Parameter Identification for Electrohydraulic Valvetrain Systems

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We consider an electrohydraulic valve system (EHVS) model with uncertain parameters that may possibly vary with time. This is a nonlinear third order system consisting of two clearly separated subsystems, one for the piston position and the other for the chamber pressure. The nonlinearities involved are flow-pressure characteristics of the solenoid valves, the pressure dynamics of the chamber due to varying volume, and a variable damping nonlinearity. We develop a parametric model that is linear in the unknown parameters of the system using filtering. We deal with a nonlinear parameterization in the variable damping term using the Taylor approximation. We design a parameter identifier, which employs a continuous-time unnormalized least-squares update law with a forgetting factor. This update law exponentially converges to the true parameters under a persistence of excitation condition, which is satisfied due to the periodic regime of operation of EHVS. We present simulation results that show good following of unknown parameters even with the presence of sensor noise. [DOI: 10.1115/1.4004780]

Keywords: automotive control, electrohydraulic valves, system identification, parameter estimation, least-squares

1 Introduction

Hydraulic actuators are used throughout industry in a wide number of applications. With a small size and a high force-to-mass ratio, the actuators have found applications in electrohydraulic positioning [1], flight control surfaces [2], and active suspension control [3]. This paper analyzes a hydraulic system, which replaces the conventional mechanical camshaft on engines. The electrohydraulic valvetrain system (EHVS) is one of the most flexible options for variable valve timing in automotive engines.

Normally, valve timings are constrained to the motion of the camshaft; however, EHVS valve timings are fully independent of the crankshaft position. This flexibility allows cycle-to-cycle control of fuel intake and waste exhaust, which leads to benefits such as improved fuel economy and lower exhaust emissions. Additionally, at low-to-moderate engine speeds, the timing can be optimized to improve full load torque as discussed by Denger and Mischker [4].

An alternate solution to the camless engine is an electromechanical valve (EMV), which is discussed in the paper by Wang et al. [5]. Like EHVS, with appropriate control schemes, EMV offers improvements to standard internal combustion engine performance as discussed by Pischinger et al. [6]. However, there are difficulties in achieving soft landings with the EMV actuator (Wang et al., [5]). Peterson and Stefanopoulou [7] address the high impact velocities via an extremum seeking method based on microphone measurements. EHVS addresses those problems with a variable damping mechanism that increases the damping as the valve comes to a close.

Because of the wide range of industrial applications, the past work in hydraulic system control has utilized various control techniques. For instance, FitzSimons and Palazzolo [8] used linear control theory and Vossoughi and Donath [9] utilized feedback linearization in their respective hydraulic control problems. Recently, quantitative feedback theory, in Pachter et al. [2] and Niksefat and Sepehri [10], and backstepping design technique (Yu et al., [11]) have been used to create robust controllers. However, it is in Alleyne and Hedrick [3] where nonlinear adaptive control is applied in the presence of parametric uncertainties. They show, experimentally, how an active nonlinear adaptive control scheme improves performance over a nonadaptive scheme. In addition, Alleyne and Liu [12] developed simplifications to their control scheme without too much performance loss. Also Yao et al. [13] utilized an adaptive robust control to overcome both parametric uncertainties and uncertain nonlinearities. With their adaptive robust motion control algorithm, they achieved more than a order of magnitude reduction in tracking errors over a proportional-integral-derivative motion controller.

Parameters vary during operation of an EHVS. For example, the bulk modulus is dependent on pressure and temperature (Yu et al. [14]). Likewise, fluid density, damping coefficients, and other parameters of the valve model are uncertain and dependent on operating conditions. For this reason, we develop a model-based on-line parameter identification scheme for EHVS. Given the number of parameters involved, achieving uniformity in their rates of convergence is nontrivial. Uniformity is critical because the slowest component of the parameter vector limits the usefulness of the entire parameter estimator. To achieve uniformity in the convergence of our parameter estimator, we employ the least-squares approach. A forgetting factor is used to keep the adaptation gains from converging to zero with time. We achieve exponential convergence of the parameter estimates thanks to the inherent persistency of excitation in automotive valve applications.

Our contribution is in developing the estimation algorithm. This development involves modeling of variable damping in the valve, a suitable parametrization of the model so that the total number of estimated parameters is minimal, suitable filtering of measured signals such that some nonlinearities arising in the model appear only as known quantities in the regressor, and simulation tests of the algorithm under a realistic valve cycle and under measurement noise.

We start with the introduction of the model in Sec. 2. In Sec. 3, we develop the variable damping model. In Sec. 4, we create parametric models of the two subsystems, which is necessary for us to approach the synthesis of parameter identifiers. In Sec. 5, we introduce the identifiers and the least-squares update law. Here, we present the algorithms that allow on-line estimation of the various parameters, and briefly discuss the role of forgetting factor in stable tracking of slowly varying parameters. In Sec. 6 and in the Appendix, we present the analysis of stability of the identifier. In Sec. 7, we present simulation results for the identifier, in the presence of slowly varying parameters and sensor noise.

2 Model of EHV System

Consider the model of the EHVS system based on the Fig. 1 given by two differential equations

$$M_t \ddot{x}_p = A_{p_1} P_1 - A_{p_2} P_s - F_o - B\dot{x}_p - RB \tag{1}$$

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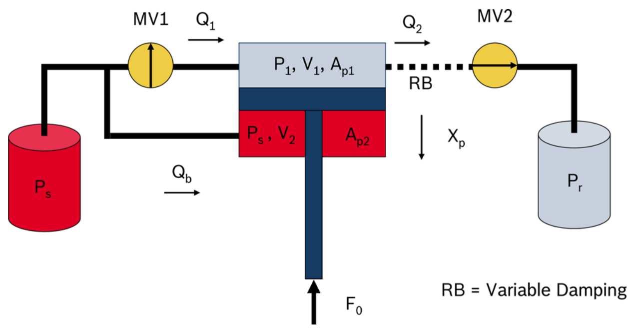

