambda = \frac{\mu}{2}$$

(31)

\[\square\]

4. Control strategy with parameter adaptation

When the parameter $\beta_s$ and the disturbance $d$ are unknown, the adaptive predictor control law is employed, which is distributed into two parts $q_1(t)$ and $q_2(t)$ as follows.

$$q(t) = q_1(t) + q_2(t)$$

(32)
where $k$ is a positive constant and $\tilde{d}(t)$ is the estimate of the disturbance $d(t)$. $\hat{p}(D, t)$ is a prediction of $p_{\text{bit}}(t + D)$ based on the measurement $p_{\text{bit}}(t)$ described by

$$\hat{p}(x, t) = e(t) + \frac{\hat{\beta}_d(t)}{v_a} \int_0^x u(y, t) \, dy, \quad \text{for all } x \in [0, D].$$

$$u(x, t) = q_1(t + x - D), \quad \text{for all } x \in [0, D].$$

where $\hat{\beta}_d$ and $\tilde{d}$ are the estimates of parameter $\beta_d$ and the disturbance $d(t)$. The update laws for the estimates $\hat{\beta}_d$ and $\tilde{d}$ are chosen based on the Lyapunov analysis as follows.

$$\dot{\hat{\beta}}_d(t) = \gamma_d \text{Proj}_{[\beta_{\min}, \beta_{\max}]}[\tau_d(t)]$$

$$\tau_d(t) = \frac{e(t) + l k}{1 + e(t)^2} \int_0^D \frac{1}{1 + x} w(x, t) \, dx$$

$$\tilde{d}(t) = \gamma_d \tilde{d}(t)$$

$$\tau_d(t) = \frac{e(t) + l k}{1 + e(t)^2} \int_0^D \frac{1}{1 + x} w(x, t) \, dx$$

with adaptation gains $\gamma_d$ and $\gamma_d$ chosen as positive constants and the positive constant $l$ satisfying $l \geq \frac{4}{v_a^2} \beta_{\max}$. $\text{Proj}(\cdot)$ is a standard projector operator in a convex set given by

$$\text{Proj}_{[\beta_{\min}, \beta_{\max}]}[\tau_d(t)] = \begin{cases} 0, & \hat{\beta}_d = \beta_{\min} \quad \text{and} \quad \tau_d < 0 \\ 0, & \hat{\beta}_d = \beta_{\max} \quad \text{and} \quad \tau_d > 0 \\ 1, & \text{else} \end{cases}$$

The transformed state of the actuator $w(x, t)$ is defined as

$$w(x, t) = u(x, t) + k \hat{p}(x, t), \quad \text{for all } x \in [0, D]$$

$$w(D, t) = 0.$$

**Theorem 2.** Let Assumption 1 hold and consider the closed-loop system (2) consisting of the adaptive controller (32)–(34) and the update laws defined by (37)–(43). There exist $\gamma_d'$ and $\gamma_d''$ such that for any $\gamma_d' \in [0, \gamma_d']$ and $\gamma_d'' \in [0, \gamma_d'']$ there exist positive constants $R$ and $\sigma$ such that,

$$\Omega(t) \leq R(e^{\sigma t} - 1)$$

where

$$\Omega(t) = ||e(t)||^2 + ||u(x, t)||^2 + ||\hat{\beta}_d(t)||^2 + ||\tilde{d}(t)||^2$$

with $||u(t)||^2 = \int_0^D u(x, t)^2 \, dx$, $\hat{p}(t) = \beta - \hat{\beta}$, and $\tilde{d}(t) = d - \tilde{d}(t)$. Furthermore, the asymptotic set-point regulation is achieved, that is

$$\lim_{t \to \infty} p_{\text{bit}}(t) = p_{\text{set}}.$$

**Proof.** Firstly, the distributed input is introduced as

$$u(x, t) = q_1(t + x - D), \quad \text{for all } x \in [0, D].$$

The system (2) can be represented in a transport PDE form

$$p_{\text{bit}} = \frac{\beta_d}{v_a} (u(0, t) + \hat{d}(t) + \tilde{d}(t) - \tilde{d}(t))$$

$$u_t(x, t) = u(x, t), \quad \text{for all } x \in [0, D]$$

$$u(D, t) = q_1(t).$$

The inverse of (42) along with (35) is given as

$$u(x, t) = w(x, t) - k e^{-\frac{\hat{\beta}_d(t)}{v_a} x} e(t) - k \int_0^x e^{-\frac{\hat{\beta}_d(t)}{v_a} (x - y)} \frac{\hat{\beta}_d}{v_a} w(y, t) \, dy, \quad \text{for all } x \in [0, D].$$

