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Brief paper

On using least-squares updates without regressor filtering in identification and adaptive control of nonlinear systems☆,☆☆

Miroslav Krstic *

Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

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ABSTRACT

In continuous-time system identification and adaptive control the least-squares parameter estimation algorithm has always been used with regressor filtering, which adds to the dynamic order of the identifier and affects its performance. We present an approach for designing a least-squares estimator that uses an unfiltered regressor. We also consider a problem of adaptive nonlinear control and present the first least-squares-based adaptive nonlinear control design that yields a complete Lyapunov function. The design is presented for linearly parametrized nonlinear control systems in 'normal form'. A scalar linear example is included which adds insight into the key ideas of our approach and allows showing that, for linear systems, our Lyapunov-LS design with unfiltered regressor, presented in the note for unnormalized least-squares, can also be extended to normalized least-squares.

1. Introduction

Least-squares update laws, with update normalization, are common in adaptive control of linear systems (Ioannou & Fidan, 2006; Ioannou & Sun, 1996; Tao, 2003). However, in adaptive control of *nonlinear* systems, least-squares estimation has received very little attention. The notable results in discrete time are (Guo, 1997; Kanellakopoulos, 1994), whereas in continuous time no designs employing least-squares adaptation have appeared since the early 1990s' 'heyday' of adaptive nonlinear control (Krstic, Kanellakopoulos, & Kokotovic, 1995; Krstic & Kokotovic, 1995, 1996; Praly, Bastin, Pomet, & Jiang, 1991).

The appeal of least-squares adaptation is that it has the capability of automatically adjusting the adaptation gain matrix (via a gain subsystem governed by a Riccati equation). The gain decays rapidly in regressor channels with a strong signal, whereas they do not decay as fast, or may even grow, in channels where the signal is weak. Simply put, least-squares almost 'magically' adjusts the adaptation rate so that all the parameter estimates converge with approximately the same speed. This results in performance and robustness advantages which have been well studied, see, for example (Berghuis, Roebbers, & Nijmeijer, 1995) where this advantage is elucidated through tests on a robotics experiment (the increased computational burden of the least-squares method is also acknowledged).

The ability of least-squares to even out the adaptation rates of different parameter vector components is not achievable with gradient, passivity-based, Lyapunov-based, or any other types of update laws, where the adaptation gains are constant—set by the designer at the beginning of the estimation experiment, and prone to a very poor guess on the part of the designer as to what the size of the signal transients in different regressor channels might be. This is particularly difficult in adaptive control applications, where the transients are virtually impossible to predict before closing the loop.

In all continuous-time applications of least-squares in adaptive control, linear (Ioannou & Fidan, 2006; Ioannou & Sun, 1996; Tao, 2003) and nonlinear (Krstic et al., 1995; Krstic & Kokotovic, 1995, 1996; Praly et al., 1991), the parametric model is first filtered to remove the time derivatives. This makes the dynamic order of adaptive controllers with least-squares estimators very high. The filtering may also make the estimators less responsive, and thus affect the performance.

In this paper we pursue an idea with which we absorb the time derivatives from the parametric model into the parameter estimate, thus removing the need for filtering. This leads to very interesting design modifications in both identification and adaptive control.

Our approach extends the ideas developed in the *immersion and invariance* approach in Astolfi and Ortega (2003), Astolfi, Karagiannis, and Ortega (2007) and Karagiannis and Astolfi (2008), where the parameter estimator employs not only an integrator but also a 'throughput' term, and where the identifier-controller design

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* Tel.: +1 858 822 1374; fax: +1 858 822 3107.
E-mail address: krstic@ucsd.edu.

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is 'modular'. We pursue least-squares adaptation which gives rise to new technical issues.

The note is organized as follows. In Section 2 we present identifier- and adaptive-controller designs with unnormalized least-squares adaptation and without regressor filtering, for a linearly parametrized nonlinear control system in 'normal form'. We do not know at this point if it is possible to extend this result to the much broader class of *strict-feedback* systems without employing overparametrization, as was done in Karagiannis and Astolfi (2008). Overparametrization in the context of least-squares adaptation would be unacceptable, as it would have to employ multiple Riccati differential equations.