Fig. 1 Schematic of the EHVS system

$$Q_1 - Q_2 = \dot{V}_1 + \frac{V_1}{\beta_e}\dot{P}_1 \tag{2}$$

and three algebraic equations

$$V_1 = V_{o_1} + A_{p_1}x_p \tag{3}$$

$$Q_1 = C_{d1}w_1x_{v1}\mathrm{sgn}(P_s - P_1)\sqrt{\frac{2|P_s - P_1|}{\rho}} \tag{4}$$

$$Q_2 = C_{d2}w_2x_{v2}\mathrm{sgn}(P_1 - P_r)\sqrt{\frac{2|P_1 - P_r|}{\rho}} \tag{5}$$

where the model variables are defined in Table 1. The system has three states, P_1, x_p, $\dot{x}_p$, two inputs that are available to the designer, x_{v_1}, x_{v_2}, and three inputs that are not available to the designer, P_s, P_r, F_0. The variables V_1, Q_1, and Q_2 algebraically depend on the states variables and inputs. The remaining quantities are system parameters, among which only the parameters V_{0_1}, M_t, A_{p_1}, and A_{p_2} are assumed to be known precisely.

This model is physics based. The solenoid valves x_{v_1} and x_{v_2} are considered to be the discrete inputs of the system. Solenoid valve x_{v_1} is normally closed, and the other solenoid valve is normally open. Pohl et al. [15] have done work in modeling the switching valve dynamics; however, in this model, those input dynamics are neglected. In order to create movement in the system, valve x_{v_2} closes while x_{v_1} opens letting high pressure into the top chamber.

Table 1 Nomenclature for the EHVS model

Nomenclature	
M_t	mass of the valve piston
$\ddot{x}_p$	acceleration of the valve piston
$\dot{x}_p$	velocity of the valve piston
x_p	position of the valve piston
A_{p1}	area of the top of the valve piston
A_{p2}	area of the bottom of the valve piston
x_{v1}	opening of solenoid valve 1
x_{v2}	opening of solenoid valve 2
V_1	chamber volume
V_{01}	chamber volume of closed valve
P_s	supply pressure
P_1	chamber pressure
P_r	reservoir pressure
C_{d1}	discharge coefficient of solenoid valve 1
C_{d2}	discharge coefficient of solenoid valve 2
w_1	area gradient of port 1
w_2	area gradient of port 2
ρ	density of hydraulic fluid
B	damping of valve piston
β_e	compressibility coefficient
F_0	dead force on valve
RB	variable damping

The difference in surface area ($A_{p1} < A_{p2}$) creates downward movement in the system. When both valves are closed, the lift is held in a steady state. To close the valve x_{v_2} is opened exposing the top chamber to the reservoir pressure and forcing upward movement. There is an inclusion of variable damping term RB, which is responsible for the soft seating of the valve. This term will be introduced in Sec. 3.

It may appear that there are seven uncertain physical parameters, β_e, B, C_{d_1}, C_{d_2}, w_1, w_2, and ρ. However, only four can be identified independently from the measured signals. The identifiable terms are the damping of the valve piston B found in Eq. (1), the compressibility coefficient β_e found in Eq. (2), and the "combination" of coefficients $C_{d_1}w_1\sqrt{\frac{2}{\rho}}$ and $C_{d_2}w_2\sqrt{\frac{2}{\rho}}$ found in Eqs. (4) and (5), respectively. The solenoid valve discharge coefficients, area gradients of the solenoid valve ports, and the density of the hydraulic fluid cannot be separately identified and therefore must be lumped together.

3 Model of Variable Damping Nonlinearity

Equation (1) contains the variable damping term RB. The purpose of variable damping is to soften the closing of the valve. The physical design of the valves (proprietary) ensures that damping is present in the "closing direction" of travel of a valve, immediately before the valve closes, namely, only for positive velocities and small displacements of the valve piston.

To model the effect of variable damping as a swift increase of damping as the valve comes to close, we introduce a phenomenological model of the variable damping in the following form:

$$RB(x_p, \dot{x}_p) = \frac{\dot{x}_p - |\dot{x}_p|}{2}\frac{D}{D_b + x_p^{2k}} \tag{6}$$

This nonlinear force acts as damping, as it is velocity dependent, but the damping value is a function of position.

The velocity term $\frac{\dot{x}_p - |\dot{x}_p|}{2}$ ensures that the variable damping force is only active during the closing of the valve. The displacement-dependent damping coefficient $\frac{D}{D_b + x_p^{2k}}$ has small values when the valve is open by an appreciable amount.

The terms (D, D_b, and k) offer flexibility in the approximation of variable damping. Figure 2 shows the effects of varying D, D_b, and k. Increasing D_b increases the distance from zero that variable damping goes into effect. The maximum gain approaches the ratio of $\frac{D}{D_b}$. Finally, a larger k creates a more step-like response.

Equation (6) gives flexibility in the phenomenological modeling of RB. However, since the unknown parameter D_b appears nonlinearly, creating a linear parametric model for estimation of D_b poses a problem. To achieve this, we take a Taylor expansion of RB about D_b at the estimate $\hat{D}_b$, which yields

