Adaptive Cancellation of Matched Unknown Sinusoidal Disturbances for LTI Systems by State Derivative Feedback

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Solutions already exist for the problem of canceling sinusoidal disturbances by measurement of state or an output for linear and nonlinear systems. In this paper, we design an adaptive controller to cancel matched sinusoidal disturbances forcing a linear time-invariant system by using only measurement of state-derivatives. Our design is based on three steps: (1) parametrization of the sinusoidal disturbance as the output of a known feedback system with an unknown output vector, (2) design of an adaptive disturbance observer and, (3) design of an adaptive controller. We prove that the equilibrium of the closed-loop adaptive system is globally uniformly asymptotically stable and locally exponentially stable. The effectiveness of the controller is illustrated with a simulation example of a second-order system. [DOI: 10.1115/1.4007708]

1 Introduction

The problem of canceling sinusoidal disturbances in dynamical systems is a fundamental control problem, with many applications such as vibrating structures [1], active noise control [2] and rotating mechanisms control [3]. The common method to approach this problem is the internal model principle for which a general solution is given in Refs. [4,5] in the case of linear systems. In the internal model approach, the disturbance is modeled as the output of a linear dynamic system which is called an exosystem. Then, the effect of the disturbance on the plant response can be completely compensated by adding a replica of the exosystem model in the feedback loop.

The output regulation problem for minimum phase, uncertain nonlinear systems is solved in Refs. [6,7], and extended for non-minimum phase plants in Ref. [8]. The regulation of a linear time-varying (LTV) system is considered in Ref. [9], and the regulation problem for time-varying known exosystem is studied in Refs. [10,11]. On the other hand, disturbance cancelation designs also exist for continuous-time linear systems [12–18], and discrete-time linear systems [19]. Moreover, designs for nonlinear systems are proposed in Refs. [20–23]. In all of these references, the controllers are designed by using measurement of state or an output.

In the last decade, the state derivative feedback control has drawn the attention of many researchers [24–29] due to its various advantages in applications. In most practical problems, especially disturbance cancelation problems, using accelerometers as sensors is easier, cheaper and more reliable than using position sensors. In this case, from the signals of the accelerometers it is possible to establish the velocities with a sufficient precision but not the displacements. Then, the system can be modeled by considering position and velocity as states and the state-derivatives are available for control design.

To the best of the authors’ knowledge, despite the fact that many papers deal with disturbance cancelation, the control design by using only measurement of state derivatives has not been considered. Employing an approach inspired by Ref. [30], we design an adaptive controller by state derivative feedback to cancel matched unknown sinusoidal disturbances forcing linear time-invariant systems. We prove global uniform asymptotic stability for the resulting plant-filter-estimator dynamics.

In Sec. 2, we introduce the problem and state our main stability theorem. In Sec. 3, we prove the theorem. A simulation example is presented in Sec. 4.

2 Problem Statement and Adaptive Controller Design

We consider the multi-input LTI system
\[
\dot{x}(t) = Ax(t) + Bu(t) + Dv(t)
\]

with the state \(x(t) \in \mathbb{R}^n\), input \(u(t) \in \mathbb{R}^m\), and sinusoidal disturbance \(v(t) \in \mathbb{R}\) given by
\[
v(t) = \sum_{i=1}^q \beta_i \sin(\omega_i t + \phi_i)
\]

where \(i \neq j \Rightarrow \omega_i \neq \omega_j, \omega_i \in \mathbb{Q}, \beta_i, \phi_i \in \mathbb{R}\).

The sinusoidal disturbance \(v\) can be represented as the output of a linear exosystem
\[
\dot{w}(t) = Sw(t)
\]

\[
v(t) = h^T w(t)
\]

where \(w \in \mathbb{R}^{2q}\) and the choice of \(S \in \mathbb{R}^{q \times q}\) and \(h \in \mathbb{R}^{2q}\) is not unique.

We make the following assumptions regarding the plant (1) and the exosystem (3) and (4):

Assumption 1. \(A\) is invertible.

Assumption 2. There exists a matrix \(R \in \mathbb{R}^m\) such that \(BR = D\).

Assumption 3. The pair \((A, B)\) is controllable.

Assumption 4. \(x\) and \(v\) are not measured but \(\dot{x}\) is measured.

Assumption 5. The pair \((h^T, S)\) is observable.

