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Control of Wing Rock Motion Using Adaptive Feedback Linearization

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The theory of adaptive control of feedback linearizable systems is applied to designing the control of wing rock motion with the extension of the technique to include tracking. The adaptation law is designed to adjust the aerodynamic parameters in the model. Precise tracking and maximum performance can be achieved if sufficient rolling moment derivative because of lateral control components are available. Case studies and simulation are carried out to illustrate the results.

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Introduction

HIGH-PERFORMANCE aircraft have mission requirements for operating in high angle of attack and sideslip. Unsteady aerodynamic effects at high angle of attack generate wing rock phenomenon on aircraft configurations incorporating slender forebodies. Delta wings with leading-edge sweeps greater than 76 deg are known to exhibit wing rock.[1,2] Typically the wing rock is one type of lateral-directional instability for airplanes flying at high angle of attack and involves mainly the roll degree of freedom. Studies in Ref. 3 have concluded that wing rock is triggered by flow asymmetries developed by negative roll damping and sustained by nonlinear aerodynamic roll damping. Considerable research has been conducted on the motion of slender delta wings to help understand the fundamental mechanisms causing wing rock. References 4 and 5 have an excellent bibliography on the experimental, theoretical, and computational aspects of aerodynamics and also on flight dynamics of wing rock motion. There are three possible approaches to suppress or prevent wing rock[3]:

1) The first approach involves attaining the wing rock free capability through a detailed aerodynamic reshaping of the basic airframe configuration.

2) The second approach introduces maneuver limiting by adopting an alpha limiter, however, this approach degrades maneuverability.

3) The third approach employs stability augmented systems (SAS) or automatic flight control systems. These have become the most effective methods for attaining strong resistance to wing rock without degrading maneuverability.

Research on the suppression of wing rock by the third approach has not been extensive. Chambers et al.[6] utilized lateral-directional control surfaces for the purpose of eliminating adverse yaw at high angles of attack and sideslip. In another case in Ref. 7 an SAS was designed to incorporate a suggested reduction of air departure/spin resistance. In both studies the main tools involved applied aerodynamic control techniques. Luo and Lan[8] projected an optimal control in conjunction with Beecham–Titchener[9] averaging technique. In their derivation of the controller,[8] it is assumed that the aerodynamic parameters are known. In a practical situation, the reliability of such an assumption is limited. Indeed, the uncertainties in the model, i.e., uncertainties in the aerodynamic moment parameters, will create an error in the computation of the control input. Fernand and Downing[10] considered discrete-time control of wing rock. They claim that stability and performance can be achieved and improved for sampled data systems by applying a discrete sliding mode technique on a discretized plant model. Singh et al.[11] employed an adaptive and neural control of the wing rock motion. In this approach, however, the control cancels the nonlinearity of the main equation (the roll moment equation) only in a very limited sense, i.e., there exists no additional dynamics and/or an actuator control surface equation.

Flight at high angle of attack, the exhibition of limit cycle behavior of the motion variables, and the creation of large amplitudes for the motion variables lead to severe requirements for the flight control system that cannot be met using conventional procedures. Nonlinear properties are essential features of the aircraft dynamics in these regimes. Techniques are needed that 1) are computationally feasible, 2) retain the nonlinearities inherent within the vehicle dynamics, 3) accommodate system uncertainties when the changes in the nominal model are large and severe, and 4) provide physical and mathematical understanding of the important flying qualities.

Nonlinear control techniques using particular linearizing transformations are currently receiving a great deal of attention among researchers. Linearizing transformations are based on the assumption that some nonlinear systems are inherently linear, i.e., they only appear to be nonlinear because they are being viewed in an inappropriate coordinate system. Nonlinear transformations are sought that take the nonlinear system into a linear representation consisting of a chain of integrators with nonlinearities matched by the input. In an ideal situation, if such a transformation does exist, control laws (nonlinear model followers) can be designed such that the states of the transformed system track the outputs of a transformed reference model.

In this paper we employ recent theoretical advances in the differential geometric formulation of nonlinear control theory in conjunction with adaptive control. We use the backstepping design with tuning functions developed in Ref. 12 for the design of global regulation and in addition extend the methodology to tracking as well. The class of systems to which this methodology is applicable includes systems meeting the definition of a parametric strict-feedback system. It is characterized via coordinate free geometry conditions that do not constrain the growth of the nonlinearities.[13] The backstepping design substantially enlarges the class of nonlinear systems with unknown parameters for which global stabilization can be achieved. Using the cited adaptive controllers for the feedback linearizable system technique inspired by the differential geometric approach, we developed a practical strategy to suppress wing rock motion. In particular, roll angle and roll rate can track desired, prespecified trajectories with good accuracy in the transient stage and achieve steady state in any desired time period. The performance, however, in particular, precise tracking in the transient stage, requires the availability of sufficient rolling moment from the aerodynamic lateral control, namely, aileron deflection angle. This design also accommodates aerodynamic parameter estimation and updating online while the control system is in operation.

The outline of this paper is as follows. We study the wing rock model development. We consider the general case of nonlinear

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control development based on the differential geometric formulation for input-state feedback linearization and adaptive control of feedback linearizable systems. We design an aerodynamic parameter estimator in conjunction with control law development. We present specific case studies to illustrate the pragmatic application of the techniques.

Wing Rock Model Development

We exploit subsequently in Eq. (7) the models developed in Refs. 14 and 15 for wing rock. Each of these references contains specific advantages: Ref. 14 specifically identified certain of the aerodynamic parameters with wind-tunnel data, whereas Ref. 15 added two additional terms to the rolling moment equation. Our Eq. (7) combines these and includes an additional factor identified with aerodynamic control effectiveness. Furthermore, we supplement Eq. (7) with Eq. (8) characterizing the actuator control surface dynamics.

In Ref. 14, the model is based on an experimental wind-tunnel wing rock developed by researchers of NASA Langley Research Center. In these tests the physical scaled model of 80-deg delta wings were sting mounted on an apparatus that allows the model to rotate freely about its body-fixed roll axis with no angular limitation. The resulting system is a single degree of freedom. The following equations hold:

$$\dot{\phi} = p \tag{1}$$

$$\dot{p} = (\bar{q}Sb/I_x)\left[C_{l_\beta}(\alpha)\beta \sin\alpha + \bar{C}_{l_p}(\alpha,\beta)(pb/2V)\right] \tag{2}$$

Where α is angle of attack in degrees, ϕ the roll angle in radians, and p the roll rate in radians per second. The constants $\bar{q}$, S, b, I_x and V are the dynamic pressure, wing reference area, wing span, roll moment of inertia, and freestream air speed, respectively. The coefficient C_{l_β} is the rolling moment stability derivative because of sideslip β. The coefficient $\bar{C}_{l_p}$ is the rolling moment derivative because of roll rate p and sideslip rate $\dot{\beta}$ as defined in Ref. 14.