In Section 3 we consider a linear scalar example, which serves two purposes, it presents the essence of the new algorithm on a notationally easy example, and it allows us to present an idea for extending the method from unnormalized least-squares algorithms to normalized least-squares (which is feasible for linear systems). Finally, in Section 4, we present a short parameter estimation example, without a control input. This example is a Van der Pol oscillator, with three unknown parameters. This example lets the reader see what the key issues are in designing a least-squares estimator for a multi-state, multi-parameter problem on an example that is easy to follow.

2. Design for systems in normal form

Consider the nonlinear parametrically uncertain system in 'normal form',

$$\dot{x}_i = x_{i+1}, \quad i = 1, \ldots, n-1 \tag{1}$$

$$\dot{x}_n = u + \phi(x) + \varphi(x)^{\mathrm{T}}\theta, \tag{2}$$

where $\theta \in \mathbb{R}^p$, $u \in \mathbb{R}$, $\phi : \mathbb{R}^n \to \mathbb{R}$, $\varphi : \mathbb{R}^n \to \mathbb{R}^p$, $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$, and ϕ, φ are locally Lipschitz. We assume that the full state x is available for measurement.

2.1. Parameter estimation

For the system (1), (2) we design a parameter estimator in the form

$$\begin{aligned}\dot{\alpha} &= -\Gamma\varphi(x)\varphi(x)^{\mathrm{T}}\alpha \\ &\quad - \Gamma\sum_{i=1}^{n-1} x_{i+1}\int_0^{x_n}\frac{\partial\varphi(x_1, \ldots, x_{n-1}, \sigma)}{\partial x_i}\mathrm{d}\sigma \\ &\quad - \Gamma\varphi(x)\left(u + \phi(x)\right)\end{aligned} \tag{3}$$

$$\dot{\Gamma} = -\Gamma\varphi(x)\varphi(x)^{\mathrm{T}}\Gamma, \qquad \Gamma(0) = \Gamma(0)^{\mathrm{T}} > 0 \tag{4}$$

$$\hat{\theta} = \alpha + \Gamma\int_0^{x_n}\varphi(x_1, \ldots, x_{n-1}, \sigma)\mathrm{d}\sigma. \tag{5}$$

Note that the estimate $\hat{\theta}(t)$ of the unknown θ is generated from another system of the same dimension, $\alpha(t)$, which is paired with a standard least-squares style Riccati equation (4).

We denote the parameter estimation error as $\tilde{\theta} = \theta - \hat{\theta}$ and state a stability/boundedness result for the identifier.

Lemma 1. *Let the maximal interval of existence of solutions of the system (1), (2) be $[0, t_f)$. Then with any $\theta \in \mathbb{R}^p$, any $\alpha(0) \in \mathbb{R}^p$, and any $p \times p$ matrix $\Gamma(0) = \Gamma(0)^{\mathrm{T}} > 0$, the functions $\left(\hat{\theta}(t), \Gamma(t)\right)$ generated by the system (3), (4), (5) are bounded and $\varphi(x(t))^{\mathrm{T}}\tilde{\theta}(t)$ is square integrable, with the bounds independent of t_f.*

Proof. From (3) and (5) we first get

$$\begin{aligned}\dot{\alpha} &= \Gamma\varphi(x)\varphi(x)^{\mathrm{T}}\Gamma\int_0^{x_n}\varphi(x_1, \ldots, x_{n-1}, \sigma)\mathrm{d}\sigma \\ &\quad - \Gamma\sum_{i=1}^{n-1} x_{i+1}\int_0^{x_n}\frac{\partial\varphi(x_1, \ldots, x_{n-1}, \sigma)}{\partial x_i}\mathrm{d}\sigma \\ &\quad - \Gamma\varphi(x)\left(u + \phi(x) + \varphi(x)^{\mathrm{T}}\hat{\theta}\right).\end{aligned} \tag{6}$$

Then, taking a derivative of (5), and with the help of (4), we obtain

$$\dot{\hat{\theta}} = \Gamma\varphi(x)\left(\dot{x}_n - u - \phi(x) - \varphi(x)^{\mathrm{T}}\hat{\theta}\right). \tag{7}$$