Using (42) and $e = p_{\text{bit}} - p_{\text{set}}$, (48) can be transformed to the following

$$\dot{e} = -k \hat{\beta}_d \frac{\hat{\beta}_d}{v_a} e(t) + \frac{\hat{\beta}_d}{v_a} u(0, t) + \frac{\hat{\beta}_d}{v_a} \tilde{d}(t) + \frac{\hat{\beta}_d}{v_a} (\tilde{d}(t) - \tilde{d}(t - D)) + \frac{\hat{\beta}_d}{v_a} w(0, t).$$
Differentiating (42) with respect to \( t \) we get

\[
w_t(x, t) = u_t(x, t) + k \frac{\hat{\beta}_u}{V_a} u(x, t) + k \frac{\hat{\beta}_a}{V_a} u(0, t) + k \frac{\hat{\beta}_s}{V_a} \int_0^x u(y, t) dy + k \frac{\hat{\beta}_s}{V_a} \hat{a}(t) + k \frac{\hat{\beta}_s}{V_a} (\hat{a}(t) - \hat{a}(t - D)) = w(x, t) + k \frac{\hat{\beta}_s}{V_a} u(0, t) + k \frac{\hat{\beta}_s}{V_a} \int_0^x u(y, t) dy + k \frac{\hat{\beta}_s}{V_a} \hat{a}(t) + k \frac{\hat{\beta}_s}{V_a} (\hat{a}(t) - \hat{a}(t - D)),
\]

where using \( u_t(x, t) = u_0(x, t) \) and \( w_t(x, t) = u_0(x, t) + k \frac{\hat{\beta}_s}{V_a} u(x, t) \).

In order to compensate the effects of the disturbance, the Lyapunov–Krasovskii functional is introduced as in [28].

\[
V_0(t) = \epsilon_1 \int_{t - D}^t \int_s^t \frac{1}{1 + \Phi(t)} (\|w(r)\|^2 + |e(r)|^2) dr ds,
\]

where \( \|w(t)\|^2 = \int_0^D w(x, t)^2 dx \) and

\[
\Phi(t) = e(t)^2 + l \int_0^D (1 + x)w(x, t)^2 dx.
\]

The derivative of \( V_0(t) \) is

\[
V_0(t) = -\epsilon_1 \int_{t - D}^t \int_s^t \frac{1}{1 + \Phi(t)} (\|w(r)\|^2 + |e(r)|^2) dr ds + \epsilon_1 \frac{D}{1 + \Phi(t)} (\|w(t)\|^2 + |e(t)|^2).
\]

Then the final control Lyapunov functional is defined as in [26].

\[
V(t) = \log(1 + \Phi(t)) + \frac{1}{\gamma_d} \beta_u(t)^2 + \frac{\beta_s}{\gamma_d V_a} \hat{a}(t)^2 + V_0(t).
\]

The derivative of \( V(t) \) is obtained as

\[
V(t) = \frac{1}{1 + \Phi(t)} (2e(t)\dot{e}(t) + l \int_0^D (1 + x)2w(x, t)w_t(x, t) dx - \frac{2}{\gamma_d} \hat{\beta}_u(t)\hat{\beta}_u(t) - \frac{2}{\gamma_d V_a} \hat{a}(t)\hat{a}(t) + V_0(t)
\]

\[
= \frac{1}{1 + \Phi(t)} \left( -2k \frac{\hat{\beta}_s}{V_a} e(t)^2 - lw(0, t)^2 - l \int_0^D w(x, t)^2 dx \right) - \epsilon_1 \int_{t - D}^t \frac{1}{1 + \Phi(t)} (\|w(r)\|^2 + |e(r)|^2) dr + \frac{\epsilon_1 D}{1 + \Phi(t)} (\|w(t)\|^2 + |e(t)|^2)
\]

