



## Brief paper

Adaptive trajectory tracking despite unknown input delay and plant parameters<sup>☆</sup>Delphine Bresch-Pietri, Miroslav Krstic<sup>\*</sup>

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## ABSTRACT

In a recent paper we presented the first adaptive control design for an ODE system with a possibly large actuator delay of unknown length. We achieved global stability under full state feedback. In this paper we generalize the design to the situation where, besides the unknown delay value, the ODE also has unknown parameters, and where trajectory tracking (rather than equilibrium regulation) is pursued.

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## 1. Introduction

Input delay arises in many forms, from an actual physical transport delay in various chemical process systems, hydraulically actuated systems, and combustion systems, to the problems where computational delay manifests itself as being equivalent to input delay. Until recently, the only results on adaptive control of systems with input delays dealt only with uncertain parameters in the ODE part of the system (Evesque, Annaswamy, Niculescu, & Dowling, 2003; Niculescu & Annaswamy, 2003; Ortega & Lozano, 1988) but not with uncertainty in the delay value itself. The importance of designing adaptive controllers for unknown delay was recognized in Diop, Kolmanovsky, Moraal, and van Nieuwstadt (2001) and Krstic and Banaszuk (2006), however only approximation-based ideas for limited classes of plants were dealt with. The robustness of adaptive backstepping control for linear ODEs to dynamic perturbations that include input delay was studied in Zhou, Wang, and Wen (2008), however the nominal system did not involve delay and the delay was not estimated nor compensated by the controller.

In a recent paper (Bresch-Pietri & Krstic, 2009) we presented the first results on delay-adaptive control for a general class of plants, under full state feedback. This result is global—including being global in the initial delay value estimate (one can arbitrarily underestimate or overestimate the delay value initially, and still achieve stabilization adaptively).

However, the result in Bresch-Pietri and Krstic (2009) contains two limitations, one made for pedagogical reasons and the other which is fundamental. The limitation for which the reason is pedagogical is in assuming that there are no unknown parameters in the ODE. This limitation is being removed by this paper. This limitation was imposed in Bresch-Pietri and Krstic (2009) to prevent the novel ideas on how to develop global adaptivity in the infinite-dimensional (delay) context from being buried under standard but nevertheless complicated details of ODE adaptive control.

The other limitation in Bresch-Pietri and Krstic (2009), which is fundamental, is in assuming that the full actuator state is measured, though the delay value is completely unknown. The physical meaning of this is that the actuator delay is modeled as a transport process, to which the control designer has physical access for measurement but the speed of propagation of this transport process is completely unknown. As we explain in Bresch-Pietri and Krstic (2009), the problem where the actuator state is not measurable and the delay value is unknown is not solvable globally, since the problem is not linearly parametrized. We show in Bresch-Pietri and Krstic (2009) how one can solve it locally, however, this is not a very satisfactory result, since it is local both in the initial state and in the initial parameter error. In other words, the initial delay estimate needs to be sufficiently close to the true delay. (The delay can be long, but it needs to be known quite closely.) Under such an assumption, one might as well use a linear controller and rely on robustness of the feedback law to small errors in the assumed delay value.

So, for this reason, in this paper we continue with a full-state feedback design, specifically assuming the measurement of the actuator state. In Bresch-Pietri and Krstic (2009) we discussed all the possible problems that one can consider with respect to the availability or unavailability of measurement of the ODE

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state, the actuator state, the knowledge (or lack of knowledge) of the delay value, and the knowledge (or lack thereof) of the ODE parameters. There is a total number of 14 distinct problem combinations. Here we focus on the most interesting one of them, with both the ODE parameters and the delay value unknown, but with full state measurement. An extension to the case where only an output (and not the complete state) of the ODE is available for measurement is easy (with the method of backstepping and Kreisselmeier observers). We don't pursue it here for consistency of concepts—since we must measure the actuator state, we might as well present a result with full measurement of the ODE state.

As in most of the research on control of unstable plants with a long actuator delay (Artstein, 1982; Evesque et al., 2003; Fiagbedzi & Pearson, 1986; Gu & Niculescu, 2003; Jankovic, 2006a,b, 2008; Klamka, 1982; Krstic, 2008a,b; Krstic & Smyshlyaev, 2008b; Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Michiels & Niculescu, 2003, 2007; Mirkin, 2004; Mondie & Michiels, 2003; Niculescu & Annaswamy, 2003; Olbrot, 1978; Richard, 2003; Tadmor, 2000; Watanabe & Ito, 1981; Watanabe, 1996; Zaccarian & Netic, 2006; Zhong, 2006a,b; Zhong & Mirkin, 2002), the essence of our approach is “predictor feedback,” which we recently showed in Krstic (2008a) and Krstic and Smyshlyaev (2008b) to be a form of backstepping boundary control for PDEs (Krstic & Smyshlyaev, 2008a,c; Smyshlyaev & Krstic, 2004, 2005) and extended to nonlinear plants (Krstic, 2008b).

In this paper we generalize the design from Bresch-Pietri and Krstic (2009) in two major ways: we extend it to ODEs with unknown parameters and extend it from equilibrium regulation to trajectory tracking. A significant number of new technical issues arise in this problem. The estimation error of the ODE parameters appears in the error models of both the ODE and of the infinite-dimensional (delay) subsystem, which is reflected also in the update law. The update law has to also deal appropriately with ensuring stabilizability with the parameter estimates, for which projection is employed. Finally, our approach for dealing with delay adaptation involves normalized Lyapunov-based tuning, a rather non-standard approach as compared to finite-dimensional adaptive control. In this framework, we need to bound numerous terms involving parameter adaptation rates (both for the delay and for the ODE parameters) in the Lyapunov analysis. Some of these terms are vanishing (when the tracking error is zero), while the others (which are due to the reference trajectory) are non-vanishing. These terms receive different treatment though both are bounded by normalization and their size is controlled with the adaptation gain.

We begin in Section 2 by defining the problem and present the adaptive control design and the main stability theorem in Section 3. Simulation results are shown in Section 4, which is then followed by the stability proof in Section 5.