Let us define

$$\theta_2 = (\bar{q}Sb/I_x)(\sin\alpha)C_{l_\beta} \tag{3}$$

$$\theta_3 = (\bar{q}Sb/I_x)(b/2V)\bar{C}_{lp} \tag{4}$$

$$\theta_4 = (\bar{q}Sb/I_x)(b/2V)(-3.82) \tag{5}$$

where $\phi = 2\beta$. Reference 15 introduces two additional terms, θ_1 and $\theta_5|\dot{\phi}|\dot{\phi}$.

There are no control surfaces in this wind-tunnel model. Therefore, we add an additional aerodynamic parameter θ_6 that defines the influence of the aerodynamic control surface, namely, the rolling moment derivative because of the aileron coefficient $C_{l_{\delta_A}}$. This is given by

$$\theta_6 = (\bar{q}Sb/I_x)C_{l_{\delta_A}}(\alpha,\beta) \tag{6}$$

$C_{l_{\delta_A}}$ is considered in our development to be a constant developed from experimental or perhaps computational aerodynamics for a particular configuration. It could also be generated from thrust vectoring in some proposed configurations. Later in this paper we conclude that good performance can be acheived if sufficient $C_{l_{\delta_A}}$ is available. In particular, $C_{l_{\delta_A}}$ for a high-performance vehicle must be large enough to accommodate large and rapid movement of vehicles at extreme flight conditions. The larger $C_{l_{\delta_A}}$ is, the better we are able to control and suppress wing rock.[16] Associated with this aerodynamic coefficient is the control surface angle δ_A. It is possible that there exists a coupling between the aerodynamic control surface and the motion variables, the roll angle and the angle of attack. Under these conditions, the numerical data from the wind tunnel in the presence of the control surface might be different. However, any such difference is not expected to be significant in the typical range where wing rock occurs.

These additions now transform Eq. (2) into the following equation:

$$\dot{p} = \theta_1 + \theta_2\phi + \theta_3 p + \theta_4|\phi|p + \theta_5|\dot{\phi}|\dot{\phi} + \theta_6\delta_A \tag{7}$$

Reference 14 provides the following wind-tunnel data at angle of attack of $\alpha = 30$ deg:

$$\theta_2 = -26.6667\,\mathrm{s}^{-1}, \qquad \theta_3 = 0.76485\,\mathrm{s}^{-1}$$
$$\theta_4 = -2.9225\,\text{rad-s}^{-1}$$

We set θ_1 and θ_5 equal to 5 and -2.5, respectively.

Finally, an aileron control surface with first-order actuator dynamics is modeled as

$$\dot{\delta}_A = -(1/\tau)\delta_A + (1/\tau)\delta_{A_{\mathrm{COM}}} \tag{8}$$

the values for τ the time constant of the actuator and θ_6 are given later in case studies 1 and 2.

Adaptive Controllers for Feedback Linearizable Systems

We utilize the method developed by Krstic et al.[12] for adaptive regulation and extend its techniques to tracking situations as well.

Consider a single-input deterministic nonlinear system defined by

$$\dot{x} = f(x,\theta) + g(x,\theta)u, \qquad x \in R^n, \qquad u \in R^1 \tag{9}$$

We consider this equation as a system that is linear in the unknown parameters:

$$\dot{x} = f_0(x) + \sum_{i-1}^{p}\theta_i f_i(x) + \left[g_0(x) + \sum_{i-1}^{p}\theta_i g_i(x)\right]u \tag{10}$$

where $\boldsymbol{\theta} = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_p]^T$ is the vector of unknown constant parameters and $\boldsymbol{f}_i, \boldsymbol{g}_i$, $0 \le i \le p$, are differentiable vector fields in a neighborhood of the origin $x = 0$, with $\boldsymbol{f}_i(0) = 0$, $1 \le i \le p$, $\boldsymbol{g}(0) \ne 0$.

To use the theory of adaptive controllers for feedback linearizable systems, the system (10) is required to be transformed into the parametric-strict feedback form

$$\begin{aligned} \dot{x}_i &= k_i x_{i+1} + \phi_i(x_1, x_2, \ldots, x_i)^T\theta, \qquad 1 \le i \le n-1 \\ \dot{x}_n &= k_x\phi_0(x) + \phi_n(x)^T\theta + k_u\beta_0(x)u \end{aligned} \tag{11}$$

With ϕ_0, β_0, and the components of ϕ_i, $1 \le i \le n$, are differentiable nonlinear functions in R^n, $\beta_0(x) \ne 0$ for all $x \in R^n$, and k_i, $1 \le i \le n-1$, k_x, k_u are nonzero constants.

The two necessary and sufficient conditions for the existence of such a transformation are[13] 1) the feedback linearization condition: the distribution

$$S^j = \operatorname{span}\left\{g_0, ad_{f_0}g_0, \ldots, ad^j_{f_0}g_0\right\}, \qquad 1 \le j \le n-1 \tag{12}$$

is involutive and of constant rank $j+1$, and 2) the parametric-strict-feedback condition

$$[Y, f_i] \in S^j \qquad \text{for all} \quad Y \in S^j, \qquad 0 \le j \le n-2, \qquad 1 \le i \le p \tag{13}$$

The objectives of the control are to force the output of the system (9) to track a definite, known, smooth, and bounded reference signal, both transiently and in steady state, while keeping all the closed-loop signals bounded, for any unknown values in the aerodynamic parameters.

The adaptive regulation and tracking are achieved using a recursive design procedure known as backstepping with tuning functions. The procedure is such that at each step the subsystem is stabilized with respect to a Lyapunov function by designing an appropriate tuning function. At the final step one develops the feedback control and the update law for the parameter estimates.

The results of this procedure are attained in terms of a regressor function ω_i, a tuning function τ_i, and a stabilizing function α_i at each ith step and, in the final step an adaptive control law and a parameter update law. Their respective equations are as follows.

Coordinate change:

$$z_i = x_i - \frac{1}{k_1 \cdots k_{i-1}} x_{di} - \frac{1}{k_{i-1}} \alpha_{i-1}, \qquad i \in 1, 2, \ldots, n \quad (14)$$

Regressors:

$$\omega_i(\bar{x}_i, \hat{\theta}, \bar{x}_{di}) = \phi_i - \left[\sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j}\phi_j\right)\right] \bigg/ k_{i-1} \quad (15)$$

Tuning functions:

$$\tau_i(\bar{x}_i, \hat{\theta}, \bar{x}_{di}) = \tau_{i-1} + \omega_i z_i \quad (16)$$

Stabilizing functions:

$$\begin{aligned}\alpha_i(\bar{x}_i, \hat{\theta}, \bar{x}_{di}) &= -k_{i-1} z_{i-1} - c_i z_i \\ &+ \sum_{j=1}^{i-1}\left(\frac{\partial \alpha_{i-1}}{\partial x_j} k_j x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial x_{dj}} x_{d(j+1)}\right) \bigg/ k_{i-1} \\ &+ \left(\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i\right) \bigg/ k_{i-1} - \omega_i^T \dot{\hat{\theta}} \\ &+ \sum_{j=2}^{i-1}\left(\frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma \omega_i z_j\right) \bigg/ k_{j-1} \end{aligned} \quad (17)$$