\[
+ \frac{1}{1 + \Phi(t)} \left( 2e(t)\hat{\beta}_u(t) - \frac{2}{\gamma_d} \hat{\beta}_u(t)\hat{\beta}_u(t) - \frac{2}{\gamma_d V_a} \hat{a}(t)\hat{a}(t) + V_0(t) \right)
\]

\[
+ 2l \int_0^D (1 + x)w(x, t) dx (\hat{a}(t) - \hat{a}(t - D)) \frac{\beta_s}{V_a} + \frac{2}{\gamma_d} \beta_s(t) \left[ \frac{\hat{\beta}_u(t)}{1 + \Phi(t)} (\hat{a}(t) + \frac{\gamma_d}{1 + \Phi(t)} e(t) + \int_0^D lk(1 + x)w(x, t) dx) \frac{1}{V_a} u(0, t) \right]
\]

\[
+ \frac{2}{\gamma_d V_a} \hat{a}(t) \left[ -\hat{a}(t) + \frac{\gamma_d}{1 + \Phi(t)} e(t) + \int_0^D lk(1 + x)w(x, t) dx \right].
\]

Considering (37)–(42) and (51) and applying the Young’s inequality and Cauchy–Schwarz inequality, we establish the following inequalities.

\[
\|w(t)\|^2 \leq A_w(\|u(t)\|^2 + |e(t)|^2)
\]

\[
\|u(t)\|^2 \leq A_u(\|w(t)\|^2 + |e(t)|^2)
\]

\[
\dot{\beta}_u(t) \leq \gamma_d A_u \frac{\|w(t)\|^2 + |e(t)|^2}{1 + \Phi(t)}
\]

\[
\|\Delta(t)\| \leq \gamma_d A_d \frac{\|w(t)\|^2 + |e(t)|^2}{1 + \Phi(t)}
\]

with

\[
A_w = 3 \left( 1 + k^2 D + k^2 D \frac{\beta_{\text{max}}}{V_a^2} \right)
\]

\[
A_u = 3 \left( 1 + k^2 D + k^2 D \frac{\beta_{\text{max}}}{V_a^2} \right)
\]

\[
A_{\beta} = 2 \sqrt{A_u(1 + lk(1 + D)) \frac{1}{V_a}}
\]

\[
A_d = 1 + lk(1 + D),
\]
Note that \( \hat{d}(t) - \hat{d}(t - D) = \int_{t-D}^{t} \hat{d}(s)ds \) from the definition of \( \hat{d}(t) \) in (39). Using the Young’s inequality and Cauchy–Schwarz inequality and (60)–(62), the following inequalities are obtained.

\[
2(e(t) + \int_{0}^{D} l_k(1 + x)w(x, t)dx)(\hat{d}(t) - \hat{d}(t - D)) \leq 2e(t) + \int_{0}^{D} l_k(1 + x)w(x, t)dx \int_{t - D}^{t} \hat{d}(s)ds \frac{\beta_a}{V_a} \leq k \frac{\beta_a}{V_a} e(t) + \frac{1}{2} \|w(t)\|^2 + \gamma_d^2 A_1 \int_{t - D}^{t} \frac{1}{\Phi(t)} (\|e(r)\|^2 + \|w(r)\|^2)dr
\]

(67)

\[
2k \frac{\beta_a}{V_a} \int_{0}^{D} (1 + x)w(x, t) \int_{y(t)}^{x} u(y, t)dydx \leq kl \frac{A_d}{V_a} \min \left\{ \frac{\|w(t)\|^2 + \|e(t)\|^2 + \|w(0, t)\|^2}{1 + \Phi(t)} (A_u + (1 + D)^2) \right\} \|e(t)\|^2 + \|w(0, t)\|^2 + \|w(t)\|^2
\]

(68)

\[
2e(t) \frac{\beta_a}{V_a} w(0, t) \leq \frac{k}{2} \frac{\beta_a}{V_a} e(t)^2 + \frac{2}{k} \frac{\beta_a}{V_a} w(0, t)^2,
\]

(69)

where

\[
A_1 = A_d \left( \frac{2l_\beta \|e\|_{\infty}}{|kV_a\beta_a}_{\min} + 2l_\beta^2 (1 + D)^2 \frac{\beta_a}{V_a} \right)
\]

(70)

\[
A_2 = \frac{1}{V_a} klA_d (A_u + (1 + D)^2)
\]