## 2. Problem formulation

We consider the following system

$$\dot{X}(t) = A(\theta)X(t) + B(\theta)U(t - D) \quad (1)$$

$$Y(t) = CX(t), \quad (2)$$

where  $X \in \mathbb{R}^n$  is the ODE state,  $U$  is the scalar input to the entire system,  $D > 0$  is an unknown constant delay, the system matrix  $A(\theta)$  and the input vector  $B(\theta)$  are linearly parametrized, i.e.,

$$A(\theta) = A_0 + \sum_{i=1}^p \theta_i A_i \quad (3)$$

$$B(\theta) = B_0 + \sum_{i=1}^p \theta_i B_i, \quad (4)$$

and  $\theta$  is an unknown but constant parameter vector that belongs to the convex set

$$\Pi = \{\theta \in \mathbb{R}^p | \mathcal{P}(\theta) \leq 0\}, \quad (5)$$

where, by assuming that the convex function  $\mathcal{P} : \mathbb{R}^p \rightarrow \mathbb{R}$  is smooth, we assure that the boundary  $\partial\Pi$  of  $\Pi$  is smooth.

**Assumption 1.** The set  $\Pi$  is bounded and known. A constant  $\bar{D}$  is known such that  $D \in ]0; \bar{D}]$ .

To make stabilization and tracking possible in the presence of unknown parameters, we make some assumptions and illustrate, with the help of an example introduced below, that these assumptions are reasonable. In the sequel,  $C^0(\Pi)$  and  $C^1(\Pi)$  is the usual notation for continuous and continuously differentiable functions (on the set  $\Pi$ ), respectively.

**Assumption 2.** We assume that the pair  $(A(\theta), B(\theta))$  is completely controllable for each  $\theta$ . Furthermore, we assume that there exists a triple of vector/matrix-valued functions  $(K(\theta), P(\theta), Q(\theta))$  such that  $K \in C^1(\Pi)$ ,  $P \in C^1(\Pi)$ ,  $Q \in C^0(\Pi)$ , the matrices  $P(\theta)$  and  $Q(\theta)$  are positive definite and symmetric, and the following Lyapunov equation is satisfied for all  $\theta \in \Pi$ :

$$P(\theta)(A + BK)(\theta) + (A + BK)(\theta)^T P(\theta) = -Q(\theta). \quad (6)$$

**Example 1.** Consider the example of a potentially unstable plant

$$\dot{X}_1(t) = \theta X_1(t) + X_2(t) \quad (7)$$

$$\dot{X}_2(t) = U(t - D) \quad (8)$$

$$Y(t) = X_1(t), \quad (9)$$

where we assume  $\Pi = [-\underline{\theta}; \bar{\theta}]$  and define

$$A(\theta) = A_0 + \theta A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \theta \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (10)$$

$$B = B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (11)$$

This is a linear system in the strict-feedback form (Krstic, Kanellakopoulos, & Kokotovic, 1995). Using the backstepping method we construct the triple  $(K, P, Q)$  as

$$K(\theta) = - \begin{pmatrix} 1 + (\theta + 1)^2 & \theta + 2 \end{pmatrix} \quad (12)$$

$$P(\theta) = \frac{1}{2} Q(\theta) = \begin{pmatrix} 1 + (1 + \theta)^2 & 1 + \theta \\ 1 + \theta & 1 \end{pmatrix}, \quad (13)$$

which satisfies the Lyapunov equation (6).

**Assumption 3.** The quantities

$$\underline{\lambda} = \inf_{\theta \in \Pi} \min \{\lambda_{\min}(P(\theta)), \lambda_{\min}(Q(\theta))\} \quad (14)$$

$$\bar{\lambda} = \inf_{\theta \in \Pi} \lambda_{\max}(P(\theta)) \quad (15)$$

exist and are known.

**Example 2 (Example 1 Continued).** One can show that the eigenvalues of  $P(\theta)$  are

$$\lambda_{\max}(P(\theta)) = \frac{2 + (\theta + 1)^2 + |\theta + 1|\sqrt{(\theta + 1)^2 + 1}}{2} \quad (16)$$

$$\lambda_{\min}(P(\theta)) = \frac{1}{\lambda_{\max}(P(\theta))}, \quad (17)$$

from which  $\underline{\lambda}$  and  $\bar{\lambda}$  are readily obtained over the set  $\Pi = [-\underline{\theta}; \bar{\theta}]$ .

**Assumption 4.** For a given smooth function  $Y^r(t)$ , there exist known functions  $X^r(t, \theta)$  and  $U^r(t, \theta)$ , which are bounded in  $t$  and continuously differentiable in the unknown argument  $\theta$  on  $\Pi$ , and which satisfy

$$\dot{X}^r(t, \theta) = A(\theta)X^r(t, \theta) + B(\theta)U^r(t, \theta) \quad (18)$$

$$Y^r(t) = CX^r(t, \theta). \quad (19)$$

**Example 3 (Example 2 Continued).** Take  $Y^r(t) = \sin(t)$ . Then, the reference trajectory pair for the state and input is

$$X^r(t, \theta) = \begin{pmatrix} \sin(t) \\ \cos(t) - \theta \sin(t) \end{pmatrix} \quad (20)$$

$$U^r(t, \theta, D) = -\sin(t + D) - \theta \cos(t + D). \quad (21)$$

Both functions are bounded in  $t$  and continuously differentiable in  $\theta$ .

### 3. Control design

To prepare for our adaptive control design, we represent the plant as

$$\dot{X}(t) = A(\theta)X(t) + B(\theta)u(0, t) \quad (22)$$

$$Y(t) = CX(t) \quad (23)$$

$$Du_t(x, t) = u_x(x, t), \quad x \in [0, 1] \quad (24)$$

$$u(1, t) = U(t), \quad (25)$$

where

$$u(x, t) = U(t + D(x - 1)). \quad (26)$$

The representation of the pure delay as a transport PDE allows a linear parameterization in the unknown delay  $D$ .

We consider reference trajectories  $X^r(t)$  and  $U^r(t)$ , such as described in Assumption 4. Let us introduce the following error variables

$$\tilde{X}(t) = X(t) - X^r(t, \hat{\theta}) \quad (27)$$

$$\tilde{U}(t) = U(t) - U^r(t, \hat{\theta}, \hat{D}) \quad (28)$$

$$e(x, t) = u(x, t) - u^r(x, t, \hat{\theta}), \quad (29)$$

with an estimate  $\hat{\theta}$  of the unknown  $\theta$ . When  $D$  and  $\theta$  are known, one can show that the control law

$$U(t) = U^r(t) - KX^r(t + D) + K \left[ e^{AD}X(t) + D \int_0^1 e^{A\hat{D}(1-y)}Bu(y, t)dy \right] \quad (30)$$

achieves exponential stability of the equilibrium  $(\tilde{X}, e) = 0$ , compensating the effects of the delay  $D$ .