Adaptive control law:

$$u = (1/k_u \beta_0)[\alpha_n + (1/k_{n-1})\dot{x}_{dn} - k_x \phi_0] \quad (18)$$

Parameter update law:

$$\dot{\hat{\theta}} = \Gamma \tau_n \quad (19)$$

where Γ is the adaptive gain matrix and

$$\bar{x}_i = (x_1, \ldots, x_i), \qquad \bar{x}_{di} = (x_{d1}, \ldots, x_{di})$$

$$z_0 = \tau_0 = \alpha_0 = 0, \qquad c_i > 0$$

The c_i are available design parameters, chosen on the basis of tradeoffs. On the one hand, the choice of large values yields more accurate tracking of state variables during transients but, consequently, the control power will be undesirably large. On the other hand, smaller values of c_i will result in lesser performance, in particular, in the transient stage. The control power will as a result be smaller. Therefore, the tradeoff must be such that the performance of the state variables (in particular, in transient stages) is satisfactory whereas the amount of control power should not be prohibitive. The derivation of these results and the norm of the parameter vector error are given in the Appendix.

Adaptive Control Development for Wing Rock Motion

We shall now focus on the design of the compensator using the methods of the preceding section to attain stabilization, tracking, and adaptation of the aerodynamic parameters for the wing rock model (6–8).

Let

$$\frac{\phi(s)_{\mathrm{DES}}}{\phi(s)_{\mathrm{COM}}} = \frac{\sigma \omega_n^2}{\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)(s + \sigma)} \quad (20)$$

be the transfer function for the specified trajectory for the desired roll angle, where ξ, ω_n, and σ represent the desired damping ratio, the desired natural frequency, and a real pole in the far left-hand s plane, respectively. In state–space form, Eq. (20) becomes

$$\dot{x}_d = a_d x_d + b_d u_d, \qquad x_d \in R^3, \qquad u_d \in R^1 \quad (21)$$

where

$$a_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\omega_n^2 & -\left(\omega_n^2 + 2\xi\omega_n\sigma\right) & -(\sigma + 2\xi\omega_n) \end{bmatrix}$$

and

$$b_d = \begin{bmatrix} 0 \\ 0 \\ \omega_n^2 \sigma \end{bmatrix}$$

The parametric strict feedback form of the wing rock model (6–8) by inspection is

$$\dot{x}_1 = k_1 x_2 + \phi_1^T(x_1)\theta \quad (22)$$

$$\dot{x}_2 = k_2 x_3 + \phi_2^T(x_1 x_2)\theta \quad (23)$$

$$\dot{x}_3 = k_x \phi_0(x) + \phi_3^T(x)\theta + k_u \beta_0 u \quad (24)$$

where

$$\phi_1(x) = 0, \qquad \phi_2^T(x, x_2) = [1 \quad x_1 \quad x_2 \quad |x_1|x_2 \quad |x_2|x_2]$$

$$\phi_3(x) = 0, \qquad \phi_0(x) = x_3$$

$$k_1 = 1, \qquad k_2 = \theta_6, \qquad k_x = -(1/\tau)$$

$$k_u = 1/\tau, \qquad \beta_0 = 1$$

$$\theta = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad \theta_5]^T$$

$$x_1 = \phi, \qquad x_2 = p, \qquad x_3 = \delta_A, \qquad \text{and} \qquad u = \delta_{A\mathrm{COM}}$$

We will illustrate the algorithm developed in the preceding section, namely, the design formulas Eqs. (14–19), by the following case studies. Together with suppressing the undesirable wing rock motion a prespecified trajectory is to be followed. The computational results in these case studies are obtained using SIMULINK[17] including the integration of Eqs. (6–8), (19), and (22–24) (with the forcing function u_d set equal to zero) by the Gear method.[18]

Case Study 1

At first we describe the nature of the control aspects of an advanced research vehicle. The aircraft is a high-angle-of-attack research vehicle flying at a selected altitude and required to bank 30 deg in about 1 s (Ref. 19). The airplane has a slender forebody and is known to exhibit wing rock at high angle of attack α. Therefore, Eqs. (1), (7), and (8) will be a proper model for this vehicle at a flight condition at which it exhibits wing rock.

For our purpose of controlling the wing rock by lateral control, we assume the aircraft is equipped with certain effectors: the primary aspect of the control would consist of slaved aileron surfaces and two differentially deflected stabilizers, and an auxiliary control could be roll thrust vectoring since the vehicle is a twin engine aircraft. It is the purpose of these effectors to attain a value of 0.75 for the rolling moment derivative with respect the lateral control, i.e.,

$$\theta_6 = k_2 = (qSb/I_x)C_{l_{\delta_{\mathrm{LAT}}}} = 0.75$$

Without the newly designed adaptive control the dynamic characteristics for the roll angle are indicated by Fig. 1. Limit cycle behavior is evident. We now design the control system for this airplane at this flight condition in accordance with the method described in the preceding section. With the introduction of the adaptive control the improvement in the roll angle and roll rate is shown in Figs. 2 and 3, respectively. The control system is initiated at $\phi(0) = 30$ deg, and subsequently the wing rock is suppressed within 1.0 s, settling time with very accurate tracking in the transient stage following the prespecified response properties. The adaptation time history for the adjustment of the aerodynamic parameters is shown in Figs. 4–8. Sensitivity analysis of the aerodynamic parameters as presented in Ref. 8 reveals that the nonlinear aerodynamic effect is derived mainly from the aerodynamic parameters θ_4 and θ_5, whereas the other aerodynamic parameters θ_1, θ_2, and θ_3 produce a lesser effect that is characterized by linear behavior. Under these conditions we have set the adaptation gain matrix Γ of Eq. (19) at $10^{-3} I_{5\times 5}$, the design constants c_1, c_2, and c_3 of Eq. (17) are 6, 7, and 8, respectively, and the initial conditions of the aerodynamic parameters $\hat{\theta}_4$ and $\hat{\theta}_5$ account for 35% of the uncertainties in these parameters.

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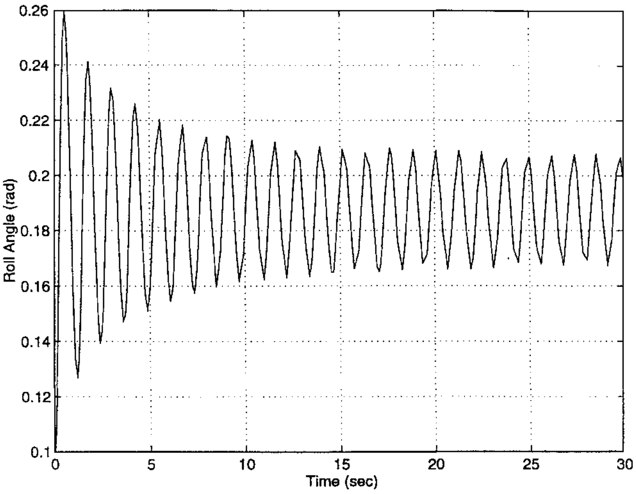

Fig. 1 Dynamic characteristics of wing rock based on numerical integration of the original equation for roll angle for case 1.