Assumption 6. The eigenvalues of \(S\) are imaginary, distinct and rational.

Assumption 7. \(q\) is known.

Assumption 8. \(S\) and \(h\) are unknown.

Assumption 9. \(\beta_i \neq 0\) for all \(i \in \{1, \ldots, q\}\).

Under Assumptions 1 and 3, there exists a control gain \(K \in \mathbb{R}^{1 \times n}\) such that \((A^{-1} + A^{-1}BK)\) is Hurwitz [24].

We state now our adaptive controller with a disturbance observer. In Sec. 3, we analyze the stability properties of the closed-loop system.

The adaptive controller for the systems (1), (3), and (4) is given by
\[
u = -K\dot{x} - R\dot{\theta}^T \hat{\xi}
\]

the update law for \(\dot{\theta}(t)\) is given by
\[
\dot{\theta} = -\gamma \hat{\xi}^T (A^{-1} D)^T \hat{P} \hat{\xi}, \quad \gamma > 0
\]

with the positive definite matrix \(P\) which is a solution of the matrix equation
\[
(A^{-1} + A^{-1} BK)^T P + P (A^{-1} + A^{-1} BK) = -2I
\]

The disturbance observer is given by
\[
\dot{\eta} = G(\eta + N(\dot{x} - Bu)) - N\dot{x}
\]

\[
\hat{\xi} = \eta + N(\dot{x} - Bu)
\]
where $G$ is a $2q \times 2q$ Hurwitz matrix with distinct poles and constitutes a controllable pair with a chosen vector $l \in \mathbb{R}^{2q}$ and $N$ is a $2q \times n$ matrix which is given by

$$N = \frac{1}{\mathcal{D}T}ID^T$$

(10)

where the given $N$ is one of the many solutions of the following equation:

$$ND = I$$

(11)

Since the matrices $G$ and $S$ have disjoint spectra, the pair $(h^T, S)$ is observable, and the pair $(G, l)$ is controllable, the Sylvester equation

$$MS - GM = lh^T$$

(12)

has a unique solution [31,32]. This fact is exploited in the proof of our stability result (Lemma 1).

We first state a theorem describing our main stability result and then we prove it using a series of technical lemmas in Sec. 3.

Theorem 1. Consider the closed-loop system consisting of the plant (1) forced by the unknown sinusoidal disturbance (2), the disturbance observers (8),(9) and the adaptive controllers (5),(6). Under Assumptions 1–8, the system’s solution $x(t) \equiv 0, \theta(t) \equiv \theta^0$ is globally uniformly asymptotically stable and locally exponentially stable. Furthermore, $\theta^0(t)z(t) - \nu(t) \to 0$ exponentially as $t \to \infty$, namely, perfect estimation of the disturbance is achieved.

3 Stability Proof

The following lemma enables us to represent the unknown sinusoidal disturbance as the output of a linear system whose input is the disturbance itself, whose state and input matrices are known, and whose output matrix is unknown. Let $G \in \mathbb{R}^{2q \times 2q}$ be a Hurwitz matrix with distinct eigenvalues and let $(G, l)$ be a controllable pair. Then, $\nu$ can be represented as the output of the model

$$\dot{z} = Gz + \nu$$

(13)

$$\nu = \theta^T \dot{z}$$

(14)

$$\theta^T = h^T (MS)^{-1}$$

(15)

Proof. By differentiating Eq. (13) with respect to time, we obtain

$$\dot{\theta} = G \dot{z} + \dot{\nu}$$

By defining the estimation error, we get

$$\delta = \dot{\nu} - \dot{\theta}$$

(16)

Differentiating $\delta$ with respect to time and in view of Eqs. (13), (8), and (9), we get

$$\dot{\delta} = G \dot{z} + \dot{\nu} - G(\eta + N(\dot{x} - Bu)) + NA \dot{x} - NA \dot{k} - \dot{ND} \dot{\nu}$$

(17)

Substituting Eq. (9) into Eq. (23), using Eq. (22) and the fact that $ND^0 = L$, we get Eq. (20). Using Eqs. (14), (22), and the fact that $\dot{z} = \dot{z}$, we obtain Eq. (19).

Lemma 1 and 2 convert the problem from cancelation of an unknown sinusoidal disturbance to an adaptive control problem. The following lemma is used in the proof of the main theorem.