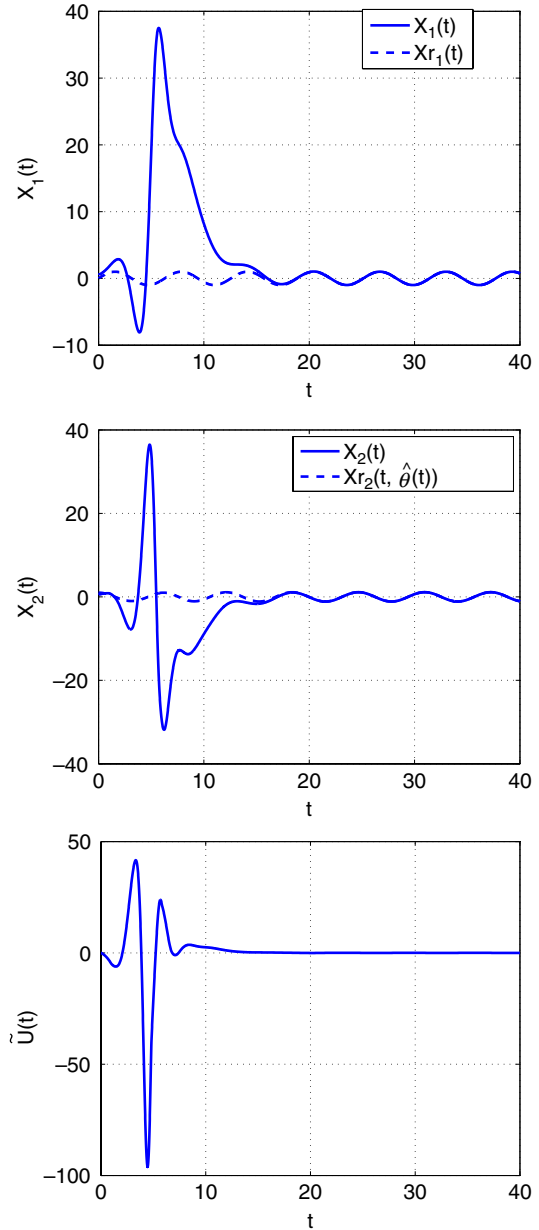
When  $D$  and  $\theta$  are unknown, we employ the control law

$$U(t) = U^r(t, \hat{\theta}, \hat{D}) - K(\hat{\theta})X^r(t + \hat{D}, \hat{\theta}) + K(\hat{\theta}) \left[ e^{A(\hat{\theta})\hat{D}(t)}X(t) + \hat{D}(t) \int_0^1 e^{A(\hat{\theta})\hat{D}(t)(1-y)}B(\hat{\theta})u(y, t)dy \right], \quad (31)$$

based on the certainty equivalence principle. The update laws for the estimates  $\hat{D}$  and  $\hat{\theta}$  are chosen based on the Lyapunov analysis (presented in Section 5) as

$$\dot{\hat{D}}(t) = \gamma_1 \text{Proj}_{[0, \bar{D}]} \{ \tau_D(t) \} \quad (32)$$

$$\dot{\hat{\theta}}(t) = \gamma_2 \text{Proj}_{\Pi} \{ \tau_\theta(t) \}, \quad (33)$$



**Fig. 1.** The system response of the system (1)–(2) with the reference trajectory (20)–(21) for  $D = 1, \theta = 0.5, \hat{\theta}(0) = 0$  and  $\hat{D}(0) = 0$ .

with adaptation gains  $\gamma_1$  and  $\gamma_2$  chosen as positive and “small enough” and with

$$\tau_D(t) = - \frac{\int_0^1 (1+x)w(x, t)K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)x}dx}{1 + \tilde{X}(t)^T P(\hat{\theta})\tilde{X}(t) + b \int_0^1 (1+x)w(x, t)^2 dx} \times \left( (A + BK)(\hat{\theta})\tilde{X}(t) + B(\hat{\theta})w(0, t) \right) \quad (34)$$

$$\tau_\theta(t) = \frac{2\tilde{X}(t)^T P(\hat{\theta})/b - \int_0^1 (1+x)w(x, t)K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)x}dx}{1 + \tilde{X}(t)^T P(\hat{\theta})\tilde{X}(t) + b \int_0^1 (1+x)w(x, t)^2 dx} \times (A_i X(t) + B_i u(0, t))_{1 \leq i \leq p}. \quad (35)$$

The matrix  $P$  is defined in Assumption 2, the standard projector operators are given by

$$\text{Proj}_{[0, \bar{D}]} \{ \tau_D \} = \tau_D \begin{cases} 0, & \hat{D} = 0 \text{ and } \tau_D < 0 \\ 0, & \hat{D} = \bar{D} \text{ and } \tau_D > 0 \\ 1, & \text{else} \end{cases} \quad (36)$$

$$\text{Proj}_\Pi\{\tau_\theta\} = \tau_\theta \begin{cases} 1, & \hat{\theta} \in \hat{\Pi} \\ & \text{or } \nabla_{\hat{\theta}} \mathcal{P}^T \tau \leq 0 \\ I - \frac{\nabla_{\hat{\theta}} \mathcal{P} \nabla_{\hat{\theta}} \mathcal{P}^T}{\nabla_{\hat{\theta}} \mathcal{P}^T \nabla_{\hat{\theta}} \mathcal{P}}, & \hat{\theta} \in \partial \Pi \\ & \text{and } \nabla_{\hat{\theta}} \mathcal{P}^T \tau > 0. \end{cases} \quad (37)$$

The transformed state of the actuator is needed in the update law and is defined as

$$w(x, t) = e(x, t) - \hat{D}(t) \int_0^x K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) e(y, t) dy - K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)x} \tilde{X}(t) \quad (38)$$

and the constant  $b$  is chosen such that

$$b \geq 4 \sup_{\hat{\theta} \in \Pi} |PB|^2(\hat{\theta}) \frac{\bar{D}}{\underline{\lambda}}. \quad (39)$$