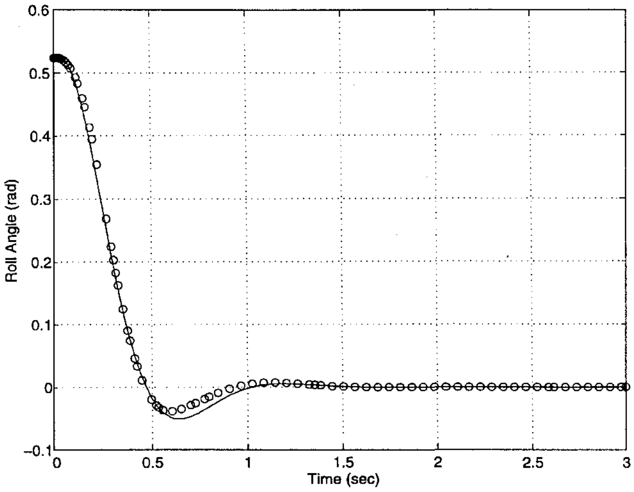

Fig. 2 Prespecified trajectory for roll angle (solid) and the roll angle response (circle) for case 1.

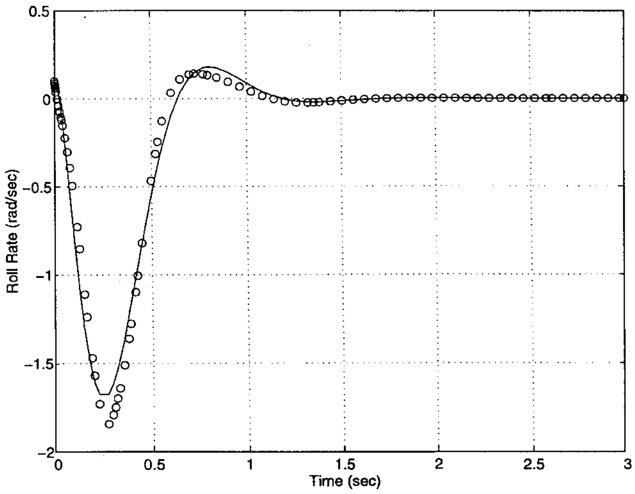

Fig. 3 Prespecified trajectory for roll rate (solid) and the roll rate response (circle) for case 1.

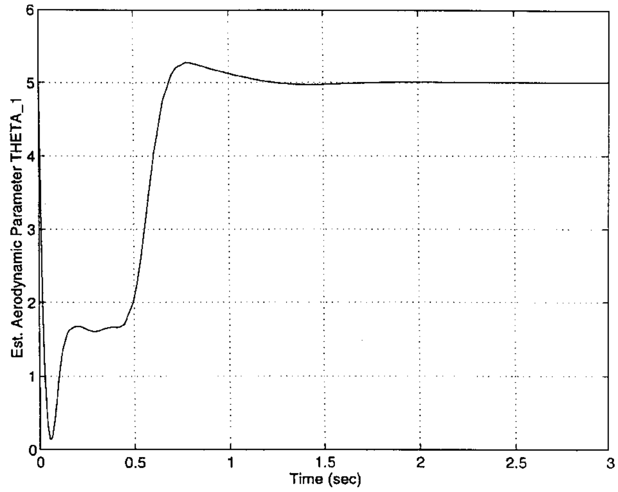

Fig. 4 Estimated aerodynamic parameter $\hat{\theta}_1$ for case 1.

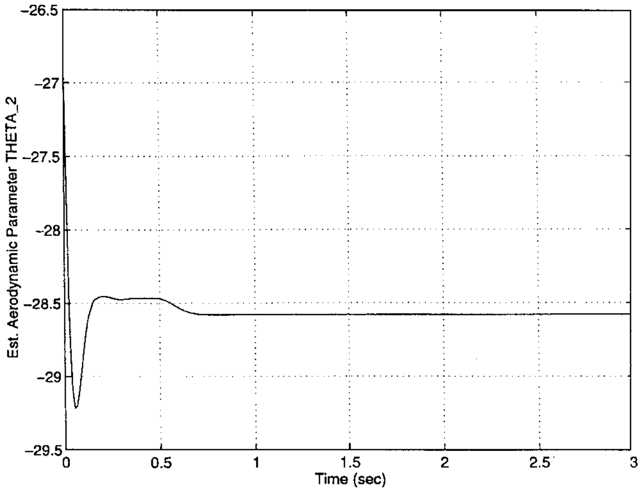

Fig. 5 Estimated aerodynamic parameter $\hat{\theta}_2$ for case 1.

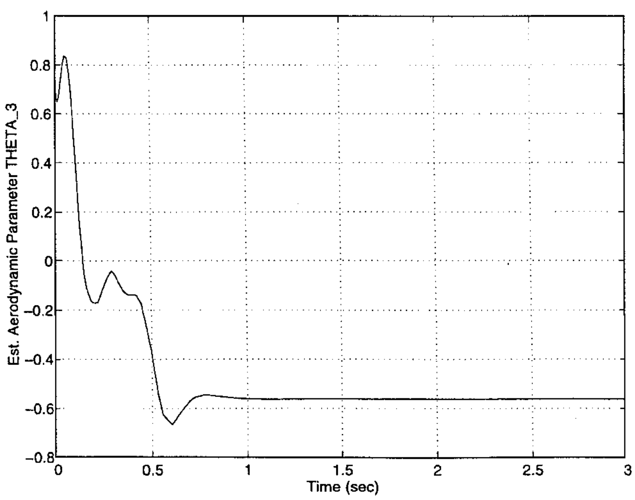

Fig. 6 Estimated aerodynamic parameter $\hat{\theta}_3$ for case 1.