From (7), using (2), we get

$$\dot{\tilde{\theta}} = -\Gamma\varphi(x)\varphi(x)^{\mathrm{T}}\tilde{\theta}. \tag{8}$$

Consider now the Lyapunov function $V_\theta = \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}$. A straightforward calculation yields

$$\dot{V}_\theta = -\left(\varphi(x)^{\mathrm{T}}\tilde{\theta}\right)^2. \tag{9}$$

By integrating this equation, the square integrability result follows. As in the proofs of Krstic et al. (1995, Lemmas 6.1 and 6.5), we can observe that $0 \le \Gamma(t) \le \Gamma(0), \forall t \ge 0$. Hence, $\left|\tilde{\theta}(t)\right| \le \left|\tilde{\theta}(0)\right| \lambda_{\max}(\Gamma(0))/\lambda_{\min}(\Gamma(0))$ for all $t \ge 0$, which establishes the boundedness result. □

Remark 2. Similar to Karagiannis and Astolfi (2008), this least-squares-based identifier absorbs the last term in (5) into a new variable α. This is related to the appearance of the derivative term $\dot{x}_n$ in (7). The estimator (7) would not be implementable but (3), (5) is.

2.2. Adaptive control

Let $y = x_1$ be the system output and let the function $y_r(t)$ be the reference trajectory which is n times differentiable. Denote $x_r(t) = \left[y_r(r), \dot{y}_r(t), \ldots, y_r^{(n-1)}(t)\right]^{\mathrm{T}}$. In this section we design an adaptive control law for achieving asymptotic tracking.

Let $0_{n-1} = [0, \ldots, 0]^{\mathrm{T}} \in \mathbb{R}^{n-1}$ and $\mathrm{e}_n = [0_{n-1}^{\mathrm{T}}, 1]^{\mathrm{T}} \in \mathbb{R}^n$. Denote $A = \begin{bmatrix} 0_{n-1} & I_{n-1} \\ 0 & 0_{n-1}^{\mathrm{T}} \end{bmatrix}$. Let $K = [k_1, \ldots, k_n] \in \mathbb{R}^n$ be chosen such that the polynomial $s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$ is Hurwitz. For any positive definite and symmetric matrix Q there exists a positive definite and symmetric matrix P such that

$$P\left(A + \mathrm{e}_n K\right) + \left(A + \mathrm{e}_n K\right)^{\mathrm{T}} P = -Q. \tag{10}$$

We are now ready to state our control law,

$$u = y_r^{(n)} + \left(K - \frac{\lambda}{2}\mathrm{e}_n^{\mathrm{T}} P\right)\tilde{x} - \phi(x) - \varphi(x)^{\mathrm{T}}\hat{\theta}, \tag{11}$$

where $\tilde{x} = x - x_r(t)$, and $\lambda > 0$, and to prove the closed-loop stability.

Theorem 3. *Consider the closed-loop system consisting of the plant (1)–(5) and the controller (11). The equilibrium $\tilde{x} = 0, \tilde{\theta} = 0$ is globally stable. Furthermore $\lim_{t\to\infty}\tilde{x}(t) = 0$, which implies, in particular, that asymptotic tracking is achieved.*

Proof. We employ the simple Lyapunov function

$$V = \lambda \tilde{x}^{\mathrm{T}} P \tilde{x} + \tilde{\theta}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta} \tag{12}$$

and show that its derivative is

$$\dot{V} = -\lambda \tilde{x}^{\mathrm{T}} Q \tilde{x} - \left(\varphi(x)^{\mathrm{T}} \tilde{\theta} - \lambda \mathrm{e}_n^{\mathrm{T}} P \tilde{x}\right)^2 . \tag{13}$$

This establishes global stability and regulation of $\tilde{x}(t)$. □

Remark 4. The incorporation of the term $-\frac{\lambda}{2}\mathrm{e}_n^{\mathrm{T}} P \tilde{x}$ in the control law allows constructing a complete Lyapunov function (12) incorporating both the Lyapunov function V_θ of the least-squares-based identifier and the Lyapunov function of the nonadaptive problem, $\tilde{x}^{\mathrm{T}} P \tilde{x}$.