**Theorem 1.** *Let Assumptions 1–4 hold and consider the closed-loop system consisting of (22)–(25), the control law (31) and the update laws defined by (32)–(39). There exists  $\gamma^* > 0$  such that for any  $\gamma \in [0, \gamma^*[$ , there exist positive constants  $R$  and  $\rho$  (independent of the initial conditions) such that, for all initial conditions satisfying  $(X^0, u^0, \hat{D}^0, \theta^0) \in \mathbb{R}^n \times L_2(0, 1) \times ]0, \bar{D}] \times \Pi$ , the following holds:*

$$\Upsilon(t) \leq R(e^{\rho T(0)} - 1), \quad \forall t \geq 0, \quad (40)$$

where

$$\Upsilon(t) = |\tilde{X}(t)|^2 + \int_0^1 e(x, t)^2 dx + \tilde{D}(t)^2 + \tilde{\theta}(t)^T \tilde{\theta}(t). \quad (41)$$

Furthermore, asymptotic tracking is achieved, i.e.,

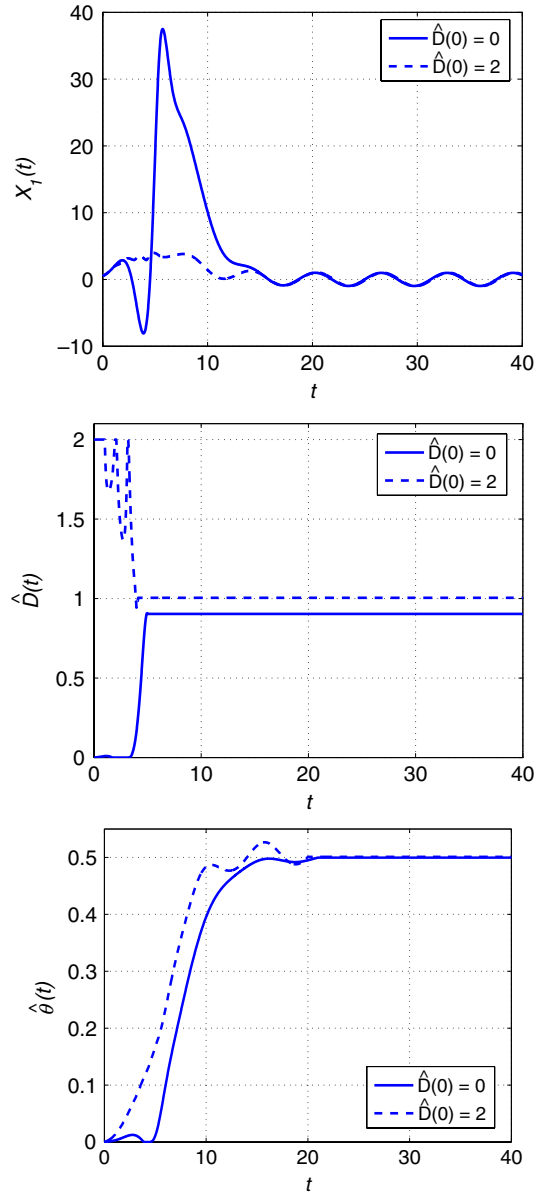
$$\lim_{t \rightarrow \infty} \tilde{X}(t) = 0, \quad \lim_{t \rightarrow \infty} \tilde{U}(t) = 0. \quad (42)$$

#### 4. Simulations

We return to the system from Examples 1–3 and present simulation results for the closed-loop system consisting of the plant (1)–(2), the control law (31) and the update laws defined through (32)–(39). We focus on the issues arising from the large uncertainties in  $D$  and  $\theta$  and from the tracking problem with the reference trajectory (20)–(21). We take  $D = 1, \theta = 0.5, \bar{D} = 2D = 2, \underline{\theta} = 0, \bar{\theta} = 2\theta = 1$ . We pick the adaptation gains as  $\gamma_1 = 10, \gamma_2 = 2.3$  and the normalization coefficient as  $b = \frac{4|PB|^2 \bar{D}}{\underline{\lambda}} = 3200$ . We show simulation results for  $X_1(0) = X_2(0) = 0.5, \hat{\theta}(0) = 0$ , and two different values of  $\hat{D}(0)$ .

Fig. 1 shows that, as Theorem 1 predicts, perfect asymptotic tracking is achieved (in a single simulation) for the output, state, and input, despite starting with an initial delay estimate of  $\hat{D}(0) = 0$ , namely, for a large parametric uncertainty and with no predictor feedback initially. Fig. 2 shows two distinct simulations, starting from two extreme initial values for the delay estimate, one at zero, and the other at twice the true delay value. The dashed simulation starts with a predictor feedback that initially overcompensates the delay, but it later adjusts the value of the delay estimate to obtain a stabilizing value of this parameter in the predictor feedback.

In Fig. 2 we observe that  $\hat{\theta}(t)$  converges to the true  $\theta$  in both simulations, whereas this is not the case with  $\hat{D}(t)$ . This is consistent with the theory. By examining the error systems (44), (45), with the help of a persistency of excitation argument, we could infer the convergence of  $\hat{\theta}(t)$  but not of  $\hat{D}(t)$ .



**Fig. 2.** The system response of the system (1)–(2) with the reference trajectory (20)–(21) for  $D = 1, \theta = 0.5, \hat{\theta}(0) = 0$  and two dramatically different initial conditions for  $\hat{D}$ :  $\hat{D}(0) = 0$  and  $\hat{D}(0) = \bar{D} = 2D = 2$ . Note that the solid plots in this figure correspond to the same simulation as in Fig. 1.