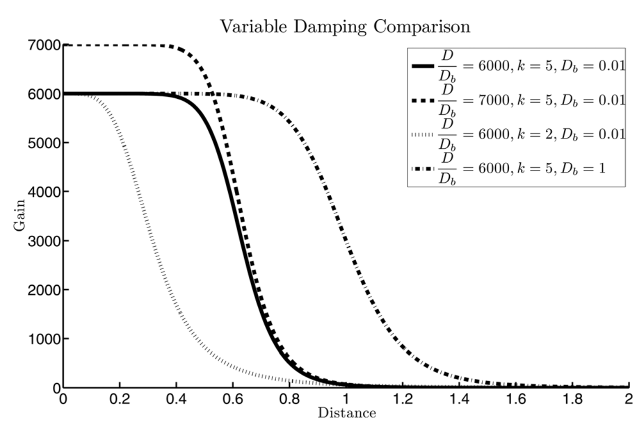

Fig. 2 Dependence of $\frac{D}{(D_b + x_p^{2k})}$ on D, D_b, k, and x_p

$$
\begin{aligned}
RB &\approx \frac{\dot{x}_p - |\dot{x}_p|}{2} D \left(\frac{1}{\hat{D}_b + x_p^{2k}} + \frac{-D_b + \hat{D}_b}{\left(\hat{D}_b + x_p^{2k}\right)^2} \right) \\
&\approx \frac{\dot{x}_p - |\dot{x}_p|}{2} D \left(\frac{2\hat{D}_b + x_p^{2k} - D_b}{\left(\hat{D}_b + x_p^{2k}\right)^2} \right) \\
&\approx \frac{\dot{x}_p - |\dot{x}_p|}{2} D \left(\frac{2\hat{D}_b + x_p^{2k}}{\left(\hat{D}_b + x_p^{2k}\right)^2} - \frac{D_b}{\left(\hat{D}_b + x_p^{2k}\right)^2} \right)
\end{aligned} \tag{7}
$$

The system is now linear in the unknown term D_b, noting that $\hat{D}_b$ is an estimate of that parameter. Substituting Eq. (7) into Eq. (1) yields

$$
\begin{aligned}
M_t \ddot{x}_p &= A_{p_1} P_1 - A_{p_2} P_s - F_o - B\dot{x}_p \\
&\quad - \frac{\dot{x}_p - |\dot{x}_p|}{2} \left[D \left(\frac{2\hat{D}_b + x_p^{2k}}{\left(\hat{D}_b + x_p^{2k}\right)^2} - \frac{D_b}{\left(\hat{D}_b + x_p^{2k}\right)^2} \right) \right]
\end{aligned} \tag{8}
$$

We use Eq. (8) as the parametric model for our identifier design and in the convergence analysis in Proposition 1. However, in our simulation tests, we use the nonlinearly parameterized model Eqs. (1) and (6).

4 Parametric Model

The system's two differential Eqs. (2) and (8) are now linear in the parameters β_e, B, D, D_b, $C_{d_1} w_1 \sqrt{2/\rho}$, and $C_{d_2} w_2 \sqrt{2/\rho}$. However Eqs. (2) and (8) involve the derivative signals $\ddot{x}_p$, $\dot{x}_p$, $\dot{P}_1$, and $\dot{V}_1$, which are noisy. Unfortunately, $\dot{x}_p$ appears nonlinearly (and nonsmoothly) in Eq. (8) and therefore must be treated as measurable in the parametric model. Filtering Eqs. (2) and (8) with a stable first order low-pass filter of the form $\frac{1}{s+\lambda}$ creates a suitable parametric model for identification.

After filtering Eq. (8) we get the parametric model

$$
\begin{aligned}
&M_t \left(\frac{\lambda}{s+\lambda} \dot{x}_p - \dot{x}_p \right) + \frac{1}{s+\lambda} \left(A_{p_1} P_1 - A_{p_2} P_s \right) = B \left(\frac{s}{s+\lambda} x_p \right) \\
&\quad + \frac{1}{s+\lambda} F_o \quad + D \left(\frac{1}{s+\lambda} \left[\frac{\left(\dot{x}_p - |\dot{x}_p|\right)\left(\hat{D}_b + x_p^{2k}\right)}{2\left(\hat{D}_b + x_p^{2k}\right)^2} \right] \right) \\
&\quad - DD_b \left(\frac{1}{s+\lambda} \left[\frac{\dot{x}_p - |\dot{x}_p|}{2\left(\hat{D}_b + x_p^{2k}\right)^2} \right] \right)
\end{aligned} \tag{9}
$$

In the absence of any information on F_0, we treat $\frac{1}{s+\lambda} F_o$ as stochastic noise; otherwise, it can be moved to the left as a known signal. Aside from B, D, and D_b all the signals in the model (9) are available and linear in the unknown parameters.

Conversion of Eq. (2) into a parametric model requires us to first rewrite it as follows:

$$
\begin{aligned}
&\frac{d}{dt} P_1 + \beta_e \frac{d}{dt} \ln V_1 \\
&\quad = \beta_e C_{d1} w_1 \sqrt{\frac{2}{\rho}} x_{v1} \operatorname{sgn}(P_s - P_1) \frac{\sqrt{|P_s - P_1|}}{V_1} \\
&\qquad - \beta_e C_{d2} w_2 \sqrt{\frac{2}{\rho}} x_{v2} \operatorname{sgn}(P_1 - P_r) \frac{\sqrt{|P_1 - P_r|}}{V_1}
\end{aligned} \tag{10}
$$

Note that the signals P_1, V_1, P_s, P_r, x_{v_1}, and x_{v_2} are available (either measured or available as control inputs). Therefore the model Eq. (10) is linear in the unknown parameters β_e, $\beta_e C_{d_1} w_1 \sqrt{\frac{2}{\rho}}$, and $\beta_e C_{d_2} w_2 \sqrt{\frac{2}{\rho}}$ but again requires some filtering due to the unavailability of the time derivatives of P_1 and $\ln V_1$. Applying the filter $\frac{1}{s+\lambda}$ to Eq. (10), we create the parametric model

$$
\begin{aligned}
&P_1 - \frac{\lambda}{s+\lambda} P_1 = \beta_e \left(\frac{\lambda}{s+\lambda} [\ln V_1] - \ln V_1 \right) \\
&\quad + \beta_e C_{d1} w_1 \sqrt{\frac{2}{\rho}} \left(\frac{1}{s+\lambda} \left[x_{v1} \operatorname{sgn}(P_s - P_1) \frac{\sqrt{|P_s - P_1|}}{V_1} \right] \right) \\
&\quad - \beta_e C_{d2} w_2 \sqrt{\frac{2}{\rho}} \left(\frac{1}{s+\lambda} \left[x_{v2} \operatorname{sgn}(P_1 - P_r) \frac{\sqrt{|P_1 - P_r|}}{V_1} \right] \right)
\end{aligned} \tag{11}
$$

5 Identifier Design

The parameter identification problem is now separated into two three-dimensional identification problems, Eq. (9) from which we estimate B, D, and DD_b, and Eq. (11) from which we estimate β_e, $\beta_e C_{d_1} w_1 \sqrt{2/\rho}$, and $\beta_e C_{d_2} w_2 \sqrt{2/\rho}$.