Another form of the control law and the parameter update law,

$$\begin{aligned} u \;=\; & y_r^{(n)} + \left(K - \frac{\lambda}{2}\mathrm{e}_n^{\mathrm{T}} P\right)\tilde{x} - \phi(x) - \varphi(x)^{\mathrm{T}}\alpha \\ & - \varphi(x)^{\mathrm{T}} \Gamma \int_0^{x_n} \varphi(x_1, \ldots, x_{n-1}, \sigma)\mathrm{d}\sigma \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{\alpha} \;=\; & \Gamma \varphi(x)\varphi(x)^{\mathrm{T}} \Gamma \int_0^{x_n} \varphi(x_1, \ldots, x_{n-1}, \sigma)\mathrm{d}\sigma \\ & - \Gamma \sum_{i=1}^{n-1} x_{i+1} \int_0^{x_n} \frac{\partial \varphi(x_1, \ldots, x_{n-1}, \sigma)}{\partial x_i}\mathrm{d}\sigma \\ & - \Gamma \varphi(x)\left(K - \frac{\lambda}{2}\mathrm{e}_n^{\mathrm{T}} P\right)\tilde{x}, \end{aligned} \tag{15}$$

allows us to observe the presence of an extra nonlinear term in the control law, and to observe that, in the presence of the adaptive controller, the update law is driven by x only.

3. Scalar example with an extension to unnormalized least-squares

The general result in Section 2 is rather complicated. It is easy to miss the main idea due to the notational overhead. In this section we consider a scalar linear example,

$$\dot{x} = u + \theta x, \tag{16}$$

which we work out in detail.

We also take the opportunity in this section to present another variant of the design, where a *normalized* version of the least-squares algorithm is employed. This extension is not limited to the scalar case, but it is limited to the *linear* case.

Before we start, we also acknowledge that the benefits of the least-squares algorithm (over gradient or other algorithms, such as passivity-based or Lyapunov-based) arise only when the dimension of the parameter vector is two or higher. Hence, we would not necessarily advocate the present approach (over the simplest Lyapunov-based approach) for the one-parameter example (16). The value of the example here is mainly educational, as it clarifies the ideas behind the least-squares design without regressor filtering, and also with normalization. (The real benefit of the least-squares design would be evident in the example $\dot{x} = u + \theta x + d$, where both θ and d are unknown constants, i.e., when the system has an unknown parameter and an unknown constant disturbance, however, we stick with the one-parameter example in this section to preserve simplicity.)

We design the parameter estimator given as

$$\dot{\alpha} = -\gamma \frac{x^2}{1+x^2}\alpha - \gamma \frac{ux}{1+x^2} \tag{17}$$

$$\dot{\gamma} = -\gamma^2 \frac{x^2}{1+x^2}, \quad \gamma(0) > 0 \tag{18}$$

$$\hat{\theta} = \alpha + \frac{\gamma}{2}\ln\left(1+x^2\right). \tag{19}$$

We stress the presence of normalization in (17) and (18), as well as the presence of the logarithmic 'throughput' term in (19).

Lemma 5. *Given the system (16) with a bounded input $u(t)$ and with any $\theta \in \mathbb{R}$, any $\alpha(0)$, and any $\gamma(0) > 0$, the solutions $(\alpha(t), \gamma(t))$ of (17), (18) are uniformly bounded and the quantities $\mathrm{d}\hat{\theta}(t)/\mathrm{d}t$ and $x(t)\tilde{\theta}(t)/\sqrt{1+x(t)^2}$ are uniformly bounded and square integrable over $t \in [0, \infty)$. Furthermore, $\mathrm{d}\hat{\theta}(t)/\mathrm{d}t$ is absolutely integrable and $\lim_{t\to\infty}\hat{\theta}(t)$ exists.*