#### 5. Proof of the main result

In this section, we prove Theorem 1. We start by considering the transformation (38) along with its inverse

$$e(x, t) = w(x, t) + \hat{D}(t) \int_0^x K(\hat{\theta}) e^{(A+BK)(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) \times w(y, t) dy + K(\hat{\theta}) e^{(A+BK)(\hat{\theta})\hat{D}(t)x} \tilde{X}(t). \quad (43)$$

Using these transformations and the models (1) and (18), the transformed system is written as

$$\dot{\tilde{X}}(t) = (A + BK)(\hat{\theta})\tilde{X}(t) + B(\hat{\theta})w(0, t) + A(\tilde{\theta})X(t) + B(\tilde{\theta})u(0, t) - \frac{\partial X^T}{\partial \hat{\theta}}(t, \hat{\theta})\dot{\hat{\theta}}(t) \quad (44)$$

$$Dw_t(x, t) = w_x(x, t) - \tilde{D}(t)p_0(x, t) - D\dot{\hat{D}}(t)q_0(x, t) - D\tilde{\theta}(t)^T p(x, t) - D\dot{\hat{\theta}}(t)^T q(x, t) \quad (45)$$

$$w(1, t) = 0, \quad (46)$$

where  $\tilde{D}(t) = D - \hat{D}(t)$  is the estimation error of the delay, the quantities

$$A(\tilde{\theta}) = \sum_{i=1}^p \tilde{\theta}_i A_i = \sum_{i=1}^p (\theta_i - \hat{\theta}_i(t)) A_i \quad (47)$$

$$B(\tilde{\theta}) = \sum_{i=1}^p \tilde{\theta}_i B_i \quad (48)$$

are linear in the parameter estimation error  $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ , and

$$p_0(x, t) = K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)x} ((A + BK)(\hat{\theta})\tilde{X}(t) + B(\hat{\theta})w(0, t)) \quad (49)$$

$$\begin{aligned} q_0(x, t) &= \int_0^x K(\hat{\theta}) \left( I + A(\hat{\theta})\hat{D}(t)(x-y) \right) e^{A(\hat{\theta})\hat{D}(t)(x-y)} \\ &\quad \times B(\hat{\theta})e(y, t) dy + K(\hat{\theta})A(\hat{\theta})xe^{A(\hat{\theta})\hat{D}(t)x}\tilde{X}(t) \\ &= \int_0^x w(y, t) \left[ K(\hat{\theta}) \left( I + A(\hat{\theta})\hat{D}(t)(x-y) \right) \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) + \hat{D}(t) \int_y^x K(\hat{\theta}) \\ &\quad \times \left( I + A(\hat{\theta})\hat{D}(t)(x-\xi) \right) e^{A(\hat{\theta})\hat{D}(t)(x-\xi)} BK(\hat{\theta}) \\ &\quad \times e^{(A+BK)(\hat{\theta})\hat{D}(t)(\xi-y)} B(\hat{\theta}) d\xi \left. \right] dy + \left[ KA(\hat{\theta})x \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)x} + \int_0^x K(\hat{\theta}) \left( I + A(\hat{\theta})\hat{D}(t)(x-y) \right) \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-y)} BK(\hat{\theta})e^{(A+BK)(\hat{\theta})\hat{D}(t)y} dy \left. \right] \tilde{X}(t). \quad (51) \end{aligned}$$

The vector-valued functions  $q(x, t)$  and  $p(x, t)$  are defined through their coefficients as follows, for  $1 \leq i \leq p$ ,

$$p_i(x, t) = K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)x} (A_i X(t) + B_i u(0, t)) \quad (52)$$

$$\begin{aligned} &= K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)x} ((A_i + B_i K(\hat{\theta}))\tilde{X}(t) + B_i w(0, t) \\ &\quad + A_i X^r(t, \hat{\theta}) + B_i u^r(0, t, \hat{\theta})) \quad (53) \end{aligned}$$

$$\begin{aligned} q_i(x, t) &= \hat{D}(t) \int_0^x \left[ \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)(x-y) \right) \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) + K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-y)} B_i \\ &\quad \times e(y, t) dy + \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)x \right) e^{A(\hat{\theta})\hat{D}(t)x} \\ &\quad \times \tilde{X}(t) - K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)x} \frac{\partial X^r}{\partial \hat{\theta}_i}(t, \hat{\theta}) + \frac{\partial u^r}{\partial \hat{\theta}_i}(x, t, \hat{\theta}) \\ &\quad - \hat{D}(t) \int_0^x K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) \\ &\quad \times \frac{\partial u^r}{\partial \hat{\theta}_i}(y, t, \hat{\theta}) dy \quad (54) \\ &= \hat{D}(t) \int_0^x w(y, t) \left[ \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)(x-y) \right) \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) + K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-y)} B_i \\ &\quad + \hat{D}(t) \int_y^x \left( \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)(x-\xi) \right) \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-\xi)} B(\hat{\theta}) + K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-\xi)} B_i \\ &\quad \times K(\hat{\theta})e^{(A+BK)(\hat{\theta})\hat{D}(t)(\xi-y)} B(\hat{\theta}) d\xi \left. \right] dy \end{aligned}$$

$$\begin{aligned} &+ \left[ \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)x \right) e^{A(\hat{\theta})\hat{D}(t)x} \right. \\ &+ \hat{D}(t) \int_0^x \left[ \left( \frac{\partial K}{\partial \hat{\theta}_i}(\hat{\theta}) + K(\hat{\theta})A_i\hat{D}(t)(x-y) \right) \right. \\ &\quad \times e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) + K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-y)} B_i \\ &\quad \times K(\hat{\theta})e^{(A+BK)(\hat{\theta})\hat{D}(t)y} dy \left. \right] \tilde{X}(t) \\ &- K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)x} \frac{\partial X^r}{\partial \hat{\theta}_i}(t, \hat{\theta}) + \frac{\partial u^r}{\partial \hat{\theta}_i}(x, t, \hat{\theta}) \\ &- \hat{D}(t) \int_0^x K(\hat{\theta})e^{A(\hat{\theta})\hat{D}(t)(x-y)} B(\hat{\theta}) \frac{\partial u^r}{\partial \hat{\theta}_i}(y, t, \hat{\theta}) dy. \quad (55) \end{aligned}$$

Now, we define the following Lyapunov function candidate

$$V(t) = D \log(N(t)) + \frac{b}{\gamma_1} \tilde{D}(t)^2 + \frac{bD}{\gamma_2} \tilde{\theta}(t)^T \tilde{\theta}(t), \quad (56)$$

where

$$N(t) = 1 + \tilde{X}(t)^T P(\hat{\theta})\tilde{X}(t) + b \int_0^1 (1+x)w(x, t)^2 dx. \quad (57)$$