(71)

\[
1 + \Phi(t) \geq \min \{1, l(1 + D)\} (\|e(t)\|^2 + \|w(t)\|^2).
\]

(72)

Using (67)–(69) and selecting \( l \geq \frac{4 \beta_a \|w\|_{\infty}}{V_a} \) and \( \epsilon_1 = \gamma_d^2 A_1 \), it is obtained

\[
\hat{V}(t) \leq - \frac{1}{1 + \Phi(t)} \left( \min \left\{ k \frac{\beta_a}{2V_a} \cdot \frac{l}{2} - \gamma_d^2 A_1 D - \frac{\gamma_d A_2}{\min \{1, l(1 + D)\}} \right\} \times (\|e(t)\|^2 + \|w(0, t)\|^2 + \|w(t)\|^2).
\]

(73)

Consequently, by choosing

\[
\gamma_1^2 = \frac{\min \left\{ k \frac{\beta_a}{2V_a} \cdot \frac{l}{2} \right\} \cdot \min \{1, l(1 + D)\}}{4A_2}
\]

(74)

\[
\gamma_2^2 = \frac{\min \left\{ k \frac{\beta_a}{2V_a} \cdot \frac{l}{2} \right\}}{4A_1 D}
\]

(75)

and \( \gamma_1 \in (0, \gamma_1^2) \) and \( \gamma_d \in (0, \gamma_2^2) \), it follows that

\[
\hat{V}(t) \leq - \frac{1}{2(1 + \Phi(t))} \min \left\{ k \frac{\beta_a}{2V_a} \cdot \frac{l}{2} \right\} (\|e(t)\|^2 + \|w(0, t)\|^2 + \|w(t)\|^2),
\]

(76)

and hence

\[
\hat{V}(t) \leq V(t), \quad \forall \ t \geq 0.
\]

(77)

which implies that \( e(t), w(0, t), w(t), w(t) \), \( \hat{d}(t) \), and \( \hat{d}(t) \) are uniformly bounded. From (32) it follows that \( q(t) \) is uniformly bounded. Using Barbalat’s lemma, we conclude that \( \lim_{t \to \infty} e(t) = 0 \) in (46). Starting from this result, now the bound stated in Theorem 2 is proved. From (57) and (60), it follows that

\[
\mid \hat{p}(t) \mid^2 + \mid \hat{u}(t) \mid^2 \leq \left( \gamma_\beta + \frac{\gamma \|V_a\|}{\beta_a} \right) V(t)
\]

(78)

\[
\|e(t)\|^2 \leq \|e(t)\| - 1)
\]

(79)

\[
\|u(t)\|^2 \leq A_u (\|w(t)\|^2 + \|e(t)\|^2) \leq \left( A_u + \frac{A_u}{t} \right) (\|e(t)\| - 1).
\]

(80)

Thus, from (45) and (59), it shows that

\[
\Omega(t) \leq \left( 1 + A_u + \frac{A_u}{t} + D \left( \gamma_\beta + \frac{\gamma \|V_a\|}{\beta_a} \right) \right) (\|e(t)\| - 1).
\]

(81)

From (57) and (59), it follows that

\[
V(0) \leq (1 + A_u (1 + D)) (\|e(0)\|^2 + \|w(0)\|^2 + \|w(t)\|^2) + \frac{1}{\gamma_\beta} \hat{p}(0) + \gamma \frac{\beta_a}{\gamma \|V_a\|} \hat{q}(0)^2.
\]

(82)
where it is chosen that $V_0(0) = 0$. Thus by setting
\[ R = 1 + A_u + \frac{A_u}{T} + D\left(\gamma_p + \frac{\gamma q V_a}{\beta_{\min}}\right) \]  
(83)
\[ \sigma = \max\left\{1 + A_w l (1 + D), \frac{1}{\gamma \beta_{\max} V_a} \right\}, \]  
(84)
we get the stability result and the bound (44) in Theorem 2. \( \square \)

5. Control strategy with time-delay on-line update law

In this section, we focus on the delay estimation in the proposed prediction-based control. Since $D$ is unknown, in addition to the predictor based controller, we must design an estimator for the unknown delay. We employ projector operators and assume a bound on the length of the delay to be known.