Proof. The proof begins by deriving the following sequence of update law representations, similar to the proof of Lemma 1:

$$\dot{\alpha} = \frac{\gamma^2}{2}\frac{x^2}{1+x^2}\ln\left(1+x^2\right) - \gamma \frac{\left(u+\hat{\theta}x\right)x}{1+x^2} \tag{20}$$

$$\dot{\hat{\theta}} = \gamma \frac{\left(\dot{x} - u - \hat{\theta}x\right)x}{1+x^2} \tag{21}$$

$$\dot{\tilde{\theta}} = -\gamma \frac{x^2}{1+x^2}\tilde{\theta}. \tag{22}$$

Then, for the Lyapunov function $V_\theta = \tilde{\theta}^2/\gamma$, we show that $\dot{V}_\theta = -\left(x\tilde{\theta}\right)^2/(1+x^2)$. The results of the lemma follow using arguments similar to those in the proofs of Krstic et al. (1995, Lemmas 6.1 and 6.5). □

We are now ready to state our control law,

$$u = -\left(1+\hat{\theta}\right)x \tag{23}$$

(which preserves the simplicity of the certainty-equivalence control for our simple example (16)) and to prove closed-loop stability.

Theorem 6. *Consider the closed-loop system consisting of (16)–(19), (23). The equilibrium $x = \tilde{\theta} = 0$ is globally stable and $\lim_{t\to\infty} x(t) = 0$.*

Proof. Consider the Lyapunov function

$$V = \ln\left(1+x^2\right) + \frac{\tilde{\theta}^2}{\gamma}. \tag{24}$$

Lyapunov functions of logarithmic form are common in discrete time adaptive systems Johansson (1983, 1989, 1995) but not in continuous-time adaptive control. The time derivative of the Lyapunov function (24) is readily shown to be

$$\dot{V} = -\left(1+\left(\tilde{\theta}-1\right)^2\right)\frac{x^2}{1+x^2}. \tag{25}$$

The regulation result follows immediately. For the proof of global stability, we first obtain

$$\ln\left(1+x(t)^2\right)^{\gamma(0)} + \tilde{\theta}(t)^2 \le \ln\left(1+x(0)^2\right)^{\gamma(0)} + \tilde{\theta}(0)^2 \tag{26}$$

and then a more conservative class-K_∞ estimate

$$v(t) \le \mathrm{e}^{h(\gamma(0))v(0)} - 1, \tag{27}$$

where $v = x^2 + \tilde{\theta}^2$ and the function $h(\cdot)$ is defined on $\mathbb{R}_+$ as

$$h(a) = \begin{cases} \frac{1}{a}, & 0 < a \le 1 \\ a, & a \ge 1. \end{cases} \tag{28}$$

□

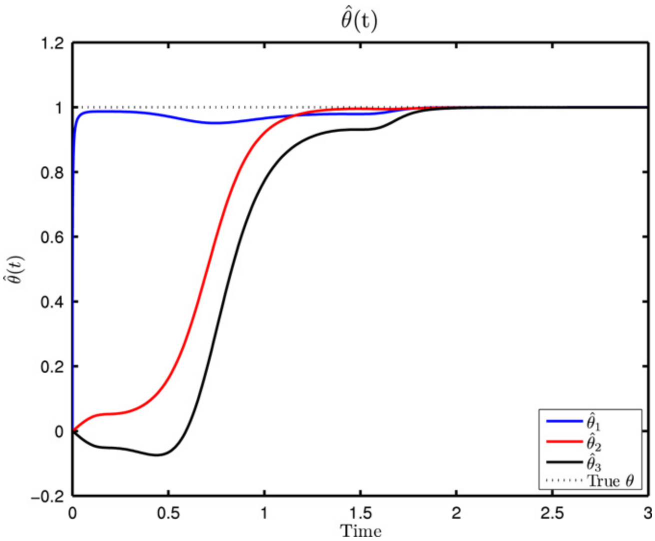

Fig. 1. Parameter estimates governed by (36)–(38). All the three true parameters are equal, $\theta_1 = \theta_2 = \theta_3 = 1$, and so are all the three estimates, $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$. Perfect convergence is achieved in less than 2 s, which is less then one third of a period of oscillation of $x(t)$.