Both problems involve vector parameterizations. First, we introduce the parameter vectors

$$
\theta_1 = \begin{bmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{13} \end{bmatrix} = \begin{bmatrix} B \\ D \\ DD_b \end{bmatrix} \tag{12}
$$

and

$$
\theta_2 = \begin{bmatrix} \theta_{21} \\ \theta_{22} \\ \theta_{23} \end{bmatrix} = \begin{bmatrix} \beta_e \\ \beta_e C_{d_1} w_1 \sqrt{\dfrac{2}{\rho}} \\ \beta_e C_{d_2} w_2 \sqrt{\dfrac{2}{\rho}} \end{bmatrix} \tag{13}
$$

The regressor vectors Φ_1 and Φ_2 are defined as

$$
\Phi_1 = \begin{bmatrix} \dfrac{s}{s+\lambda} x_p \\ \dfrac{1}{s+\lambda} \left[\dfrac{\left(\dot{x}_p - |\dot{x}_p|\right)\left(2\hat{D}_b + x_p^{2k}\right)}{2\left(\hat{D}_b + x_p^{2k}\right)^2} \right] \\ -\dfrac{1}{s+\lambda} \left[\dfrac{\dot{x}_p - |\dot{x}_p|}{2\left(\hat{D}_b + x_p^{2k}\right)^2} \right] \end{bmatrix} \tag{14}
$$

and

$$
\Phi_2 = \begin{bmatrix} \dfrac{\lambda}{s+\lambda} [\ln V_1] - \ln V_1 \\ \dfrac{1}{s+\lambda} \left[x_{v1} \operatorname{sgn}(P_s - P_1) \dfrac{\sqrt{|P_s - P_1|}}{V_1} \right] \\ -\dfrac{1}{s+\lambda} \left[x_{v2} \operatorname{sgn}(P_1 - P_r) \dfrac{\sqrt{|P_1 - P_r|}}{V_1} \right] \end{bmatrix} \tag{15}
$$

Combining the regressor and the parameter vectors, we get the quantity

$$
Y = \Phi^T \theta \tag{16}
$$

which for the two respective problems is defined as

$$
Y_1 = M_t \left(\frac{\lambda}{s+\lambda} \dot{x}_p - \dot{x}_p \right) + \frac{1}{s+\lambda} \left(A_{p_1} P_1 - A_{p_2} P_s \right) \tag{17}
$$

and

$$Y_2 = P_1 - \frac{\lambda}{s+\lambda} P_1 \tag{18}$$

To estimate the parameter vectors, we employ the *unnormalized least-squares with forgetting factor update law* (Ioannou and Sun, [16])

$$\dot{\hat{\theta}} = \Gamma\Phi\left(Y - \Phi^T\hat{\theta}\right) \tag{19}$$

$$^*\dot{\Gamma} = \beta\Gamma - \Gamma\Phi\Phi^T\Gamma \tag{20}$$

where the second equation is a Riccati equation for the gain matrix $\Gamma(t)$ with an initial condition $\Gamma(0)$ chosen as positive definite and symmetric and $\beta < 0$ is the forgetting factor. Clearly, for our two parametric models, we employ two identifiers

$$\dot{\hat{\theta}}_1 = \Gamma_1\Phi_1\left(Y_1 - \Phi_1^T\hat{\theta}_1\right), \quad \dot{\Gamma}_1 = \beta_1\Gamma_1 - \Gamma_1\Phi_1\Phi_1^T\Gamma_1 \tag{21}$$

and

$$\dot{\hat{\theta}}_2 = \Gamma_2\Phi_2\left(Y_2 - \Phi_2^T\hat{\theta}_2\right), \quad \dot{\Gamma}_2 = \beta_2\Gamma_2 - \Gamma_2\Phi_2\Phi_2^T\Gamma_2 \tag{22}$$

Looking at Fig. 3, the identifier has four main parts: the filter which provides the values for Φ and Y, the estimation error, the Riccati equation which determines the values for Γ, and the update law which provides the estimate $\hat{\theta}$.

We show in Sec. 6 that the update law guarantees that $\hat{\theta}(t)$ converges to the true value θ (in the absence of noise) if the regressor vector $\Phi(t)$ is "persistently exciting." The square-wave character of the input signals in the EHVS application helps ensure persistence of excitation.

6 Identifier Stability

With $\beta = 0$, Eq. (20) is referred to as the "pure" least-squares algorithm. In a pure least-squares algorithm, the Riccati Eq. (20) will typically tend to become singular as time advances. In an environment in which the unknown parameters are static this would be acceptable, however, with EHVS there is a possibility of slow changes in the true parameters. To keep $\Gamma(t)$ from becoming illconditioned the forgetting factor is employed, with $\beta < 0$.