**Assumption 2.** There exist known constants $D_{\min}$ and $D_{\max}$ such that $D \in [D_{\min}, D_{\max}]$.

The system (1) is represented with a transport PDE form as
\[ p_{\text{est}} = \frac{\beta_a}{V_a} (u(0, t) + \hat{d}(t) + \hat{d}(t) - d(t - D)) \]  
(85)
\[ D_{\text{est}} = u_\Gamma(x, t), \quad \text{for all } x \in [0, 1] \]  
(86)
\[ u(1, t) = q(t). \]  
(87)

Note that with this representation, it is given
\[ u(x, t) = q(t + D(x - 1)), \quad \text{for all } x \in [0, 1]. \]  
(88)

Following the control design in previous section, the control law and the adaptive predictor control law are developed as follows.
\[ q(t) = q_1(t) + q_2(t) \]  
(89)
\[ q_1(t) = -k \hat{p}(t) \]  
(90)
\[ q_2(t) = -\hat{d}(t). \]  
(91)

where $k$ is a positive constant and $\hat{d}(t)$ is the estimate of the disturbance $d(t)$. $\hat{p}(t)$ is a prediction of $p_{\text{est}}(t)$ based on the measurement $p_{\text{set}}(t)$ described by
\[ \hat{p}(t) = e(t) + \hat{d}(t) \frac{\hat{\beta}_a(t)}{V_a} \int_0^1 u(y, t) dy, \]  
(92)
where $\hat{d}(t), \hat{\beta}_a(t)$ and $\hat{d}(t)$ are the estimates of delay $D$, parameter $\beta_a$ and the disturbance $d(t)$. The update laws for the estimates are chosen based on the Lyapunov analysis as follows.
\[ \frac{\dot{\hat{\beta}}_a(t)}{V_a} = \gamma \hat{\beta}_a \text{Proj}[^{[\beta_{\min}, \beta_{\max}]}, \beta_a], \]  
(93)
\[ \tau = \frac{e(t) + l k \hat{d}(t) \int_0^1 (1 + x) w(x, t) dx}{1 + e(t)^2 + l \hat{d}(t) \int_0^1 (1 + x) w(x, t)^2 dx} \]  
(94)
\[ \dot{\hat{d}}(t) = \gamma d \tau_d(t), \]  
(95)
\[ \tau_d(t) = \frac{e(t) + l k \hat{d}(t) \int_0^1 (1 + x) w(x, t) dx}{1 + e(t)^2 + l \hat{d}(t) \int_0^1 (1 + x) w(x, t)^2 dx}, \]  
(96)
\[ \hat{D}(t) = \gamma D \text{Proj}[^{[D_{\min}, D_{\max}]}, D], \]  
(97)
\[ \tau_D(t) = \frac{k \int_0^1 (1 + x) w(x, t) dx}{1 + e(t)^2 + a \int_0^1 (1 + x) w(x, t)^2 dx} \]  
(98)

with adaptation gains $\gamma_p$, $\gamma_d$, and $\gamma_D$ chosen as positive constants and the positive constant $l$ satisfying $l \geq \frac{4 \hat{\beta}_{\max}}{1 + V_a}$. Proj{.} is a standard projector operator. The transformed state of the actuator $w(x, t)$ is defined as
\[ w(x, t) = u(x, t) + k e(t) + k \hat{d}(t) \frac{\hat{\beta}_a(t)}{V_a} \int_0^1 u(y, t) dy, \quad \text{for all } x \in [0, 1] \]  
(99)

**Theorem 3.** Let Assumptions 1 and 2 hold. Then system (1) in presence of unknown input delay $D$, uncertain parameter $\beta_a$ and unknown disturbance $d$, with the controller (89)–(91) and the update laws (93)–(98) achieves exponential set-point regulation, that is
\[ \lim_{t \to \infty} p_{\text{set}}(t) = p_{\text{set}}. \]  
(100)
Table 1
Wellbore configuration in IRIS drilling simulator.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drillpipe outer diameter</td>
<td>0.1390 (m)</td>
</tr>
<tr>
<td>Drillpipe inner diameter</td>
<td>0.1183 (m)</td>
</tr>
<tr>
<td>Annulus inner diameter</td>
<td>0.4508 (m)</td>
</tr>
<tr>
<td>Oil density</td>
<td>805 (kg/m³)</td>
</tr>
<tr>
<td>Water density</td>
<td>1000 (kg/m³)</td>
</tr>
<tr>
<td>High gravity density</td>
<td>4200 (kg/m³)</td>
</tr>
<tr>
<td>Oil-water ratio</td>
<td>3.610</td>
</tr>
<tr>
<td>Drilling fluid mixture density</td>
<td>1670 (kg/m³)</td>
</tr>
</tbody>
</table>

