For solvable DAE systems, a general methodology was developed wherein the DAE system was modified by a dynamic state feedback compensator such that the resulting system was solvable and possessed a control invariant state space, thereby allowing the derivation of standard state-space realizations. For the feedback-modified system, a space-state realization was derived that can be used as the basis for controller synthesis. Extension of the proposed methodology for nonlinear DAE systems will be explored in future work.

REFERENCES


Adaptive Nonlinear Output-Feedback Schemes with Marino–Tomei Controller

Miroslav Krstić and Petar V. Kokotović

Abstract—Three new adaptive nonlinear output-feedback schemes are presented. The first scheme employs the tuning functions design. The other two employ a novel estimation-based design consisting of a strengthened controller-observer pair and observer-based and swapping-based identifiers. They remove restrictive growth and matching conditions present in the previous output-feedback nonlinear estimation-based designs and allow a systematic improvement of transient performance.

I. INTRODUCTION

In the last few years, adaptive control of nonlinear systems has emerged as an exciting research area. Early efforts focused on the state-feedback problem and resulted in a systematic design procedure called adaptive backstepping [5]. The more challenging output-feedback problem was then addressed for systems with nonlinearities which depend on the output only. This problem was first solved under restrictive structural and growth conditions on the nonlinearities [3, 4]. Subsequently, the growth restrictions were removed [6], but the structural restriction remained: the output nonlinearities were not allowed to precede the control input.

The removal of this structural restriction by Marino and Tomei in [15] was a breakthrough in adaptive nonlinear output-feedback control. This was achieved by merging the filtered transformations of [13] and [14] with the adaptive backstepping of [5] and using a novel compensation of the estimation error effects. An alternative approach for the same class of systems was presented in [7]. The nonlinear systems considered in [15] and [7] are still the largest class for which asymptotic tracking of arbitrary smooth reference signals can be achieved. A more general class of systems was considered in [16], but only for the set-point regulation problem.

In [15], the authors view their adaptive scheme as an existence result because of its complexity and high-dynamic order which are primarily due to the overparameterization inherited from the original adaptive backstepping procedure [5]. The overparameterization amounts to employing $\rho$ different update laws for the same parameter vector, $\rho$ being the relative degree of the plant. Another drawback of the scheme in [15] is that it is restricted to the unnormalized gradient update law. Furthermore, in the setting of [15], the passivity of the observer error system could not be exploited to design a simple observer-based identifier.

The three new adaptive schemes proposed in this paper remove these drawbacks. They achieve minimal parameterization in two different ways. The first scheme, presented in Section III, employs the “tuning functions” technique developed in [9]. In this scheme we modify the Marino–Tomei controller with terms which compensate the mismatch between the actual update law and the tuning functions. The other two adaptive schemes avoid overparameterization by using

Manuscript received September 24, 1993; revised November 9, 1994. This work was supported in part by the National Science Foundation under Grant ECS-9203491 and the Air Force Office of Scientific Research under Grant F-49622-92-J-0495.

M. Krstić is with the Department of Mechanical Engineering, University of Maryland, College Park, MD 20742 USA.
P. V. Kokotović is with the Department Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106 USA.

Publisher Item Identifier S 0018-9286(96)00982-8.
a novel estimation-based approach. They are motivated by our recent state-feedback results [10], [11]. The "observer-based" scheme in Section V has a simple identifier which exploits the passivity of the observer error system. This scheme is still restricted to an unnormalized gradient update law, as are [15] and the tuning functions scheme. The "swapping-based" scheme in Section VI removes this last restriction and can incorporate any standard update law: gradient, least-squares, normalized, or unnormalized.

Until recently, the estimation-based approach to adaptive nonlinear control [18] was unable to guarantee global boundedness without restrictive growth or matching conditions. This is now accomplished by strengthening the controller-observer pair with nonlinear damping including the $\kappa$-terms of [8], so that its boundedness properties are achieved independently of the identifier. This strengthening guarantees boundedness of all closed-loop states whenever $\theta$ is bounded, $\theta \in L_\infty$, and either $\theta \in L_\infty$ or $\theta \in L_2$. The identifiers, in turn, independently guarantee that $\theta \in L_\infty$ and either $\theta \in L_\infty$ (swapping-based scheme) or $\theta \in L_2$ (observer-based scheme).

In Section VII we show analytically that the schemes given in this paper can be used for systematically improving transient performance. Notation: $X_{(i)}$ and $X_j$ denote the $i$th row and the $j$th column of matrix $X$, respectively.