Case Study 2

We consider an aircraft similar to an AFTI/F-16. The Advanced Fighter Technology Integration (AFTI) vehicle is a highly modified F-16 aircraft. It has been changed to allow flight demonstration of the benefits possible from the integration of advanced technology features. We consider the aircraft to have a slender delta wing such that its behavior is modeled by Eqs. (1), (7), and (8). We calculate the aerodynamic parameters θ_1–θ_5 based on the airplane parameters for the AFTI/F-16. For 0.6 Mach and 39,000 flight altitude, these airplane parameters are as follows: $\bar{q}$ (dynamic pressure) = 158.81 lb/ft^2, S (wing reference area) = 300.0 ft^2, c (wing mean aerodynamic cord) = 11.32 ft, c (wing span) = 30.0 ft, V_T (trim velocity) = 596.91 ft/s, W (weight) = 21,018.0 lb, and inertia I_x = 10033.4 slug-ft.^{2}

The resulting values for the aerodynamic parameters then become $\theta_2 = -32.748$, $\theta_3 = 1.436$, and $\theta_4 = -5.481$, with $\theta_1 = 4$

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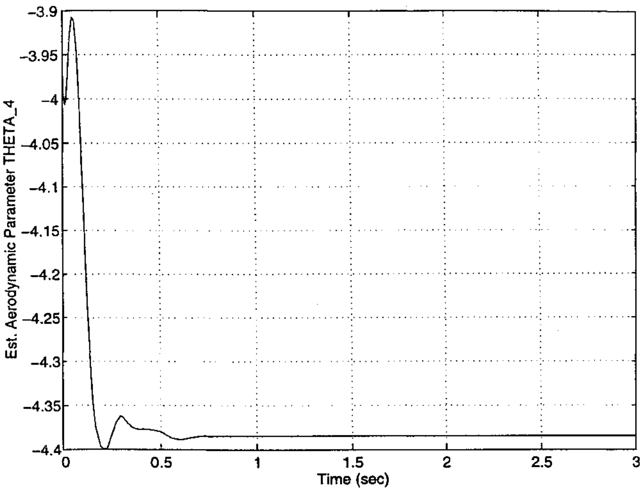

Fig. 7 Estimated aerodynamic parameter $\hat{\theta}_4$ for case 1.

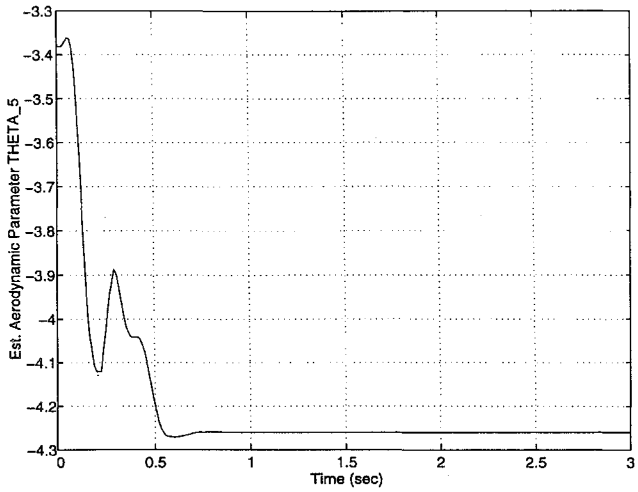

Fig. 8 Estimated aerodynamic parameter $\hat{\theta}_5$ for case 1.

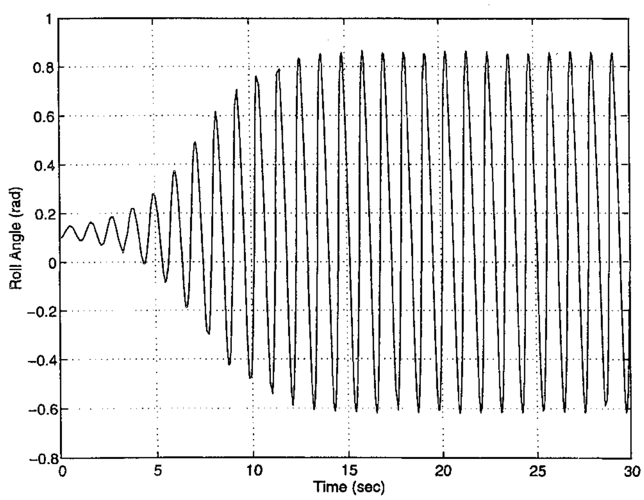

Fig. 9 Dynamic characteristics of wing rock based on numerical integration of the original equation for roll angle for case 2.

and $\theta_5 = 0.1$. In addition, the nondimensional aerodynamic rolling moment derivative because of the aileron control deflection coefficient $C_{l_{\delta_A}}$ is -0.003489 (1/deg), where the aileron control surface actuator model is deflection limit ± 21.5 deg, and rate limit 80 deg/s with time constant 0.0495 s lag.

We design a wing leveler or bank angle hold autopilot for this aircraft to suppress the wing rock with the dynamic characteristics for the roll angle as shown in Fig. 9. The performance objective is specified as before by a target frequency and damping ratio. By using the algorithm, the adaptive control is computed. Simulation results are shown by the following. Figure 10 shows that the control system will capture the system and function for an initial roll angle as high as $\phi(0) = 46$ deg. It also demonstrates achievement of a steady state with a transient stage of 1.5 s.

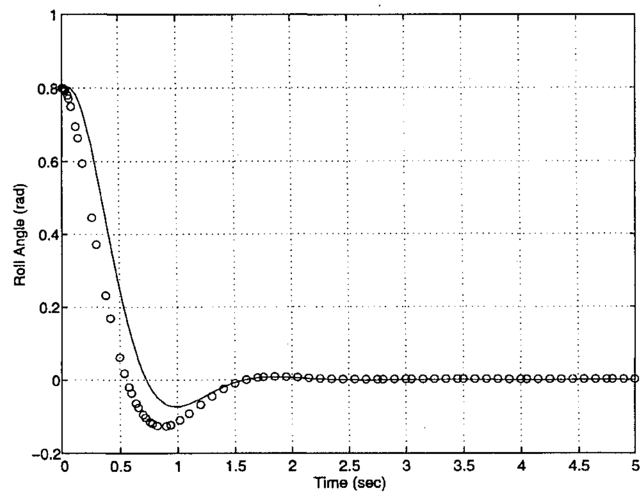

Fig. 10 Prespecified trajectory for roll angle (solid) and the roll angle response (circle) for case 2.

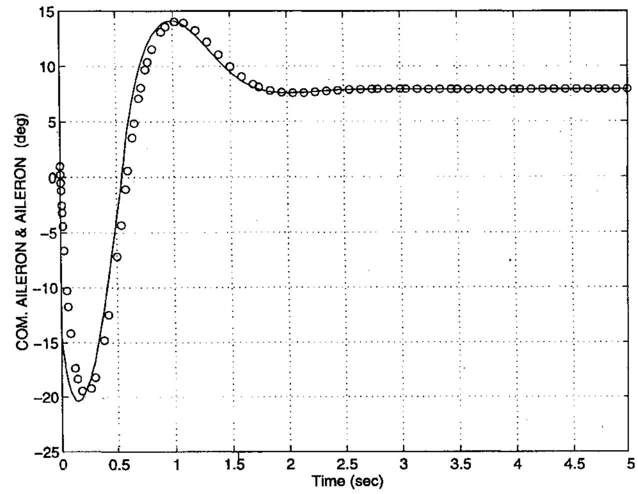

Fig. 11 Aileron deflection response (circle) and adaptive control; the commanded aileron deflection (solid) for case 2.