Taking a time derivative of  $V(t)$ , we obtain

$$\begin{aligned} \dot{V}(t) &= -\frac{2b}{\gamma_1} \tilde{D}(t)(\dot{\hat{D}}(t) - \gamma_1 \tau_D(t)) \\ &\quad - \frac{2bD}{\gamma_2} \tilde{\theta}(t)^T (\dot{\hat{\theta}}(t) - \gamma_2 \tau_\theta(t)) \\ &\quad + \frac{D}{N(t)} \left( \sum_{i=1}^p \dot{\hat{\theta}}_i(t) \left( \tilde{X}(t)^T \frac{\partial P}{\partial \hat{\theta}_i}(\hat{\theta})\tilde{X}(t) \right. \right. \\ &\quad \left. \left. - \tilde{X}(t)^T P(\hat{\theta}) \frac{\partial X^r}{\partial \hat{\theta}_i}(t, \hat{\theta}) \right) - \tilde{X}(t)^T Q(\hat{\theta})\tilde{X}(t) \right. \\ &\quad \left. + 2\tilde{X}(t)^T P(\hat{\theta})B(\hat{\theta})w(0, t) - \frac{b}{D} \|w\|^2 - \frac{b}{D} w(0, t)^2 \right. \\ &\quad \left. - 2b\dot{\hat{D}}(t) \int_0^1 (1+x)w(x, t)q_0(x, t) dx \right. \\ &\quad \left. - 2b\dot{\hat{\theta}}(t)^T \int_0^1 (1+x)w(x, t)q(x, t) dx \right), \quad (58) \end{aligned}$$

where we have used an integration by parts. Using the assumptions that  $\hat{D}(0) \in ]0; \bar{D}]$  and  $\hat{\theta}(0) \in \Pi$ , the update laws (32)–(33), and the properties of the projection operator, we get

$$\begin{aligned} \dot{V}(t) &\leq \frac{D}{N(t)} \left( \sum_{i=1}^p \dot{\hat{\theta}}_i(t) \left( \tilde{X}(t)^T \frac{\partial P}{\partial \hat{\theta}_i}(\hat{\theta})\tilde{X}(t) \right. \right. \\ &\quad \left. \left. - \tilde{X}(t)^T P(\hat{\theta}) \frac{\partial X^r}{\partial \hat{\theta}_i}(t, \hat{\theta}) \right) - \lambda_{\min}(Q(\hat{\theta}))|\tilde{X}(t)|^2 \right. \\ &\quad \left. + 2\tilde{X}(t)^T P(\hat{\theta})B(\hat{\theta})w(0, t) - \frac{b}{D} \|w\|^2 - \frac{b}{D} w(0, t)^2 \right. \\ &\quad \left. - 2b\dot{\hat{D}}(t) \int_0^1 (1+x)w(x, t)q_0(x, t) dx \right. \\ &\quad \left. - 2b\dot{\hat{\theta}}(t)^T \int_0^1 (1+x)w(x, t)q(x, t) dx \right), \quad (59) \end{aligned}$$

and, substituting the expressions of (32) and (33) and using (39) with the Young inequality, we obtain

$$\dot{V}(t) \leq -\frac{D}{2N(t)} \left( \lambda_{\min}(Q)|\tilde{X}|^2 + \frac{b}{D} \|w\|^2 + 2\frac{b}{D} w(0, t)^2 \right)$$



$$\begin{aligned}
 &+ 2Db\gamma_1 \frac{\int_0^1 (1+x)|w(x,t)||p_0(x,t)|dx}{N(t)} \\
 &\times \frac{\int_0^1 (1+x)|w(x,t)||q_0(x,t)|dx}{N(t)} \\
 &+ D\gamma_2 \sum_{i=1}^p \left( \frac{\int_0^1 (1+x)|w(x,t)||p_i(x,t)|dx}{N(t)} \right. \\
 &\left. + 2|\tilde{X}(t)^T P(\hat{\theta})/b| |A_i X(t) + B_i u(0,t)| \right) \\
 &\times \frac{1}{N(t)} \left( \left| \tilde{X}(t)^T \frac{\partial P}{\partial \hat{\theta}_i}(\hat{\theta}) \tilde{X}(t) \right| + \left| \tilde{X}(t)^T P(\hat{\theta}) \right. \right. \\
 &\left. \left. \times \frac{\partial X^r}{\partial \hat{\theta}_i}(t, \hat{\theta}) \right| + 2b \int_0^1 (1+x)|w(x,t)||q_i(x,t)|dx \right). \quad (60)
 \end{aligned}$$

Fuhermore, each signal depending on  $\hat{\theta}$ , namely  $A, B, K, P, \partial P/\partial \hat{\theta}_i, \partial X^r/\partial \hat{\theta}_i$  and  $\partial u^r/\partial \hat{\theta}_i$ , is given as continuous in  $\hat{\theta}$ . Since  $\hat{\theta}$  remains in  $\Pi$ , a closed and bounded subset of  $\mathbb{R}^p$ , each signal is bounded in terms of  $\hat{\theta}$  and admits a finite upper bound. We denote  $M_A = \sup_{\hat{\theta} \in \Pi} |A(\hat{\theta})|$  and define  $M_P, M_B, M_K, M_{A+BK}, M_{\partial K/\partial \hat{\theta}}$  similarly. Therefore, using Cauchy–Schwartz and Young inequalities, along with (49), (50), (52) and (54), we get

$$\begin{aligned}
 &\int_0^1 (1+x)|w(x,t)||p_0(x,t)|dx \\
 &\leq M_0(|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2) \quad (61)
 \end{aligned}$$

$$\int_0^1 (1+x)|w(x,t)||q_0(x,t)|dx \leq M_0(|\tilde{X}(t)|^2 + \|w(t)\|^2) \quad (62)$$

$$\begin{aligned}
 &\times \left( \int_0^1 (1+x)|w(x,t)||p_i(x,t)|dx + 2|\tilde{X}(t)^T P(\hat{\theta})/b| \right. \\
 &\left. \times |A_i X(t) + B_i u(0,t)| \right) \\
 &\leq M_i(|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2 + \|w(t)\|) \quad (63)
 \end{aligned}$$

$$\begin{aligned}
 &\times \int_0^1 (1+x)|w(x,t)||q_i(x,t)|dx \\
 &\leq M_i(|\tilde{X}(t)|^2 + \|w(t)\|^2 + \|w(t)\|), \quad (64)
 \end{aligned}$$

where  $M_0$  and  $M_i$  ( $1 \leq i \leq p$ ) are sufficiently large constants given by

$$\begin{aligned}
 M_0 &= M_K \max \{M_{A+BK} + M_A, 2M_K((1 + M_A\bar{D}) \\
 &\times (M_B + M_B M_K(1 + \bar{D})) + M_A)\} e^{(M_A + M_{A+BK})\bar{D}} \quad (65)
 \end{aligned}$$