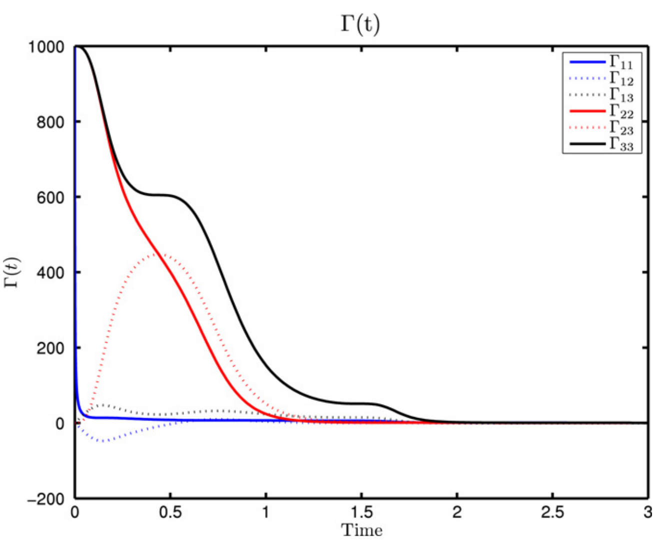

Fig. 2. The six components of the matrix $\Gamma(t)$, starting from initial condition $\Gamma(0) = \mathrm{diag}\{1000, 1000, 1000\}$. They decay to zero in the same amount of time as the estimates converge to the true parameter values.

We summarize the complete adaptive compensator in the most explicit and self-contained form (with $\hat{\theta}$ eliminated):

$$u = -\left(1 + \frac{\gamma}{2}\ln\left(1+x^2\right) + \alpha\right)x \tag{29}$$

$$\dot{\alpha} = \gamma\frac{x^2}{1+x^2}\left(1 + \frac{\gamma}{2}\ln\left(1+x^2\right)\right) \tag{30}$$

$$\dot{\gamma} = -\gamma^2\frac{x^2}{1+x^2}. \tag{31}$$

4. Parameter estimation for a Van der Pol system

We consider the Van der Pol oscillator example,

$$\ddot{x} + \left(\theta_3 x^2 - \theta_2\right)\dot{x} + \theta_1 x = 0, \tag{32}$$

where we denote $\theta = [\theta_1, \theta_2, \theta_3]^{\mathrm{T}}$ and assume that

$$\theta_1, \theta_3 > 0 \tag{33}$$

to ensure boundedness of solutions. This system has a globally attractive limit cycle.

The state space model of this system is

$$\dot{x}_1 = x_2 \tag{34}$$

$$\dot{x}_2 = \left[-x_1, x_2, -x_1^2 x_2\right]\theta. \tag{35}$$

Following the construction in Section 2.1, the parameter estimator is given by

$$\dot{\alpha} = -\Gamma\begin{bmatrix}-x_1\\ x_2\\ -x_1^2x_2\end{bmatrix}\left[-x_1, x_2, -x_1^2x_2\right]\alpha + \Gamma\begin{bmatrix}x_2^2\\ 0\\ x_1x_2^3\end{bmatrix} \tag{36}$$

$$\dot{\Gamma} = -\Gamma\begin{bmatrix}-x_1\\ x_2\\ -x_1^2x_2\end{bmatrix}\left[-x_1, x_2, -x_1^2x_2\right]\Gamma \tag{37}$$

$$\hat{\theta} = \alpha + \Gamma\begin{bmatrix}-x_1x_2\\ \frac{1}{2}x_2^2\\ -\frac{1}{2}x_1^2x_2^2\end{bmatrix}. \tag{38}$$

Lemma 7. *Given the system (32) under the assumption (33) and with any $\theta_2 \in \mathbb{R}$, any $\alpha(0) \in \mathbb{R}^3$, and any 3×3 matrix $\Gamma(0) = \Gamma(0)^{\mathrm{T}} > 0$, the solutions $(\alpha(t), \Gamma(t))$ of (36), (37) are uniformly bounded and the quantities $\mathrm{d}\hat{\theta}(t)/\mathrm{d}t$ and $-x_2(t)\tilde{\theta}_1(t) + x_2(t)\tilde{\theta}_2(t) - x_1(t)x_2(t)^2\tilde{\theta}_3(t)$ are uniformly bounded and square integrable over $t \in [0, \infty)$. Furthermore, $\mathrm{d}\hat{\theta}(t)/\mathrm{d}t$ is absolutely integrable and $\lim_{t\to\infty}\hat{\theta}(t)$ exists.*