Figure 11 shows the aileron deflection angle δ_A and the commanded aileron deflection $\delta_{A_{COM}}$, i.e., the required adaptive control, each as a function of time. Since the initial condition $\phi(0)$ is relatively large, one expects a large control input for the wing rock suppression. Under these conditions, we have set the adaptation gain matrix Γ of Eq. (19) at $10^{-4} I_{5\times 5}$, and the design constants c_1, c_2, and c_3 of Eq. (17) are 7, 6, and 5, respectively. The initial conditions of the aerodynamic parameters $\hat{\theta}_4$ and $\hat{\theta}_5$ account for 25% uncertainties in these parameters.

Conclusions

We have shown the feasibility both theoretically and pragmatically of applications of recent theoretical advances in the differential geometric formulation of nonlinear control theory. The theory provides a design approach to achieve an adaptive control for feedback linearizable systems. We have applied it to the problem of suppressing wing rock with an extension of the technique to include tracking. The designed control and parameter update laws can track as a prespecified model system with remarkable accuracy reaching steady state within a transient stage of any desired time length. Thus, assigned maneuvers can be maintained without degradation simultaneously with the suppression of wing rock. Of course, such performance requires the availability of sufficient rolling moment because of the lateral control. It may be that control augmentation by thrust vectoring to boost the rolling moment because of the lateral control to desired levels will be necessary for some aircraft.

The design procedure accommodates aerodynamic parameter estimation and updating online while the control system is in operation.

The attainment of this adaptive control with these properties is not only a theoretical exercise, we present illustrations by designing such control laws for an AFTI/F-16 class aircraft and a high-angle-of-attack research vehicle similar to F-18/HARV under certain prescribed conditions.

Appendix: Details of Procedure

In this appendix we present the details of applying adaptive control procedure of Ref. 12 for parametric strict feedback form systems with an extension to tracking.

Step 1

Introducing $z_1 = x_1 - x_{d1}$ and $z_2 = x_2 - (x_{d2} + \alpha_1)/k_1$, we write $\dot{x}_1 = k_1 x_2 + \phi_i^T \theta$ as

$$\dot{z}_1 = k_1 z_2 + \alpha_1 + \phi_1^T \theta \tag{A1}$$

and use α_1 as a control to stabilize Eq. (A1) with respect to the Lyapunov function $V_1 = \frac{1}{2} z_1^2 + \frac{1}{2}(\hat{\theta} - \theta)^T \Gamma^{-1} (\hat{\theta} - \theta)$. Then,

$$\dot{V}_1 = z_1 \left(k_1 z_2 + \alpha_1 + \phi_1^T \hat{\theta} \right) + (\hat{\theta} - \theta)^T \Gamma^{-1} (\dot{\hat{\theta}} - \Gamma z_1 \phi_1) \tag{A2}$$

If x_2 were our actual control, we would let $z_2 = 0$, that is, $k_1 x_2 = x_{d2} + \alpha_1$. Then, we would eliminate $\hat{\theta} - \theta$ from $\dot{V}_1$ with the update law $\dot{\hat{\theta}} = \Gamma \tau_1$, where

$$\tau_1(x_1) = \phi_1(x_1) z_1 \tag{A3}$$

To make $\dot{V}_1 = -c_1 z_1^2$, we would choose

$$\alpha_1(x_1, \hat{\theta}) = -c_1 z_1 - \phi_1^T (x_1) \hat{\theta} \tag{A4}$$

Since x_2 is not our control, we have $z_2 \neq 0$, and we do not use $\dot{\hat{\theta}} = \Gamma \tau_1$ as an update law. However, we retain τ_1 as our first stabilizing function. Thus, we postpone the decision about $\hat{\theta}$ and tolerate the presence of $\hat{\theta} - \theta$ in $\dot{V}_1$:

$$\dot{V}_1 = -c_1 z_1^2 + k_1 z_1 z_2 + (\theta - \hat{\theta}) \Gamma^{-1} (\Gamma \tau - \dot{\hat{\theta}}) \tag{A5}$$

The second term $z_1 z_2$ in $\dot{V}_1$ will be canceled at the next step. The closed-loop form of Eq. (A1) with Eq. (A4) is

$$\dot{z}_1 = -c_1 z_1 + k_1 z_2 + (\theta - \hat{\theta}) \phi(x_1) \tag{A6}$$

Step 2

Introducing $z_3 = x_3 - x_{d3}/k_1 k_2 - \alpha_2/k_2$, we rewrite $\dot{x}_2 = k_2 x_3 + \phi_2^T (x_1 x_2) \theta$ as

$$\begin{aligned} \dot{z}_2 &= k_2 z_3 + \alpha_2 + \phi_2^T \theta \\ &\quad - \left[\frac{\partial \alpha_1}{\partial x_1} \left(k_1 x_2 + \phi_1^T \theta \right) + \frac{\partial \alpha_1}{\partial x_{d1}} x_{d2} \right] \Big/ k_1 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \Big/ k_1 \end{aligned} \tag{A7}$$

and use α_2 as a control to stabilize Eqs. (A6) and (A7) with respect to $V_2 = V_1 + \frac{1}{2} z_2^2$. Then,

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 + z_2 \Bigg[k_1 z_1 + k_2 z_3 + \alpha_2 \\ &\quad - \left(\frac{\partial \alpha_1}{\partial x_1} k_1 x_2 + \frac{\partial \alpha_1}{\partial x_{d1}} x_{d2} \right) \Big/ k_1 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \Big/ k_1 + \omega_2^T \hat{\theta} \Bigg] \\ &\quad + (\hat{\theta} - \theta)^T \Gamma^{-1} (\dot{\hat{\theta}} - \Gamma \tau_2) \end{aligned} \tag{A8}$$

where $\omega_2 = \phi_2 - (\partial \alpha_1 / \partial x_1) \phi_2 / k_1$.

If x_3 were our actual control, we would let $z_3 = 0$ and eliminate $\hat{\theta} - \theta$ from $\dot{V}_2$ with the update law $\dot{\hat{\theta}} = \Gamma \tau_2$, where

$$\tau_2 \left(x_1, x_2, \hat{\theta}, x_{d1}, x_{d2} \right) = \phi_1 z_1 + \left(\phi_2 - \frac{\partial \alpha_1}{\partial x_1} \phi_1 \Big/ k_1 \right) z_2 \tag{A9}$$

Then, to make $\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2$, we would design α_2 such that the bracketed term multiplying z_2 equals $-c_2 z_2$, namely,

$$\begin{aligned} \alpha_2(x_1, x_2, \hat{\theta}, x_{d1}, x_{d2}) &= -k_1 z_1 - c_2 z_2 \\ &\quad + \frac{1}{k_1} \left(\frac{\partial \alpha_1}{\partial x_1} k_1 x_2 + \frac{\partial \alpha_1}{\partial x_{d1}} x_{d2} \right) \\ &\quad + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 \Big/ k_1 - \left(\phi_2 - \frac{\partial \alpha_1}{\partial x_1} \phi_1 \Big/ k_1 \right)^T \hat{\theta} \end{aligned} \tag{A10}$$