Lemma 2.

$$MS \rightarrow \frac{1}{\mathcal{D}T}ID^T$$

(18)

where

$$\dot{\nu} = \sum_{i=1}^{q} \beta_i \cos(\omega_i t + \phi_i)$$

(19)

with $\beta_i = \beta_i \omega_i$.

By solving Eq. (25), we get

$$\frac{\ddot{z}}{z}(t) = e^{Gt} \frac{\ddot{z}}{z}(0) + \int_{0}^{t} e^{G(t-s)} \dot{\nu} ds$$

(20)

Since $G$ has distinct eigenvalues and is Hurwitz, it is diagonalizable. Using a Jordan decomposition of the matrix $G$, we can write

$$G = L\Lambda L^{-1}$$

(21)

where $L$ is the square $2q \times 2q$ matrix whose $i$th column is the $i$th eigenvector of $G$ and $\Lambda$ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues of $G$.

Defining $L^{-1} x = l$, substituting Eq. (28) into Eq. (27) and using the property $e^{L^{-1} t} = Le^{L^{-1} t}$, we get

$$\frac{\ddot{z}}{z}(t) = Le^{L^{-1} t} \frac{\ddot{z}}{z}(0) + \int_{0}^{t} e^{-L^{-1} t} \dot{\nu} ds$$

(22)

By computing the integral in Eq. (29), we obtain

$$\frac{\ddot{z}}{z}(t) = L(e^{\Lambda t} \frac{\ddot{z}}{z}(0)) + \sum_{i=1}^{q} \frac{\beta_i}{\omega_i} \left( -\omega_i \cos(\omega_i t + \phi_i) + \omega_i \sin(\omega_i t + \phi_i) \right)$$

(23)

and

$$C_c = L^{-1} \frac{\ddot{z}}{z}(0) - \Psi(0)$$

(24)

Proof. By differentiating Eq. (22) with respect to time and using Eqs. (20) and (21), the estimate $\dot{z}$ can be represented as the solution of

$$\ddot{z} = G \ddot{z} + \dot{\nu}$$

(25)

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Since $\dot{\nu}$ is a sufficiently rich signal order of $2\eta$ and $(G, f)$ is a controllable pair, $\zeta$ is persistently exciting [33]. Therefore, there exist positive $\rho^*$ and $\theta_0$ such that for all $\rho > \rho^*$ and $t_0 \geq 0$ the following holds

$$\int_{t_0}^{t_0 + \rho} \dot{\zeta}(t)\dot{\zeta}^T(t)dt \geq \rho \theta_0 I$$

(33)

Under Assumption 6, the frequencies of $\dot{\nu}$ can be represented as

$$\omega_i = \frac{\text{num}(\omega_i)}{\text{den}(\omega_i)}, \text{num}(\omega_i), \text{den}(\omega_i) \in \mathbb{Z}^+, \quad i = 1, \ldots, q$$

Then, $\rho$ that given by

$$\rho = \frac{\text{lcm}(\text{num}(\omega_1), \ldots, \text{num}(\omega_q)) \times \text{lcm}(\text{den}(\omega_1), \ldots, \text{den}(\omega_q))}{2\pi}$$

(34)

where lcm is the abbreviation of the least common multiple, satisfies Eq. (33) if $\theta \in \mathbb{Z}^+$ is chosen sufficiently large for given $\rho^*$ and $\omega_1, \ldots, \omega_q$. Since $\Psi(t)$ defined by Eq. (31) has a period $\rho$ and incorporates only zero-mean functions, it follows that

$$\int_{t_0}^{t_0 + \rho} \Psi(t)dt = 0$$

(35)

Substituting Eqs. (30)–(35) into Eq. (24), we get

$$Q_0(\rho, t_0) \geq \text{LH}$$

(36)

where

$$\text{LH} = \rho \theta_0 I - \frac{1}{\rho} \Gamma(\rho)\Gamma^T(\rho)$$

(37)

and $c_i$ denotes the $i$th row of the vector $C_i/\lambda_i$.