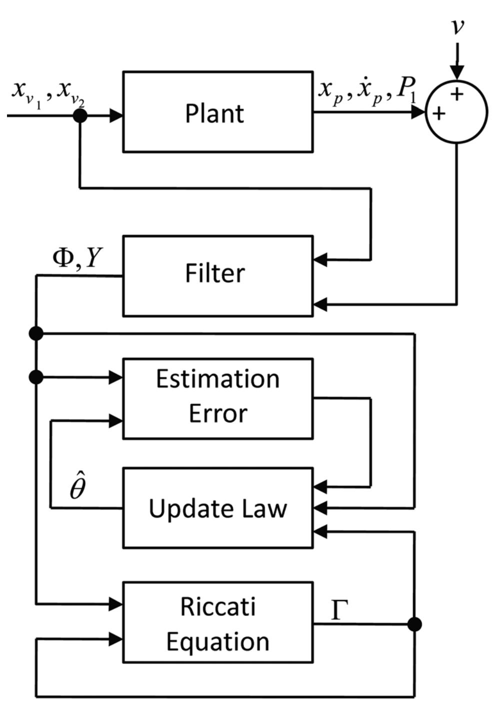

Fig. 3 Estimator block diagram

Table 2 Parameter values used in the simulation

	Parameter Values	
M_t	1	Mass
A_{p1}	20	Area
A_{p2}	10	Area
x_{v1}, x_{v2}	1	Length
V_{01}	5	Volume
P_s	250	Pressure
P_r	10	Pressure
C_{d1}	10	Dimensionless
C_{d2}	10	Dimensionless
w_1, w_2	5	Length
ρ	10	Density
B	50	Damping
β_e	250	Pressure
D	60	Dimensionless
D_b	0.01	Dimensionless
k	5	Dimensionless

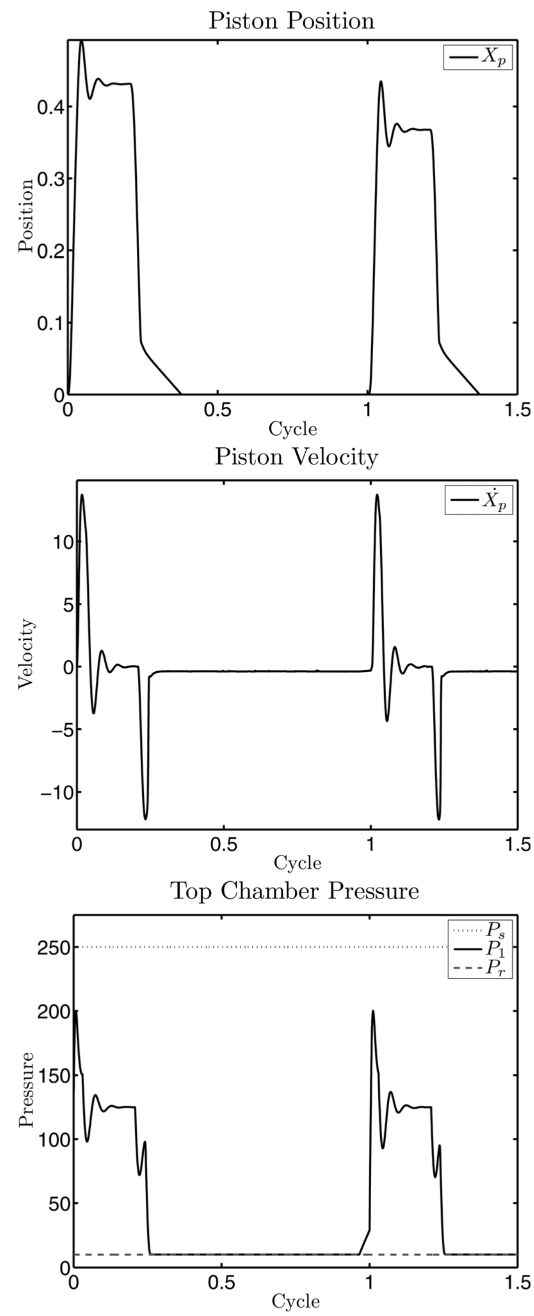

Fig. 4 Measured system states in transient

Having $\beta < 0$, the problem of $\Gamma(t)$ becoming arbitrarily small in a direction no longer exists. However, $\Gamma(t)$ now can grow without bound since $\dot{\Gamma}$ may satisfy $\dot{\Gamma} > 0$ since $\beta\Gamma < 0$ and $\Gamma\Phi\Phi^T\Gamma$ is only positive semidefinite. One solution to this issue is to modify the algorithm by placing an upper bound on Γ. However, such modifications are not necessary when $\Phi \in \mathcal{L}_\infty$ and Φ is persistently exciting (Ioannou and Sun, [16], p. 213). The persistence of excitation property of Φ guarantees that over an interval of time the integral of $-\Gamma\Phi\Phi^T\Gamma$ is a negative definite matrix that counteracts the positive definite effect of $\beta\Gamma$.

Proposition 1. *If $\Phi(t)$ is uniformly bounded and persistently exciting, i.e., the exist constants $\alpha_0 < 0$ and T_0 such that*

$$\frac{1}{T_0}\int_t^{t+T_0} \Phi(\tau)\Phi^T(\tau)d\tau \geq \alpha_0 I, \quad \forall t \geq 0 \tag{23}$$

then for all $\hat{\theta}(0) \in \mathbb{R}^m$, the following holds:

$$\left|\tilde{\theta}(t)\right|^2 \leq M\left|\tilde{\theta}(0)\right|^2 \mathrm{e}^{-\beta t} \tag{24}$$

where

$$\tilde{\theta} = \theta - \hat{\theta} \tag{25}$$

and

$$M = \frac{\dfrac{1}{\lambda_{\min}(\Gamma_0)} + \dfrac{\sup_{t\geq 0}\lambda_{\max}\left\{\Phi(t)\Phi(t)^T\right\}}{\beta}}{\min\left\{\alpha_0 T_0, \dfrac{1}{\lambda_{\max}(\Gamma_0)}\right\} e^{-\beta T_0}} \tag{26}$$

Please refer to Sec. 9 for a proof of Proposition 1

Remark 1. According to Proposition 1, exponential convergence occurs when $\beta < 0$. In the case of a pure least-squares algorithm, where $\beta = 0$, convergence of $\hat{\theta}(t)$ to θ is not guaranteed to be exponential even when Φ is PE. It has been shown (Ioannou and Sun, [16], p. 169) that

$$\Gamma(t) \leq \frac{1}{(t - T_0)\alpha_0} I \tag{27}$$

$$\left|\tilde{\theta}(t)\right| \leq \frac{\Gamma^{-1}(0)}{(t - T_0)\alpha_0}\left|\tilde{\theta}(0)\right| \tag{28}$$

for all $t < T_0$. Hence, only a polynomial decay rate is guaranteed for the pure least-squares algorithm, even in the presence of persistency of excitation. This is in contrast to the gradient algorithm, with its fixed and positive definite adaptation gain matrix Γ, where PE yields exponential convergence.