Since x_3 is not our control, we have $z_3 \neq 0$, and we do not use $\dot{\hat{\theta}} = \Gamma \tau_2$ as an update law. However, we retain τ_2 as our second tuning function. The resulting $\dot{V}_2$ is

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 - c_2 z_2^2 + k_2 z_2 z_3 \\ &\quad + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} z_2 \Big/ k_1 + (\theta - \hat{\theta})^T \Gamma^{-1} \right) (\Gamma \tau_2 - \dot{\hat{\theta}}) \end{aligned} \tag{A11}$$

The first two terms in $\dot{V}_2$ are negative definite, the third term will be canceled at the next step, and the last term is tolerated at this step, as the decision about $\hat{\theta}$ is again postponed. The closed-loop form of Eq. (A7) with Eq. (A10) is

$$\begin{aligned} \dot{z}_2 &= -k_1 z_1 - c_2 z_2 + k_2 z_3 \\ &\quad + (\theta - \hat{\theta})^T \omega_2 + \left[\frac{\partial \alpha_1}{\partial \hat{\theta}} (\Gamma \tau_1 - \dot{\hat{\theta}}) \right] \Big/ k_1 \end{aligned} \tag{A12}$$

Step 3

Introducing $z_4 = x_4 - x_{d4}/k_1 k_2 k_3 - \alpha_3/k_3$, we rewrite $\dot{x}_3 = k_3 x_4 + \phi_3^T (x_1, x_2, x_3) \theta$ as

$$\begin{aligned} \dot{z}_3 &= k_3 z_4 + \alpha_3 + \phi_3^T \theta - \left(\frac{\partial \alpha_2}{\partial x_1} \left(k_1 x_2 + \phi_1^T \theta \right) + \frac{\partial \alpha_2}{\partial x_{d1}} x_{d2} \right) \Big/ k_2 \\ &\quad - \left[\frac{\partial \alpha_3}{\partial x_2} \left(k_2 x_3 + \phi_2^T \theta \right) + \frac{\partial \alpha_2}{\partial x_{d2}} x_{d3} \right] \Big/ k_2 - 1 \Big/ k_2 \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} \end{aligned} \tag{A13}$$

and use α_3 as a control to stabilize the (z_1, z_2, z_3) system with respect to $V_3 = V_2 + \frac{1}{2} z_3^2$. Then,

$$\begin{aligned} \dot{V}_3 &= -c_1 z_1^2 - c_2 z_2^2 \\ &\quad + z_3 \Bigg[k_2 z_2 + k_3 z_4 + \alpha_3 - \left(\frac{\partial \alpha_2}{\partial x_1} k_1 x_2 + \frac{\partial \alpha_2}{\partial x_{d1}} x_{d2} \right) \Big/ k_2 \\ &\quad - \left(\frac{\partial \alpha_2}{\partial x_2} k_2 x_3 + \frac{\partial \alpha_2}{\partial x_{d2}} x_{d3} \right) \Big/ k_2 - \left(\frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) \\ &\quad + \omega_3^T \hat{\theta} - \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \omega_3 z_2 \right) \Bigg] + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}} z_2 \Big/ k_1 \right) \\ &\quad + (\Gamma \tau_3 - \dot{\hat{\theta}}) + (\hat{\theta} - \theta)^T \Gamma^{-1} (\dot{\hat{\theta}} - \Gamma \tau_3) \end{aligned} \tag{A14}$$

where

$$\omega_3 = \phi_3 - \left(\frac{\partial \alpha_2}{\partial x_1} \phi_1 + \frac{\partial \alpha_2}{\partial x_2} \phi_2 \right) \Big/ k_2$$

If x_4 were our actual control, we would let $z_4 \equiv 0$ and eliminate $\hat{\theta} - \theta$ from $\dot{V}_3$ with the update law $\dot{\hat{\theta}} = \Gamma \tau_3$, where

$$\tau_3 = \tau_2 + \omega_3 z_3 \tag{A15}$$

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Then, to make $\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2$, we would design α_3 such that the bracketed term multiplying z_3 equals $-c_3z_3$, namely,

$$\alpha_3(x_1, x_2, x_3, \hat{\theta}, x_{d1}, x_{d2}, x_{d3}) = -k_2z_2 - c_3z_3 + \left(\frac{\partial\alpha_2}{\partial x_1}k_1x_2 + \frac{\partial\alpha_2}{\partial x_{d1}}x_{d2}\right)\Big/ k_2 + \left(\frac{\partial\alpha_2}{\partial x_2}k_2x_3 + \frac{\partial\alpha_2}{\partial x_{d2}}\right)\Big/ k_2 + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\tau_3\Big/ k_2 - \omega_3^T\hat{\theta} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\omega_3z_2/k_1 \tag{A16}$$

Since x_4 is not our control, we have $z_4 \neq 0$, and we do not take $\dot{\hat{\theta}} = \Gamma\tau_3$ as an update law. However, we retain τ_2 as our third tuning function and α_3 as our third stabilizing function. The resulting $\dot{V}_3$ is

$$\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - c_3c_3^2 + z_2z_3 + k_3z_3z_4 + \left(\frac{\partial\alpha_2}{\partial\hat{\theta}}z_2\Big/ k_1 + \frac{\partial\alpha_1}{\partial\hat{\theta}}z_3\Big/ k_2 + (\theta-\hat{\theta})^T\Gamma^{-1}\right) + (\Gamma\tau_3 - \dot{\hat{\theta}}) \tag{A17}$$

The first three terms in $\dot{V}_3$ are negative definite, the fourth term will be canceled at the next step, and the last term is tolerated at this step, as then decision about $\dot{\hat{\theta}}$ is again postponed. The closed-loop form of Eq. (A13) with Eq. (A16) is

$$\dot{z}_3 = -k_2z_2 - c_3z_3 + k_3z_4 + (\theta-\hat{\theta})^T\omega_3 + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Big/ k_2(\Gamma\tau_3 - \dot{\hat{\theta}}) + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\omega_3z_2$$

Step *i*

Introducing $z_{i+1} = x_{i+1} - x_{d(i+1)}/k_1, \ldots, k_i - \alpha_i/k_i$, we rewrite $\dot{x}_i = k_ix_{i+1} + \phi(x_1, \ldots, x_i)^T\theta$ as

$$\dot{z}_i = k_iz_{i+1} + \alpha_i + \phi_i^T\theta - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\left[\left(k_kx_{k+1} + \phi_k^T\theta\right)/k_{i-1}\right] - \left(\frac{\partial\alpha_{i-1}}{\partial x_{dk}}x_{d(k+1)}\right)\Big/ k_{i-1} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}\Big/ k_{i-1} \tag{A18}$$

and use α_i as a control to stabilize the $(z_1, z_2, \ldots, z_i)$ system with respect to $V_i = V_{i-1} + \frac{1}{2}z_i^2$. Then,

$$\dot{V}_i = -\sum_{k=1}^{i-1}c_kz_k^2 + \sum_{k=2}^{i-1}z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\Gamma\tau_i - \dot{\hat{\theta}})\Big/ k_{k-1} \times\left[k_{i-1}z_{i-1} + k_iz_{i+1} + \alpha_i - \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}k_kx_{k+1} + \frac{\partial\alpha_{i-1}}{\partial x_{dk}}x_{dk+1}\Big/ k_{i-1}\right) - \left(\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\right)\dot{\hat{\theta}}\Big/ k_{i-1} + \omega_i^T\hat{\theta} - \sum_{k=2}^{i-1}\left(\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_iz_k\right)\Big/ k_{k-1}\right] + (\hat{\theta}-\theta)\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\tau_i) \tag{A19}$$

where

$$\omega_i = \phi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_1}{\partial x_1}\phi_1\Big/ k_{i-1}$$