Since $L$ is full rank, $Q_0$ satisfies the inequality (24) if $\mu^T\Theta_H > 0$ for all nonzero $\mu \in \mathbb{R}^{2d}$. Using Eq. (37), we have

$$\mu^T(\rho^*) \Theta_H = \rho \theta_0 (\mu_1^2 + \ldots + \mu_d^2) - \frac{1}{\rho^*} \left( c_0^2e^{i\lambda t_0}(e^{i\rho t_0} - 1) + \ldots + c_d^2e^{i\lambda t_0}(e^{i\rho t_0} - 1) \right)^2$$

(39)

By using Cauchy-Schwarz’s inequality and by noting that $\lambda_i < 0, |(e^{i\rho t_0} - 1)| \leq 1$, we have

$$\mu^T(\rho^*) \Theta_H \geq \rho \theta_0 \left( \mu_1^2 + \ldots + \mu_d^2 \right) - \frac{1}{\rho^*} \left( \frac{(c_0^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2 + \ldots + (c_d^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2}{(c_0^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2 + \ldots + (c_d^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2} \right)$$

(40)

Since $\rho \theta_0$ is monotone increasing and $\frac{1}{\rho^*} \frac{(c_0^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2 + \ldots + (c_d^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2}{(c_0^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2 + \ldots + (c_d^2e^{i\lambda t_0})(e^{i\rho t_0} - 1)^2}$ is monotone decreasing with respect to $\rho$ for all fixed $t_0$, one can find a $\rho$ using Eq. (34) such that for all $t_0 \geq 0$, Eq. (24) holds.

**Proof of Theorem 1:** We represent the closed-loop system as a LTV system which is given by

$$\dot{\zeta} = E(t)\zeta + F(t)\delta$$

(41)

where

$$E(t) = \begin{bmatrix} A_{cl} & D_{cl}^{2T} \\ \gamma_j(A^{-1}D)^T P_{cl} & \gamma_j(D^T)^T P_{cl} D_{cl}^{2T} \end{bmatrix}$$

(42)

$$F(t) = \begin{bmatrix} 0 \\ \gamma_j(A^{-1}D)^T PD_{cl}^{2T} \end{bmatrix}$$

(43)

$$\zeta = \begin{bmatrix} x^T \\ \dot{\theta}^T \end{bmatrix}$$

(44)

$$\delta = \theta - \dot{\theta}$$

(45)

with

$$A_{cl} = (A^{-1} + A^{-1}BK)^{-1}$$

(46)

$$D = A_{cl} A^{-1}D$$

(47)

We first show that the equilibrium $\zeta = 0$ of the homogenous part of the LTV system (41) is exponentially stable. Toward that end, we choose the following Lyapunov function

$$V = \frac{1}{2} \zeta^T P_c \zeta$$

(48)

where

$$P_c = \begin{bmatrix} P & 0 \\ 0 & \frac{1}{\gamma} I \end{bmatrix}$$

(49)

Taking the derivative of $V$, we get

$$\dot{V} = \frac{1}{2} \zeta^T \begin{bmatrix} A_{cl}^TP + PA_{cl} & (A_{cl}^TPA_{cl}^{-1} + P)D_{cl}^{2T} \\ \gamma_jD^T(A_{cl}^TPA_{cl}^{-1} + P) & \gamma_jD^T(D_{cl}^T(A_{cl}^{-1}P + PA_{cl}^{-1})D_{cl}^{2T}) \end{bmatrix} \zeta$$

(50)

By pre and postmultiplying Eq. (7) by $A_{cl}^T$ and $A_{cl}$ and using the fact that $A_{cl} = (A^{-1} + A^{-1}BK)^{-1}$, we obtain

$$A_{cl}^TP + PA_{cl} = -2A_{cl}^TPA_{cl}$$

(51)

Premultiplying Eq. (7) by $A_{cl}^T$, we get

$$A_{cl}^TPA_{cl}^{-1} + P = -2A_{cl}^TPA_{cl}$$

(52)

Postmultiplying Eq. (7) by $A_{cl}$, we get

$$P + A_{cl}^TPA_{cl} = -2A_{cl}$$

(53)

Substituting Eqs. (7) and (51)–(53) into Eq. (50), we get

$$\dot{V} = -\frac{1}{2} \zeta^T \begin{bmatrix} A_{cl}^TPA_{cl}^{-1} & \gamma_jD^T \end{bmatrix} \begin{bmatrix} \gamma_jD^T & 0 \\ 0 & \gamma_jD^T \end{bmatrix} \zeta$$

(54)

Defining

$$C^T(t) = \begin{bmatrix} A_{cl} & D_{cl}^{2T} \end{bmatrix}$$

(55)

we get

$$\dot{V} = -\frac{1}{2} \zeta^T (E(t)P_c + P_c E(t))\zeta = -\zeta^T C(t)C(t)\zeta$$

(56)

Therefore, it follows that $P_c$, as defined in Eq. (49), satisfies the following inequality:

$$E(t)P_c + P_c E(t) + \alpha C^T(t)C(t) \leq 0$$

(57)

for some $\alpha > 0$.