II. MARINO-TOMEI FILTERED TRANSFORMATIONS AND OBSERVER

Problem Statement: As in [15], we consider SISO nonlinear systems transformable into the output-feedback canonical form

$$\dot{x} = Ax + \phi(y) + \Phi(y)u + [0\ 1]^T x, \quad x \in \mathbb{R}^n$$

$$y = x_1,$$  

(1)

where $a = [a_1, \ldots, a_q]^T \in \mathbb{R}^q$, $b = [b_m, b_0]^T \in \mathbb{R}^{m+1}$ are vectors of unknown constant parameters, and

$$A = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_0,1 \\ \vdots \\ \phi_0,n \end{bmatrix}$$

(2)

$$\Phi = \begin{bmatrix} \phi_{1,1} & \cdots & \phi_{1,n} \\ \vdots \\ \phi_{q,1} & \cdots & \phi_{q,n} \end{bmatrix}.$$  

It is assumed that $\phi_{i,1}, 0 \leq j \leq q, 1 \leq i \leq n$, and $\sigma$ are smooth. Geometric conditions which characterize the class of nonlinear systems that can be transformed into this form have been given in [15]. The class of systems which are globally stabilizable by output feedback is not much broader than (1). It was shown in [17] that the system $\dot{x} = x_2, \dot{x}_2 = x_3 + u, y = x_1$ cannot be globally stabilized by dynamic output feedback for $n > 2$.

Assumption 2.1: The polynomial $B(s) = b_m s^n + \cdots + b_1 s + b_0$ is Hurwitz, and $\sigma(y) \neq 0$ if $y \in \mathbb{R}$. The sign of $b_m$ is known.

Assumption 2.2: The reference signal $y(t)$ and its first $p$ derivatives are known and bounded, and $y^p_{\infty}(t)$ is piecewise continuous.

Filtered Transformations: The filtered transformations employ the input filters

$$v_i = \frac{1}{s + \lambda} \sigma(y)u, \quad i = 1, \ldots, \rho - 1$$

(3)

and the output filters

$$\dot{\hat{\xi}} = A\hat{\xi} + B_{\phi}(y), \quad \hat{\xi} \in \mathbb{R}^{n-1}$$

(5)

where $A_1$ and $B_1$ are given by

$$A_1 = \begin{bmatrix} -I & 0 \\ 0 & 0 \end{bmatrix}, \quad B_1 = [A_1, \epsilon_1]$$

(7)

and the vectors $\bar{l}$ and $l$ are defined via the coefficients of polynomial $(s + \lambda)^{n-1}$

$$l = \begin{bmatrix} 1, (n-1) \lambda, \ldots, (n-1) \lambda^{n-2}, \lambda^{n-1} \end{bmatrix}^T$$

(8)

For later use, (4) is rewritten in the compact form

$$\dot{\hat{\xi}} = A_1 \hat{\xi} + \hat{\xi}(sI - A_0)^{-1}z = \frac{1}{s + \lambda} Z.$$  

(9)

Lemma 2.1 [15]: For (1) and (3)-(6), there exist a vector $\beta(b) \in \mathbb{R}^n$ and a matrix $S(\theta) \in \mathbb{R}^{n \times [(n+2)(n-1)]}$ where

$$\theta = [b_m, \beta(b)^T, \theta^T]^T$$

(10)

such that the parameter-dependent coordinate transformation

$$\chi = x - S(\theta)[v^T, \mu^T, \xi^T, \cos(\Xi)^T]^T$$

(11)

takes (1) into the "adaptive observer form"

$$\dot{\hat{\chi}} = A \hat{\chi} + l(\omega_0 + \omega^T \theta)$$

$$y = \hat{\chi}_1$$

(12)

where $\omega_0$ and the "regressor" $\omega$ are given by

$$\omega_0 = \phi_1(y) + \xi_1$$

(13)

$$\omega^T = [v^T, \mu^T, \phi_1, \Xi^T].$$

(14)

Observer: Once they have brought (1) to the adaptive observer form, Marino and Tomei design the observer for the transformed state $\chi$

$$\dot{\hat{\chi}} = A \hat{\chi} + K_o(y - \hat{\chi}_1) + l(\omega_0 + \omega^T \theta)$$

(15)

where $K_o$ is chosen so that $A_o = A - K_o \sigma_0^T$ satisfies

$$\det(sI - A_o) = (s + c_0)(s + \lambda)^{n-1}$$

(16)

namely, $e_1^T(sI - A_o)^{-1}l = 1/(s + c_0)$. The observer error $\epsilon = \chi - \hat{\chi}$ is governed by

$$\dot{\epsilon} = A_o \epsilon + l(\overline{\omega}^T \dot{\theta}).$$

(17)

III. TUNING FUNCTIONS SCHEME

The tuning function's design [9] is applied to the system consisting of the states $y, v_1, \ldots, v_{p-1}$. The design steps are only briefly outlined.