7 Simulations

We now present the simulations done of the plant Eqs. (1)–(5). In this simulation, initial conditions were set to be $x_p(0) = 0$, $\dot{x}_p(0) = 0$, and $P_1(0) = p_s/2$. The parameter values used in the simulation are given in Table 2. These values were chosen such that the simulations produce the same qualitative behavior as expected of the actual system. The true system values are proprietary.

Looking at the system behavior (Fig. 4), the effect of variable damping is evident. At the end of a closing event, the velocity slows considerably. This creates a soft landing for the valve.

After verifying the stable operation of the system, we now move to verify the condition (23) for exponential convergence of the identifier. We numerically check if the regressor Φ of each identifier is PE. Taking Eq. (23) and shifting the bounds of integration results in

$$\frac{1}{T_0}\int_{t-T_0}^{t} \Phi(\tau)\Phi^T(\tau)d\tau \geq \alpha_0 I \tag{29}$$

which is satisfied if the following sufficient condition is satisfied

$$\lambda_{\min}\left\{\int_{t-T}^{t} \Phi(\tau)\Phi^T(\tau)d\tau\right\} > 0 \tag{30}$$

Looking at Fig. 5, both regressors satisfy the condition of Eq. (30). Given that PE holds, the identifiers exponentially converge to the true parameters in the absence of noise.

After verifying the convergence of the identifier, we employ the update law Eqs. (19) and (20) where the regressor vectors Φ are defined by Eqs. (14) and (15) and the quantities Y are defined by Eqs. (17) and (18).

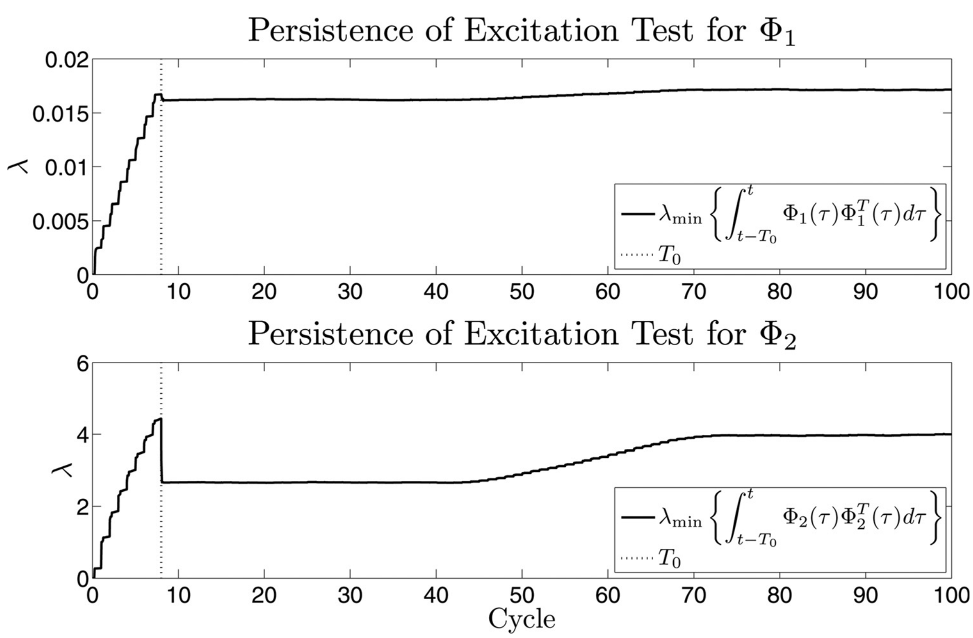

Fig. 5 Results of the test to determine the persistence of excitation of the regressors

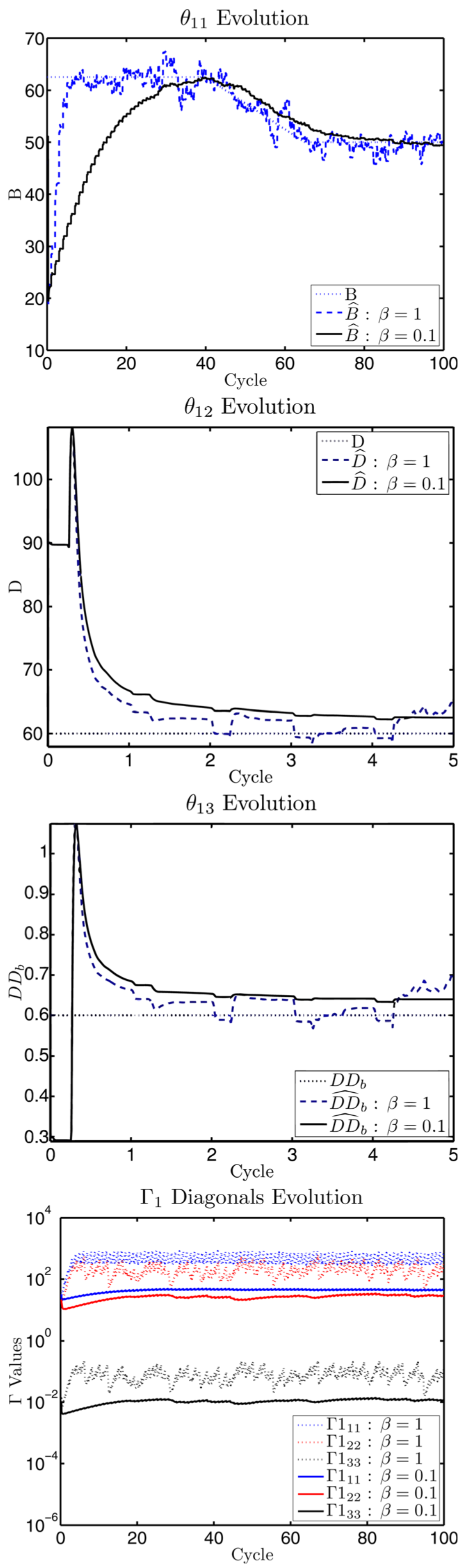

Fig. 6 Parameters estimates $\hat{\theta}_1$ and the associated Γ_1 diagonal terms