Proof. The proof uses the Lyapunov function $V_\theta = \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}$, the boundedness of $(x_1(t), x_2(t))$, and employs the standard arguments as in Krstic et al. (1995); Praly et al. (1991). □

Figs. 1–4 show the simulation results for the estimator (36)–(38). These simulations show excellent convergence, to the true parameter values. Persistency of excitation is ensured due to the fact that the plant is in a limit cycle, thus containing more than one spectral component in the solution, and due to the fact that the parametrization involves only three unknown parameters.

Fig. 2 displays the property that we discuss in the second paragraph of Introduction, namely, that the least-squares estimator automatically adjusts the *relative* values of the entries in the adaptation gain matrix, to balance out the convergence rates of the different entries of the parameter vectors, even in the presence of vastly different signals strengths in the regressor vector (please observe the growth in the cross terms of the adaptation matrix $\Gamma(t)$, particularly of $\Gamma_{23}(t)$; this balances out the convergence rates of $\hat{\theta}_2(t)$ and $\hat{\theta}_3(t)$). All the entries of $\Gamma(t)$ eventually converge to zero, as is always the case with the standard LS algorithm in the presence of persistent excitation. A standard remedy for this issue, if perceived as undesirable (when the true parameters are expected to change with time) is the 'forgetting factor' or 'covariance resetting'.

5. Conclusions

We have introduced an approach for designing least-squares parameter estimators in continuous time which do not require regressor filtering. We have also presented the first least-squares-based adaptive control design that results in a complete Lyapunov function.

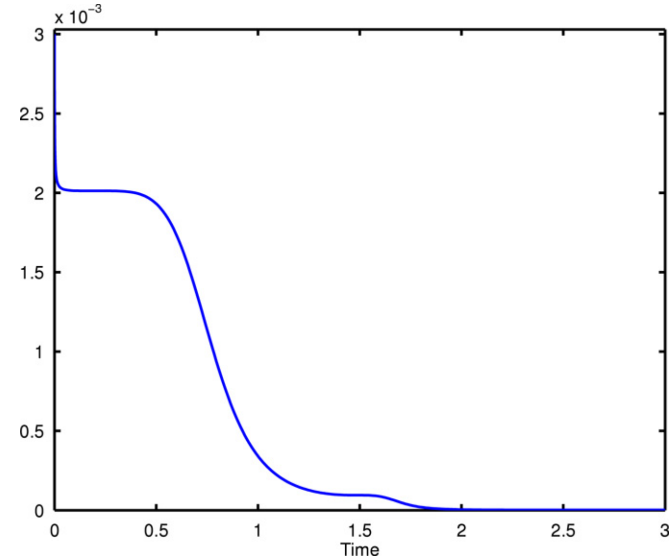

Fig. 3. Lyapunov function $V_\theta = \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}$ as a function of time. It decreases monotonically to zero.

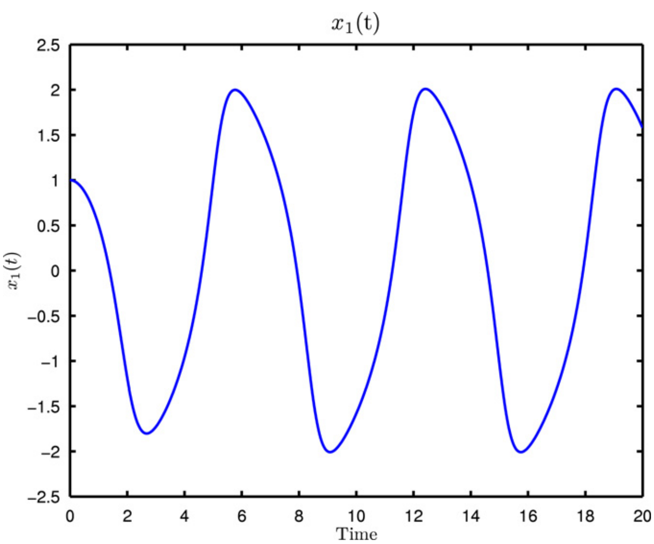

Fig. 4. Output $x_1(t)$, which is the same as the first component of the regressor vector. Note that the response here is shown over a much longer time interval than in Figs. 1–3.