The equilibrium $\zeta = 0$ of the homogenous part of Eq. (41) is exponentially stable if $(C(t), E(t))$ is a uniformly completely observable (UCO) pair [34]. For a bounded $H(t)$, the pairs $(C(t),$
$E(t)$ and $(C(t), E(t) + H(t)C(t)^T)$ have the same UCO property [34]. Choosing
\[ H(t) = \begin{bmatrix} -I \\ -\hat{\xi}(A^{-1}D)^T \end{bmatrix} \] (58)
we write the system corresponding to the pair $(C, E + HC^T)$ as
\[ \dot{Y} = 0 \] (59)
\[ y = C^T(t)Y \] (60)
The state transition matrix of Eq. (59) is $\Phi = I$. Therefore, $(C, E + HC^T)$ is a UCO pair if there exist positive constants $\bar{z_1}, \bar{z_2}, \rho$ such that the observability Gramian satisfies
\[ \bar{z}_I \geq \int_{t_0}^{t_0 + \rho} C(t)C^T(t)dt \geq \bar{z}_1 I \] (61)
for all $t_0 \geq 0$. Since $\hat{\xi}$ is bounded, recalling Eq. (55), the upper bound of Eq. (61) is satisfied. We now prove the lower bound in Eq. (61). Calculating the integral in Eq. (61), we get
\[ X = \int_{t_0}^{t_0 + \rho} C(t)C^T(t)dt = \left[ A^T_{cl}D A_{cl} \right]_{t_0}^{t_0 + \rho} \] (62)
\[ = \int_{t_0}^{t_0 + \rho} \hat{\xi}d\hat{\xi} \int_{t_0}^{t_0 + \rho} D\hat{\xi}d\hat{\xi} dt \]
Let $S_k$ be the Schur complement of $A^T \bar{A}_h \rho$ in $X$, where
\[ S_k = \int_{t}^{t_0 + \rho} \hat{\xi}D^T \hat{\xi} dt - \frac{1}{\rho} \int_{t}^{t_0 + \rho} \hat{\xi}d\hat{\xi} A_{cl}^{-1} A_{cl}^T \hat{\xi} dt \]
\[ = D^T \hat{\xi} \int_{t}^{t_0 + \rho} \hat{\xi}d\hat{\xi} \int_{t}^{t_0 + \rho} \hat{\xi} d\hat{\xi} dt \] (63)
Since $A^T \bar{A}_h \rho$ is positive definite, $X$ is positive definite if and only if $S_k$ is positive definite. Since $D^T \hat{\xi}$ is a positive scalar, according to Lemma 3 there exists a positive $\rho$ such that for all $t_0 > 0, S_k > 0$. Hence, $(C, E + HC^T)$ is UCO, which implies that $(C, E)$ is UCO. Therefore, the state transition matrix $\Phi(t, t_0)$ corresponding to $E(t)$ in Eq. (41) satisfies
\[ \| \Phi(t, t_0) \| \leq \kappa_0 e^{-\gamma_0(t-t_0)} \] (64)
for some positive constants $\kappa_0, \gamma_0$. Since $G$ is Hurwitz, we have that
\[ |\delta(t)| = |e^{(s(t)-t_0)}| \leq \kappa_1 e^{-\gamma_1|s|} |\delta(t)| \] (65)
for some positive constants $\kappa_1, \gamma_1$. The solution of Eq. (41) is written as
\[ |\hat{z}(t)| = \Phi(t, 0) |\hat{z}(0)| + \int_0^t \Phi(t, \tau) F(\tau) \delta(\tau) d\tau \] (66)
Using Eqs. (64)–(66), we get
\[ |\hat{z}(t)| \leq \kappa_0 e^{-\gamma_0} |\hat{z}(0)| + \int_0^t \kappa_0 e^{-\gamma_0(t-\tau)} |F(\tau)| |\hat{z}(\tau)| \delta(\tau) d\tau \]
\[ \leq \kappa_0 e^{-\gamma_0} |\hat{z}(0)| + \kappa_0 e^{-\gamma_0} |\delta(0)| \sup_{0 \leq \tau \leq t} |F(\tau)| \int_0^t e^{(s(t)-t_0)} \frac{1}{2} \min(\gamma(t), \gamma) t_0 \frac{1}{2} t_0 \min(\gamma(t), \gamma) \frac{1}{2} t_0 \] (67)
\[ \kappa_1 = \kappa_1 e^{-\gamma_1} |\hat{z}(0)| + \kappa_2 e^{-\gamma_2} |\hat{z}(0)| + \kappa_1 \sup_{0 \leq \tau \leq t} |F(\tau)| \int_0^t e^{(s(t)-t_0)} \frac{1}{2} \min(\gamma(t), \gamma) t_0 \frac{1}{2} t_0 \min(\gamma(t), \gamma) \frac{1}{2} t_0 \] (68)
Using the fact that $\hat{z} = \hat{\xi}$ and substituting Eq. (17) into Eq. (22), we get
\[ \hat{\xi}(t) = MSw(t) - \delta(t) \] (69)
Substituting Eq. (69) into Eq. (9) and using Eq. (12) and the fact that $\ddot{x} - Bu = Ax + Du$ and $ND = I$, we obtain
\[ \eta(t) - GMw(t) = -NAx(t) - \delta(t) \] (70)
By virtue of (44)
\[ \eta(t) - GMw(t) = -[NA, 0, I] \begin{bmatrix} \hat{\xi} \\ \delta \end{bmatrix} \]
Since $\hat{\xi}(t) = \begin{bmatrix} x'(0) \\ \hat{\theta}'(0) - H'(MS)^{-1} \end{bmatrix}$ and $\delta(0) = -NAx(0) - \eta(0) + GMw(0)$, following Eqs. (65) and (67), we get that the solution $x(t) \equiv 0, \hat{\theta}(t) \equiv (MS)^{-1} h, \eta(t) \equiv GMw(t)$ is globally uniformly asymptotically stable and locally exponentially stable. Furthermore, according to Lemma 2, $\hat{\theta}'(t) \hat{\xi}(t) - \nu(t) \to 0$ exponentially as $t \to \infty$, namely, perfect estimation of the disturbance is achieved.