and there exist $\gamma_1^*, \gamma_2^*$, and $\gamma_2^*$ such that for any $\gamma_1^* \in [0, \gamma_1^*], \gamma_2^* \in [0, \gamma_2^*]$, and $\gamma_2^* \in [0, \gamma_2^*]$ there exist positive constants $R$ and $\sigma$ such that,

$$\Omega(t) \leq R(e^{\sigma t(0)} - 1)$$

(101)

where

$$\Omega(t) = ||\dot{e}(t)||^2 + ||w(t)||^2 + ||\dot{\beta}_a(t)||^2 + ||\dot{d}(t)||^2$$

(102)

with $||u(t)||^2 = \int_0^1 u(t)^2 dx$, $\tilde{\beta}(t) = \beta - \hat{\beta}, \hat{d}(t) = d - \dot{d}(t)$ and $\dot{\hat{d}}(t) = d - \ddot{d}(t)$.

**Proof.** A Lyapunov functional is defined as the following

$$V(t) = D \log(1 + \Xi(t)) + \frac{1}{\gamma_1^*} \tilde{\beta}_a(t)^2 + \frac{\beta_a}{\gamma_2^*} \dot{\beta}_a^2 + \frac{d\beta^2_a(t)}{\gamma_2^*} + V_0(t)$$

(103)

$$\Xi(t) = e(t)^2 + d \int_0^1 (1 + x)w^2(x, t)dx,$$

(104)

$$V_0(t) = \epsilon_1 \dot{D}(t) \int_0^1 \frac{1}{1 + \Phi(r)} (||w(r)||^2 + ||e(r)||^2) dr,$$

(105)

Taking the time derivative of the above function and following the same procedure in the previous section, we get

$$\dot{V}(t) \leq - \frac{D}{(1 + \Xi(t))} M(||e(t)||^2 + ||w(0, t)||^2 + ||w(0)||^2).$$

(106)

and hence $\dot{V}(t)$ is negative definite and thus $V(t) \leq V(0)$. It implies that $e(t), w(0, t), w(t), \tilde{\beta}_a(t), ||\dot{\beta}_a(t)||, ||\dot{d}(t)||$ and $||\ddot{d}(t)||$ are uniformly bounded. From (89) it follows that $q(t)$ is uniformly bounded. Using Barbalat’s lemma, we conclude that $\lim_{t \to \infty} r(t) = 0$ in (100). Then we get the stability result and the bound (101) in Theorem 3. The proof is similar to the proof of Theorem 2. □

6. Simulation results

In this section, the proposed methodology is evaluated using a high fidelity well flow model WeMod [37], which is a high fidelity drilling simulator developed by the International Research Institute of Stavanger. IRIS WeMod solves a set of partial differential equations for transient two-phase flow referred to as the drift-flux formulation [38]. The simulator has been proven through several onshore and offshore tests [39]. The well geometry is summarized in Table 1.

The simulations are conducted based on two approaches, proposed adaptive predictor control and PI control. Two cases are considered: Case 1 is the BHP regulation during flowrate changes (draw-down test) and Case 2 is the BHP regulation during the pressure set-point changes. The BHP measurement is achieved in 1 s sampling rate. The objective is to control the BHP at a set point within ±5 (bar). In the normal drilling operation, the mud pump and the backpressure pump are operated manually. The physical devices for controlling the bottomhole pressure is the choke opening $z_c$, and the flow rate of the backpressure pump. The control signal relates to $q(t) = q_{pump} + q_{ch}$. We will employ a fast flow controller to achieve the desired flow through the choke.