$$\begin{aligned}
 M_i &= \max \{|A_i| + |B_i| M_K + |B_i| + 2M_P/b, \\
 &2 \sup_{(t, \hat{\theta}) \in \mathbb{R} \times \Pi} (|A_i| |X^r(t, \hat{\theta})| + |B_i| |u^r(0, t, \hat{\theta})|), \\
 &2M_K \sup_{(t, \hat{\theta}) \in \mathbb{R} \times \Pi} \left| \frac{\partial X^r}{\partial \hat{\theta}}(t, \hat{\theta}) \right| + 2 \sup_{(t, \hat{\theta}) \in \mathbb{R} \times \Pi} \left| \frac{\partial u^r}{\partial \hat{\theta}}(t, \hat{\theta}) \right| \\
 &\times (1 + \hat{D} M_K M_B), \left( (M_{\partial K/\partial \hat{\theta}} + M_K |A_i| \bar{D}) M_B + |A_i| M_K \right) \\
 &\times (2\bar{D} + 2\bar{D} M_K M_B + M_K)\} e^{(M_A + M_{A+BK})\bar{D}}. \quad (66)
 \end{aligned}$$

Consequently, if we define

$$M'_p = \max_{1 \leq i \leq p} \sup_{\hat{\theta} \in \Pi} \left| \frac{\partial P(\hat{\theta})}{\partial \hat{\theta}_i} \right| \quad (67)$$

$$M_r = \max_{1 \leq i \leq p} \sup_{\hat{\theta} \in \Pi, t \geq 0} \left| \frac{\partial X^r(t, \hat{\theta})}{\partial \hat{\theta}_i} \right| \quad (68)$$

using (61)–(64) in (60), we obtain

$$\begin{aligned}
 \dot{V}(t) &\leq -\frac{D}{2N(t)} \left( \lambda_{\min}(Q) |\tilde{X}|^2 + \frac{b}{D} \|w\|^2 + 2\frac{b}{D} w(0,t)^2 \right) \\
 &+ 2Db\gamma_1 M_0^2 \frac{|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2}{N(t)} \\
 &\times \frac{|\tilde{X}(t)|^2 + \|w(t)\|^2}{N(t)} + D\gamma_2 \sum_{i=1}^p \frac{M_i}{N(t)} \left( |\tilde{X}(t)|^2 + \|w(t)\|^2 \right. \\
 &\left. + w(0,t)^2 + \|w(t)\| \right) \frac{1}{N(t)} \left( M'_p |\tilde{X}(t)|^2 + M_r |\bar{P}| |\tilde{X}(t)| \right. \\
 &\left. + 2bM_i (|\tilde{X}(t)|^2 + \|w(t)\|^2 + \|w(t)\|) \right). \quad (69)
 \end{aligned}$$

Bounding the cubic and quadric terms with the help of  $N(t)$ , we arrive at

$$\begin{aligned}
 \dot{V}(t) &\leq -\frac{D}{2N(t)} \left( \underline{\lambda} |\tilde{X}(t)|^2 + \frac{b}{D} \|w(t)\|^2 + 2\frac{b}{D} w(0,t)^2 \right) \\
 &+ \frac{2Db\gamma_1 M_0^2}{\min\{\underline{\lambda}, b\}} \frac{|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2}{N(t)} \\
 &+ D\gamma_2 \sum_{i=1}^p M_i \left( M'_p \left( \frac{1}{\underline{\lambda}} + \frac{1}{2 \min\{1, b\}} \right) \right. \\
 &\left. + M_p M_r \left( \frac{1}{2} + \frac{1}{2 \min\{1, \underline{\lambda}\}} \right) \right. \\
 &\left. + 2bM_i \left( \frac{1}{\min\{\underline{\lambda}, b\}} + \frac{1}{2 \min\{1, b\}} + 1 \right) \right) \\
 &\times \frac{|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2}{N(t)}. \quad (70)
 \end{aligned}$$

Defining the following constant,

$$m = \frac{2 \max \left\{ bM_0^2, \sum_{i=1}^p M_i (M'_p + M_p M_r + 3bM_i) \right\}}{\min\{1, \underline{\lambda}, b\}} \quad (71)$$

we finally obtain

$$\begin{aligned}
 \dot{V}(t) &\leq -\frac{D}{2N(t)} \left( \min \left\{ \underline{\lambda}, \frac{b}{D} \right\} - 2(\gamma_1 + \gamma_2)m \right) \\
 &\times (|\tilde{X}(t)|^2 + \|w(t)\|^2 + w(0,t)^2). \quad (72)
 \end{aligned}$$

Consequently, by choosing

$$\gamma^* = \frac{\min\{\underline{\lambda}, b/D\}}{4bm} \quad (73)$$

and  $(\gamma_1, \gamma_2) \in [0; \gamma^*]^2$ , we make  $\dot{V}(t)$  negative semidefinite and hence

$$V(t) \leq V(0), \quad \forall t \geq 0. \quad (74)$$

Starting from this result, we now prove the results stated in Theorem 1. From the transformation (38) and its inverse (43), we obtain these two inequalities

$$\|w(t)\|^2 \leq r_1 \|e(t)\|^2 + r_2 |\tilde{X}(t)|^2 \quad (75)$$

$$\|e(t)\|^2 \leq s_1 \|w(t)\|^2 + s_2 |\tilde{X}(t)|^2, \quad (76)$$

where  $r_1, r_2, s_1, s_2$  are sufficiently large positive constants given by

$$r_1 = 3 \left( 1 + \bar{D}^2 M_K^2 e^{2M_{A+BK}\bar{D}} M_B^2 \right) \quad (77)$$

$$r_2 = 3M_K^2 e^{2M_{A+BK}\bar{D}} \quad (78)$$

$$s_1 = 3 \left( 1 + \bar{D}^2 M_K^2 e^{2M_A \bar{D}} M_B^2 \right) \quad (79)$$

$$s_2 = 3M_K^2 e^{2M_A \bar{D}}. \quad (80)$$

Furthermore, from (56) and (76), it follows that

$$\bar{D}(t)^2 + \tilde{\theta}(t)^T \tilde{\theta}(t) \leq \frac{\gamma_1 + \gamma_2}{b} V(t) \quad (81)$$

$$\|\tilde{X}(t)\|^2 \leq \frac{1}{\underline{\lambda}} (e^{V(t)/D} - 1) \quad (82)$$

$$\|e(t)\| \leq \frac{s_1}{b} (e^{V(t)/D} - 1) + s_2 \|\tilde{X}(t)\|. \quad (83)$$