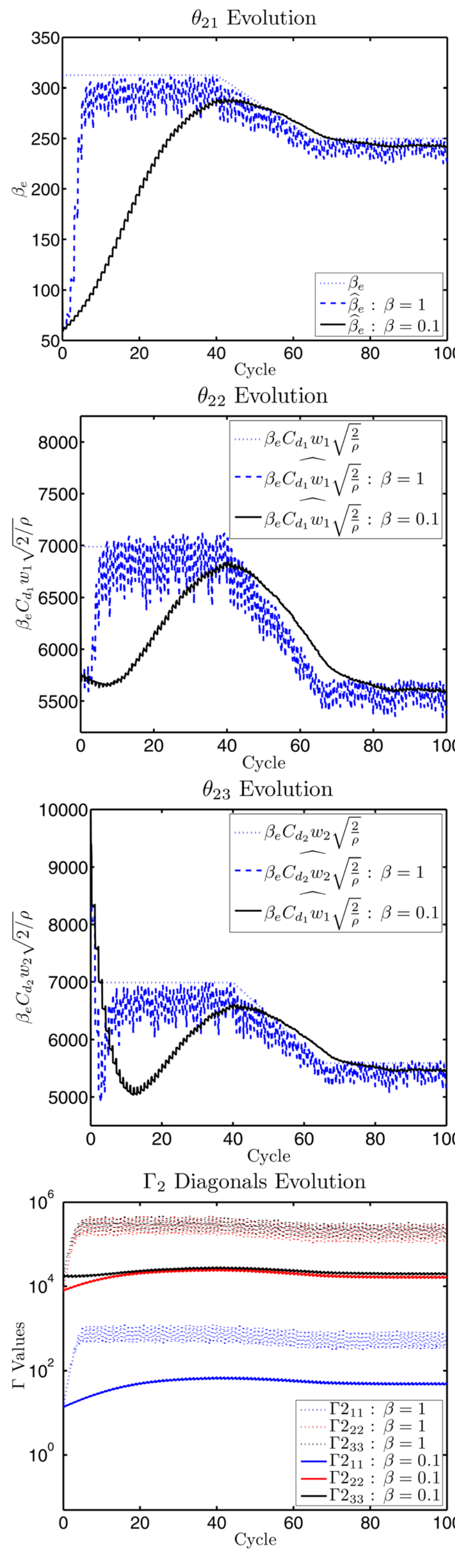

Fig. 7 Parameters estimates $\hat{\theta}_2$ and the associated Γ_2 diagonal terms

The initial value for the estimate $\hat{D}(0)$ must be chosen with care. In the parameter vector (12), there is no explicit estimate of $\hat{D}_b$, only of $\hat{DD}_b$. Since the regressor vector Φ_1 relies on the estimate $\hat{D}_b$, we take

$$\hat{D}_b = \frac{\hat{DD}_b}{\hat{D}}$$

This implies that as $\hat{D}$ approaches 0 the value $\hat{D}_b$ will approach infinity. Therefore, if $\hat{D}(0)$ is chosen too small there can be a failure in the identifiers due to division by zero. The same care is not necessary for the other estimates.

In the simulation, all true parameters except D and D_b are allowed to vary with time. After forty cycles there is a gradual variation of 20% in the true parameters. In addition to the parameter variation, sensor noise is injected into the state measurements, which is on the order of 1% of the maximum of the state measurement. Despite these violations of the idealized conditions of the theory (noise and nonconstant parameters), most estimates converge to within 5% of the true value within twenty cycles. However, it is notable that both θ_{12} (i.e., $\hat{D}$) and θ_{13} (i.e., $\hat{D}D_b$) converge within five cycles. As the true parameters change, the estimates follow the variation and stay within the 5%. These results are shown in Figs. 6 and 7.

Looking at Figs. 6 and 7, we see the trade-off in the choice of the forgetting factor β. A larger β increases the speed of convergence at the cost of an increased sensitivity to noise. Therefore, if it is known beforehand how quickly the parameters vary, β could be tuned appropriately.

In addition to the trade-off in the parameter estimates, Γ is also affected by the choice of β. A larger β will reduce the time it takes Γ to settle into stable oscillations, but also a larger β will also increase the mean value of the stable oscillations. One phenomenon that we do not show here is the evolution of the off-diagonal terms of the gain matrices, which grow symmetrically and self-adjust to the excitation levels in the different channels of the regressor vectors similarly to the diagonals of the matrices.

8 Conclusions

In this paper, we developed a successful adaptive on-line parameter estimator that operates in the presence of noise for the EHVS. We first developed a model of the system and included an approximation of the variable damping. Then we proceeded to create the parametric model through low-pass filtering of the states. Lastly we used a least-squares estimator with forgetting factor to identify the unknown parameters, even in the presence of measurement noise and slowly varying parameters and despite the nonlinear parameterization in the onset parameter of variable damping. This estimator provides a basis for a future model-based control design.

As noted by Berghuis et al. [17], while least-squares adaptation is fast relative to a gradient based method, the least-squares algorithm is computational complex. However in a gradient based methods, gain tuning of large sets of adjustable parameters becomes a matter of heuristics, since the relative strength of different regressor channels may not be a priori known, which is especially the case when an identifier is used in adaptive control. If the least-squares method is computationally too complex to run in real time, the gain matrix Γ obtained as a long-time limit in simulations with the least-squares algorithm can provide insight into the choice of an appropriate constant gain matrix in a gradient based method.

Appendix

This section contains a detailed proof of Proposition 1.