The model parameters are calculated that the volume of annulus is $V_a = 419.8$ (m³), and the compressibility parameter $\beta_a$ is within a region $[0.5 \times 10^5, 2 \times 10^5]$. The controller is turned on at 30 s with initials of $\tilde{\beta}_a(0) = 1.55 \times 10^5$ and $\dot{\beta}_a(0) = 8 \times 10^{-4}$. The control parameters and adaptation gains in the proposed adaptive predictor control are chosen as $k = 0.7, l = 3000, \gamma_1^* = 2 \times 10^{-4}, \gamma_2^* = 1 \times 10^{-7}, \gamma_3^* = 3 \times 10^{-2}$. The control parameters for PI control are chosen as $k_p = 0.7, k_i = 0.035$. The controller is turned on at $t = 30$ s.

6.1. Case 1: Pressure control during flowrate changes

In the draw-down test, the backpressure pump is set at a fixed rate $q_{ch} = 400$ (l/min) and the flow rate through the mud pump $q_{pump}$ changes from 2000 (l/min) to 500 (l/min) with the step 500 (l/min). Two examples are considered: case 1(a) is 2 delay in the control input and case 1(b) is 8 s delays in control input, where the time delay is unknown in the control design. $D$ is within a region $[1, 3]$ for case 1(a) and within a region $[6, 9]$ in case 1(b). The initials are selected as $\dot{D}(0) = 1$ for case 1(a) and $\dot{D}(0) = 7$ for case 1(b). The objective is to control the BHP at a set point $p_{set} = 630$ (bar) within ±5 (bar).

Figs. 4 and 5 show the flow rates through the mud pump and backpressure pump, the choke, and the BHP regulation using PI control and proposed predictor control. Fig. 6 shows the estimates of parameter $\dot{\beta}_a$, disturbance $d$ and the time delay $D$. Using proposed predictor control, the pressure can be controlled at the set-point 630 ± 5 (bar). Using PI control, the pressure cannot be controlled within the desired
6. Bound when there is 8 s delay. Also PI control brings a large pressure surge before pressure returns back to the set-point, which the deviation from the set-point is not within the desired ±5 (bar) region. Clearly, the proposed predictor control has good performance in the pressure deviation and tracking speed compared with PI control. The pressure deviations from the desired set-point is much smaller and the tracking speed is faster for the proposed predictor control.

6.2. Case 2: Pressure control during set-point changes

For the second use case, we will present BHP control during set-point changes. The BHP set-point \( p_{\text{set}} \) is chosen as 630 (bar), 640 (bar), and 660 (bar) at different times. The mud pump and the backpressure pump are set at fixed rates as \( q_{\text{pump}} = 2000 \) (l/min) and \( q_{\text{bpp}} = 400 \) (l/min), separately. We consider 5 s delay in the input and \( D \) is within a region [1, 6]. The initial is selected as \( \hat{D}(0) = 4 \). The control parameters are fixed for both controllers when changing of set-point. The input and output data for the drilling operation are presented in Fig. 7, which shows the flow rates through the mud pump, backpressure pump and the choke, and the BHP regulation using PI control and proposed
predictor control. Clearly, using proposed predictor control, the pressure deviation from the desired set-point is much smaller and the tracking speed is faster than PI control. Fig. 8 shows the estimates of parameter, disturbance and time delay.

6.3. Discussion

Throughout the section, the control parameters are fixed for both cases. The simulation results show that the proposed predictor control is robust to the different drilling operations. According to the typical drilling operating conditions, a set of simulations is iteratively performed to find the good parameters. A rigorous methodology for the optimal tuning of parameters is difficult defined since the real system is nonlinear and time-varying for drilling operation. The further research on tuning of control parameters should be investigated for drilling operation so that the appropriate observer can guarantee the system obtaining the fast response and robustness. The tuning of control parameters remains an open problem for drilling operation.

7. Conclusion

In this paper, we address the adaptive predictor feedback design for a MPD system in the presence of unknown parameters, disturbance and time-delay. An adaptive predictor feedback control algorithm is developed to maintain BHP at the desired set-point and compensate the effects of time-delay and disturbance during drilling, where the adaptation employs Lyapunov update law design with normalization. The stabilization of the dynamic system and the asymptotic tracking are demonstrated by the proposed predictor control. The proposed method is evaluated using a high fidelity drilling simulator and cases from a North Sea drilling operation are simulated. Two cases are described including pressure regulation during draw-down test and set-point changes. The results show that the proposed adaptive predictor control obtains the pressure regulation within the desired region and compensates the effects of time-delay in MPD.

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