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References

Astolfi, A., & Ortega, R. (2003). Immersion and invariance: A new tool for stabilization and adaptive control of nonlinear systems. *IEEE Transactions on Automatic Control*, *48*, 590–606.

Astolfi, A., Karagiannis, D., & Ortega, R. (2007). Towards applied nonlinear adaptive control *9th IFAC workshop on adaptation and learning in adaptive control and signal processing*.

Berghuis, H., Roebbers, H., & Nijmeijer, H. (1995). Experimental comparison of parameter estimation methods in adaptive robot control. *Automatica*, *31*, 1275–1285.

Guo, L. (1997). On critical stability of discrete-time adaptive nonlinear control. *IEEE Transactions on Automatic Control*, *42*, 1488–1499.

Ioannou, P., & Fidan, B. (2006). *Adaptive control tutorial*. SIAM.

Ioannou, P., & Sun, J. (1996). *Robust adaptive control*. Prentice Hall.

Johansson, R. (1983). Lyapunov functions for adaptive systems, *Proc. 22nd IEEE conference on decision and control* (pp. 449–454).

Johansson, R. (1989). Global Lyapunov stability and exponential convergence of direct adaptive control. *International Journal of Control*, *50*, 859–869.

Johansson, R. (1995). Supermartingale analysis of minimum variance adaptive control. *Control Theory and Advanced Technology*, *10*, 993–1013.

Kanellakopoulos, I. (1994). A discrete-time adaptive nonlinear system. *IEEE Transactions on Automatic Control*, *39*, 2362–2365.

Karagiannis, D., & Astolfi, A. (2008). Nonlinear adaptive control of systems in feedback form: An alternative to adaptive backstepping. *Systems & Control Letters*, *57*, 733–739.

Krstic, M., Kanellakopoulos, I., & Kokotovic, P. (1995). *Nonlinear and adaptive control design*. New York: Wiley.

Krstic, M., & Kokotovic, P. V. (1995). Adaptive nonlinear design with controller-identifier separation and swapping. *IEEE Transactions on Automatic Control*, *40*, 426–441.

Krstic, M., & Kokotovic, P. V. (1996). Modular approach to adaptive nonlinear stabilization. *Automatica*, *32*, 625–629.

Praly, L., Bastin, G., Pomet, J.-B., & Jiang, Z.P. (1991). Adaptive stabilization of nonlinear systems. In P.V. Kokotovic (Ed.) *Foundations of adaptive control* (pp. 347–434).

Tao, G. (2003). *Adaptive Control Design and Analysis*. Wiley.

M. Krstic is the Sorenson Distinguished Professor and Director of the Center for Control Systems and Dynamics (CCSD) at UC San Diego. He received his Ph.D. in 1994 from UC Santa Barbara and was Assistant Professor at the University of Maryland until 1997. He is a coauthor of the books Nonlinear and Adaptive Control Design (1995), Stabilization of Nonlinear Uncertain Systems (1998), Flow Control by Feedback (2002), Real-time Optimization by Extremum Seeking Control (2003), Control of Turbulent and Magnetohydrodynamic Channel Flows (2007), and Boundary Control of PDEs: A Course on Back-stepping Designs (2008). Krstic received the Axelby and Schuck paper prizes, NSF Career, ONR Young Investigator, and PECASE awards, the UCSD Research Award, is a Fellow of IEEE and IFAC, and has held the appointment of Springer Distinguished Professor of Mechanical Engineering at UC Berkeley. His editorial service includes IEEE TAC, Automatica, Systems and Control Letters, and Int. J. Adaptive Contr. Sig. Proc. He has served as the Vice-President for Technical Activities of IEEE Control Systems Society, member of its Board of Governors, and the Chair of its Fellow Evaluation Committee.