Proof. Taking the inverse of Eq. (20) we have

$$\dot{\Gamma}^{-1} = -\beta\Gamma^{-1} + \Phi\Phi^T \tag{A1}$$

with initial condition

$$\Gamma_0^{-1} = \Gamma_0^{-T} \tag{A2}$$

which yields

$$\Gamma^{-1}(t) = \mathrm{e}^{-\beta t}\Gamma_0^{-1} + \int_0^t \mathrm{e}^{-\beta(t-\tau)}\Phi(\tau)\Phi^T(\tau)d\tau\,.$$

Using the condition that $\Phi(t)$ is persistently exciting we can show that for all $t \geq T_0$

$$\begin{aligned}\Gamma^{-1}(t) &\geq \int_0^t \mathrm{e}^{-\beta(t-\tau)}\Phi(\tau)\Phi^T(\tau)d\tau \\ &= \int_{t-T_0}^t \mathrm{e}^{-\beta(t-\tau)}\Phi(\tau)\Phi^T(\tau)d\tau \\ &\quad + \int_0^{t-T_0} \mathrm{e}^{-\beta(t-\tau)}\Phi(\tau)\Phi^T(\tau)d\tau \\ &\geq \mathrm{e}^{-\beta T_0}\alpha_0 T_0 I\end{aligned} \tag{A3}$$

For $t \leq T_0$, we have

$$\begin{aligned}\Gamma^{-1}(t) &\geq \mathrm{e}^{-\beta t}\Gamma_0^{-1} \geq \mathrm{e}^{-\beta T_0}\Gamma_0^{-1} \\ &\geq \frac{1}{\lambda_{\max}(\Gamma_0)}\mathrm{e}^{-\beta T_0} I\end{aligned} \tag{A4}$$

Conditions Eqs. (A3) and (A4) imply that

$$\Gamma^{-1}(t) \geq \gamma_1 I \tag{A5}$$

for all $t \geq 0$, with

$$\gamma_1 = \min\left\{\alpha_0 \mathrm{T}_0, \frac{1}{\lambda_{\max}(\Gamma_0)}\right\}\mathrm{e}^{-\beta \mathrm{T}_0} \tag{A6}$$

For the upper limit we can use the boundedness of Φ and establish

$$\begin{aligned}\Gamma^{-1}(t) &\leq \Gamma_0^{-1} + \sup_{t\geq 0}\lambda_{\max}\left\{\Phi(t)\Phi(t)^T\right\}\int_0^t \mathrm{e}^{-\beta(t-\tau)}d\tau I \\ &\leq \gamma_2 I\end{aligned} \tag{A7}$$

where

$$\gamma_2 = \frac{1}{\lambda_{\min}(\Gamma_0)} + \frac{\sup_{t\geq 0}\lambda_{\max}\left\{\Phi(t)\Phi(t)^T\right\}}{\beta} \tag{A8}$$

Combining (A5) and (A7) we get

$$\gamma_1 I \leq \Gamma^{-1}(t) \leq \gamma_2 I \tag{A9}$$

with $\gamma_1 > 0$, $\gamma_2 > 0$ and therefore,

$$\gamma_2^{-1} I \leq \Gamma(t) \leq \gamma_1^{-1} I$$

This guarantees $\Gamma, \Gamma^-1 \in \mathcal{L}_\infty$.

With $\tilde{\theta} = \theta - \hat{\theta}$ and $Y = \Phi^T\theta$ we get

$$Y - \Phi^T\hat{\theta} = \Phi^T\theta - \Phi^T\hat{\theta} = \Phi^T\tilde{\theta} \tag{A10}$$

which, after substitution into Eq. (19), yields

$$\dot{\tilde{\theta}} = -\Gamma\Phi\Phi^T\tilde{\theta} \tag{A11}$$

Now proceed with the following Lyapunov function

$$V = \tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{A12}$$

Taking the derivative of Eq. (A12) in time yields

$$\dot{V} = 2\dot{\tilde{\theta}}^T\Gamma^{-1}\tilde{\theta} + \tilde{\theta}^T\dot{\Gamma}^{-1}\tilde{\theta}$$

Substituting Eqs. (A1) and (A11) into the previous equation results in

$$\begin{aligned}\dot{V} &= -2\tilde{\theta}^T\Phi\Phi^T\Gamma\Gamma^{-1}\tilde{\theta} + \tilde{\theta}^T\left(-\beta\Gamma^{-1} + \Phi\Phi^T\right)\tilde{\theta} \\ &= -\tilde{\theta}^T\Phi\Phi^T\tilde{\theta} - \tilde{\theta}^T\beta\Gamma^{-1}\tilde{\theta} \leq 0\end{aligned} \tag{A13}$$

Since $-\tilde{\theta}^T\Phi\Phi^T\tilde{\theta}$ is negative definite and β is a positive scalar value we bound (A13) by

$$\dot{V} \leq -\beta V \tag{A14}$$

which could be rewritten as

$$V(t) \leq V_0\mathrm{e}^{-\beta t}, \quad \forall t \geq 0 \tag{A15}$$

Now we take the original Lyapunov equation (A12) and combine it with the results of (A9) to bound $V(t)$ as follows

$$\gamma_1 \left|\tilde{\theta}(t)\right|^2 \leq V(t) \leq \gamma_2 \left|\tilde{\theta}(t)\right|^2 \tag{A16}$$

Knowing that the upper bound of $V(t)$ can be described with (A15), from (A16) we get

$$\left|\tilde{\theta}(t)\right|^2 \leq \frac{1}{\gamma_1} V(t) \leq \frac{1}{\gamma_1} V_0 \mathrm{e}^{-\beta t} \tag{A17}$$

Finally, substituting $\gamma_2 \left|\tilde{\theta}(0)\right|^2$ as the maximum value of V_0, we get the result

$$\left|\tilde{\theta}(t)\right|^2 \leq \frac{\gamma_2}{\gamma_1} \left|\tilde{\theta}(0)\right|^2 \mathrm{e}^{-\beta t}$$

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