The reason why we are not claiming global exponential stability is that the quantities $\sup_{0 \leq \tau \leq t} |F(\tau)|, \kappa_0, \gamma_0$ while bounded, actually depend on the initial conditions $x(0), \hat{\theta}(0), \eta(0)$, as can be observed by tracing the derivation of these quantities throughout the paper and in the quoted sources.

4 Simulation Results

We illustrate the performance of our controller with a second-order system with $A = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ the unknown disturbance $\nu(t) = 3 \sin(t + \pi/5)$ and initial condition...
The control gain \( K \) is chosen such that the eigenvalues of \((A^{-1} + A^{-1}BK)\) are \(-2\) and \(-1\). For the update law, we choose \( \gamma = 1 \). Finally, the controllable pair \((G, I)\) is chosen as \( G = \begin{bmatrix} 0 & 1 \\ -1.95 & -2.8 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ R = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \) and \( N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \).

From Figs. 1 and 2, one can observe that \( x(t) \) exponentially converges to zero and the unknown disturbance is perfectly estimated, as Theorem 1 predicts.

The placement of the poles of \((A^{-1} + A^{-1}BK)\) as well as the update gain \( \gamma \) affect the convergence of the states and the estimation. Increasing the absolute value of the real part of the poles provides high control gain. The update gain \( \gamma \) should be chosen proportionally high or low with respect to control gain to provide an optimum convergence rate.

5 Conclusions

In the present work we design an adaptive controller by state derivative feedback to cancel matched unknown sinusoidal disturbances forcing a linear time-invariant system. We prove that the closed-loop system’s solution \( x(t) \equiv 0, \theta(t) \equiv (MS)^{-1} h, \eta(t) \equiv GMw(t) \) is globally uniformly asymptotically stable and locally exponentially stable. The effectiveness of our controller is demonstrated with a numerical example.

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