Thus, from the definition of  $\mathcal{V}(t)$ , it is easy to show that

$$\mathcal{V}(t) \leq \left( \frac{1+s_2}{\underline{\lambda}} + \frac{s_1}{b} + \frac{(\gamma_1 + \gamma_2)D}{b} \right) (e^{V(t)/D} - 1). \quad (84)$$

Besides, using (75), we also obtain

$$V(0) \leq \left( D(\bar{\lambda} + s_2 b + 2s_1 b) + b \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right) \right) \mathcal{V}(0). \quad (85)$$

Finally, if we define

$$R = \frac{1+s_2}{\underline{\lambda}} + \frac{s_1}{b} + \frac{(\gamma_1 + \gamma_2)D}{b} \quad (86)$$

$$\rho = \bar{\lambda} + s_2 b + 2s_1 b + \frac{b}{D} \left( \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right), \quad (87)$$

we obtain the global stability result given in Theorem 1.

We now prove tracking. From (74), we obtain the uniform boundedness of  $\|\tilde{X}(t)\|$ ,  $\|w(t)\|$ ,  $\hat{D}(t)$  and  $\|\hat{\theta}(t)\|$ . From (43), we obtain that  $\|e(t)\|$  is also uniformly bounded in time. From (31), we get the uniform boundedness of  $U(t)$  and consequently of  $\tilde{U}(t)$  for  $t \geq 0$ . Thus, we get that  $u(0, t)$  and  $e(0, t)$  are uniformly bounded for  $t \geq D$ . Besides, from (33) and (52), we obtain the uniform boundedness of  $\|\dot{\hat{\theta}}(t)\|$  for  $t \geq D$ . Finally, with (44), we obtain that  $d\tilde{X}(t)^2/dt$  is uniformly bounded for  $t \geq D$ . As  $|\tilde{X}(t)|$  is square integrable, from (72), we conclude from Barbalat's lemma that  $\tilde{X}(t) \rightarrow 0$  when  $t \rightarrow \infty$ .

Besides, from (72), we get the square integrability of  $\|w(t)\|$ . From (76), we obtain the square integrability of  $\|e(t)\|$ . Consequently, with (31), we obtain the square integrability of  $\tilde{U}(t)$ . Furthermore,

$$\begin{aligned} \frac{d\tilde{U}(t)^2}{dt} &= 2\tilde{U}(t) \left( K(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)} \dot{\tilde{X}}(t) + \dot{\hat{D}}(t) G_0(t) \right. \\ &\quad \left. + \sum_{i=1}^p \dot{\hat{\theta}}_i(t) G_i(t) + \frac{\hat{D}}{D} H(t) \right) \end{aligned} \quad (88)$$

with

$$\begin{aligned} G_0(t) &= K(\hat{\theta}) \left[ A(\hat{\theta}) e^{A(\hat{\theta})\hat{D}(t)} \tilde{X}(t) + \int_0^1 (I + A(\hat{\theta})\hat{D}(t)) \right. \\ &\quad \left. \times (1-y) e^{A(\hat{\theta})\hat{D}(t)(1-y)} B(\hat{\theta}) e(y, t) dy \right] \end{aligned} \quad (89)$$

$$\begin{aligned} G_i(t) &= \frac{\partial K}{\partial \hat{\theta}}(\hat{\theta}) \left[ e^{A(\hat{\theta})\hat{D}(t)} \tilde{X}(t) + \hat{D}(t) \int_0^1 e^{A(\hat{\theta})\hat{D}(t)(1-y)} \right. \\ &\quad \left. \times B(\hat{\theta}) e(y, t) dy \right] + K(\hat{\theta}) \left[ A_i \hat{D}(t) e^{A(\hat{\theta})\hat{D}(t)} \tilde{X}(t) \right. \\ &\quad \left. + \hat{D}(t) \int_0^1 [A_i \hat{D}(t)(1-y) e^{A(\hat{\theta})\hat{D}(t)(1-y)} B(\hat{\theta}) \right. \end{aligned}$$

$$\left. + e^{A(\hat{\theta})\hat{D}(t)(1-y)} B_i \right] e(y, t) dy \quad (90)$$

$$\begin{aligned} H(t) &= K(\hat{\theta}) \left[ B(\hat{\theta}) \tilde{U}(t) - e^{A(\hat{\theta})\hat{D}(t)} B(\hat{\theta}) e(0, t) \right. \\ &\quad \left. + \int_0^1 A(\hat{\theta}) \hat{D}(t) e^{A(\hat{\theta})\hat{D}(t)(1-y)} B(\hat{\theta}) e(y, t) dy \right]. \end{aligned} \quad (91)$$

The signals  $\dot{\hat{D}}, \dot{\hat{\theta}}_1, \dots, \dot{\hat{\theta}}_p$  are uniformly bounded over  $t \geq 0$ , according to (32)–(33). By using the uniform boundedness of  $\tilde{X}(t), \tilde{X}'(t), \|e(t)\|, \tilde{U}(t)$  over  $t \geq 0$  and of  $e(0, t)$  for  $t \geq D$  and the uniform boundedness of all the signals which are functions of  $\hat{\theta}$  for  $t \geq 0$ , we obtain the uniform boundedness of  $d\tilde{U}(t)^2/dt$  over  $t \geq D$ . Then, with Barbalat's lemma, we conclude that  $\tilde{U}(t) \rightarrow 0$  when  $t \rightarrow \infty$ .

## 6. Conclusions

In this paper we presented a result on global adaptive tracking for ODEs with unknown parameters and long unknown actuator delays, with full state feedback. The control design employs predictor feedback, modified from stabilization to tracking, whereas adaptation employs our relatively novel Lyapunov update law design with normalization. A simulation example with a second order linear plant illustrates the effectiveness of the design.

Future work should focus on systems with simultaneous state and input delay. The non-adaptive design (Jankovic, 2008) may be a good starting point for an interesting class of systems, and the clf tools (Karafyllis & Jiang, 2008) for systems with state delays may be instrumental.

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