If x_{i+1} were our actual control, we would let $z_{i+1} = 0$ and eliminate $\hat{\theta} - \theta$ from $\dot{V}_i$ with the update law $\dot{\hat{\theta}} = \Gamma\tau_i$, where

$$\tau_i(x_1, \ldots, x_i, \hat{\theta}) = \tau_{i-1} + \omega_iz_i \tag{A20}$$

Then to make

$$\dot{V}_i = -\sum_{k=1}^{i}c_kz_k^2$$

we would design α_i such that the bracketed term multiplying z_i equals $-c_iz_i$, namely,

$$\alpha_i(x_1, \ldots, x_i, \theta, x_{d1}, \ldots, x_{di}) = -k_{i-1}z_{i-1} - c_iz_i + \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}k_kx_{k+1} + \frac{\partial\alpha_{i-1}}{\partial x_{dk}}x_{d(k+1)}\right)\Big/ k_{i-1} + \frac{\partial\alpha_{i-1}}{\partial\theta_k}\Gamma\tau_i\Big/ k_{i-1} + \omega_i^T\hat{\theta} + \sum_{k=2}^{i-1}\left(\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_iz_k\right)\Big/ k_{k-1} \tag{A21}$$

Since x_{i+1} is not our control, we have $z_{i+1} \neq 0$, and we do not use $\dot{\hat{\theta}} = \Gamma\tau_i$ as an update law. However, we retain τ_i as our ith tuning function and α_i as our ith stabilizing function. The resulting $\dot{V}_i$ is

$$\dot{V}_i = -\sum_{k=1}^{i}c_kz_k^2 + k_iz_iz_{i+1} + \left[\sum_{k=1}^{i-1}\left(z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Big/ k_{k-1}\right) + (\theta - \hat{\theta}^T)\Gamma^{-1}\right](\Gamma\tau_i - \dot{\hat{\theta}}) \tag{A22}$$

The closed-loop form of Eq. (A18) with Eq. (A21) is

$$\dot{z}_i = -k_{i-1}z_{i-1} - c_iz_i + k_iz_{i+1} + (\theta-\hat{\theta})^T\omega_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\Gamma\tau_i - \dot{\hat{\theta}})\Big/ k_{i-1} + \sum_{k=2}^{i-1}\left(z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_i\right)\Big/ k_{k-1} \tag{A23}$$

Step *n*

Introducing $z_n = x_n - x_{dn}/k_1, \ldots, k_{n-1} - \alpha_{n-1}/k_n$, we rewrite $\dot{x}_n = k_x\phi_0 + \phi_n^T\theta + k_u\beta_0u$ as

$$\dot{z}_n = k_x\phi_0 + \phi_n^T\theta + k_u\beta_0u - \left[\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_{dk}}\left(k_kx_{k+1} + \theta^T\phi_k\right) + \frac{\partial\alpha_{n-1}}{\partial x_{dk}}x_{d(k+1)}\right]\Big/ k_{n-1} - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}\Big/ k_{n-1} \tag{A24}$$

We now design actual update law $\dot{\hat{\theta}} = \Gamma\tau_n$ and feedback control u to stabilize the full z system with respect to Lyapunov function $V_n = V_{n-1} + \frac{1}{2}z_n^2$. Our goal is to make $\dot{V}_n$ nonpositive:

$$\dot{V}_n = -\sum_{k=1}^{n-1}c_kz_k^2 + \sum_{k=2}^{n-1}z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\Gamma\tau_n - \dot{\hat{\theta}}) + z_n\left[k_{n-1}z_n + k_u\beta_0u + k_x\phi_0 - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}k_kx_{k+1} + \frac{\partial\alpha_{n-1}}{\partial x_{dk}}x_{d(k+1)}\Big/ k_{n-1} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}\Big/ k_{n-1} + \omega_n^T\hat{\theta} - \sum_{k=2}^{n-1}\left(\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_nz_k\right)\Big/ k_{k-1}\right] + (\hat{\theta}-\theta)^T\Gamma^{-1}(\dot{\hat{\theta}} - \Gamma\tau_n) \tag{A25}$$

where

$$\omega_n = \phi_n - \left(\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\phi_k\right)\Big/ k_{n-1}$$

To eliminate $(\hat{\theta} - \theta)$ from $\dot{V}_n$, we choose the update law

$$\dot{\hat{\theta}} = \Gamma\tau_n \tag{A26}$$

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and the control u such that the bracketed term multiplying z_n equals $-c_n z_n$:

$$u = \frac{1}{k_u \beta_0(x)} \Bigg[-k_{n-1} z_{n-1} - c_n z_n - k_x \phi_0(x) + \sum_{k=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_k} k_k x_{k+1} + \frac{\partial \alpha_{n-1}}{\partial x_{dk}} x_{dk-1} \right) \Big/ k_{n-1} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \Gamma \tau_n \Big/ k_{n-1} - \omega_n^T \hat{\theta} + \sum_{k=2}^{n-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \omega_n z_k \Big/ k_k \Bigg] \tag{A27}$$

Thus, we have

$$\dot{V}_n = -\sum_{k=1}^{n} c_k z_k^2$$

With Eq. (A27) the closed-loop form of Eq. (A24) becomes

$$\dot{z}_n = -k_{n-1} z_{n-1} - c_n z_n + (\theta - \hat{\theta})^T \omega_i + \sum_{k=2}^{n-1} \left(\frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma \omega_n z_k \right) \Big/ k_{n-1}$$

To reduce the parameter error vector $\theta - \hat{\theta}(t)$, we have $(\theta - \hat{\theta})^T \Gamma^{-1}(\theta - \hat{\theta}) \leq 2V_n(t) \leq 2V_n(0)$, that is,

$$(\theta - \hat{\theta})^T \Gamma^{-1} (\theta - \hat{\theta}) \leq z(0)^T z(0) + [\theta - \hat{\theta}(0)]^T \Gamma^{-1} [\theta - \hat{\theta}(0)] \tag{A28}$$

This bound shows that a possibility for reducing $\theta - \hat{\theta}(t)$ lies in $z(0)$, and $z(0)$ can be set to zero by an appropriate initialization of the reference trajectory.

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