Step 1: The first component of the state of the error system is the tracking error $z_1 = y - \hat{y}_r$. The first stabilizing function is designed as

$$\alpha_1(X, t) = -\zeta(c + d)z_1 + \chi_1 \hat{\chi}_1 \triangleq \chi_{t\chi}$$

(18)

where $X \triangleq (y, \hat{\chi}_1, \hat{\xi}, \Xi, \mu, \hat{\theta}, \hat{\zeta})$, the quantity $\zeta$ is an estimate of $\epsilon = 1/b_m$, and

$$\chi_1 = -\hat{\chi}_2 - \omega_0 - \overline{\omega}^T \dot{\theta} + \dot{\hat{\chi}}_r$$

(19)

where the truncated regressor is given by

$$\overline{\omega}^T = [0, \mu^T, \phi_1, \Xi^T].$$

(20)

We design the first tuning function as

$$\tau_1 = \Gamma(\overline{\omega}z_1 + \gamma_1 \omega_1), \quad \gamma_1 > 0.$$
Step 2: Introducing \( z_2 = v_1 - \alpha_1 \), the second stabilizing function

\[
\alpha_2(X, v_1, t) = -c + d \left( \frac{\partial \alpha_1}{\partial y} \right)^2 z_2 + \tilde{\alpha}_2 + \sigma_2 \tag{22}
\]

consists of

\[
\tilde{\alpha}_2 = -b_m z_2 + \lambda v_1 + \frac{\partial \alpha_1}{\partial y} (\hat{\chi}_2 + \omega + \omega^T \hat{\theta})

+ \frac{\partial \alpha_1}{\partial \hat{x}} [A\hat{x} + K_0(y - \hat{x}_1) + l(\omega + \omega^T \hat{\theta})]

+ \frac{\partial \alpha_1}{\partial \hat{\xi}} (A\hat{\xi} + B\hat{\phi}) + \frac{\partial \alpha_1}{\partial \hat{\varepsilon}} (A\hat{\varepsilon} + B\hat{\Phi})

+ \frac{\partial \alpha_1}{\partial y_1} (\lambda u_1 + e_1 v_1) + \frac{\partial \alpha_1}{\partial y_2} \hat{v}_n + \frac{\partial \alpha_1}{\partial y_3} \hat{y}_n
\]

and the compensating function

\[
g_2 = \frac{\partial \alpha_1}{\partial \hat{\theta}} \tau_2 + \frac{\partial \alpha_1}{\partial \hat{\xi}} \tau_1
\]

where the second tuning function is designed as

\[
\tau_2 = \tau_1 + \Gamma \left( -\frac{\partial \alpha_1}{\partial y} \omega + z_1 e_1 \right) z_2. \tag{25}
\]

Step \( i (3 \leq i \leq \rho) \): Subsequent components of the state of the error system are defined as \( z_i = v_{i-1} - \alpha_{i-1} \). The stabilizing functions

\[
\alpha_i(X, v_1, \ldots, v_{i-1}, t) = -c + d \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 z_i + \tilde{\alpha}_i + \sigma_i \tag{26}
\]

consist of

\[
\tilde{\alpha}_i = -z_{i-1} + \lambda v_{i-1} + \frac{\partial \alpha_{i-1}}{\partial y} (\hat{\chi}_2 + \omega + \omega^T \hat{\theta})

+ \frac{\partial \alpha_{i-1}}{\partial \hat{x}} [A\hat{x} + K_0(y - \hat{x}_1) + l(\omega + \omega^T \hat{\theta})]

+ \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}} (A\hat{\xi} + B\hat{\phi}) + \frac{\partial \alpha_{i-1}}{\partial \hat{\varepsilon}} (A\hat{\varepsilon} + B\hat{\Phi})

+ \frac{\partial \alpha_{i-1}}{\partial y_{i+1}} (\lambda u_{i+1} + e_{i+1} v_{i+1})

+ \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_j} (-\lambda u_j + v_{i+j}) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_j} (\hat{y}_{i-1})_{(j)} \tag{27}
\]

and the compensating functions

\[
\sigma_i = \frac{\partial \alpha_{i-1}}{\partial \hat{x}} \tau_i + \frac{\partial \alpha_{i-1}}{\partial \hat{y}} \omega \tau_i
\]

\[
\tau_i = \tau_{i-1} + \Gamma \left( -\frac{\partial \alpha_{i-1}}{\partial y} \omega \right) \tau_i. \tag{29}
\]

At the end of the recursion, the last stabilizing function \( \alpha_\rho \) is used for the actual control law

\[
u = \frac{1}{\sigma(y)} \alpha_\rho. \tag{30}
\]

The update laws are designed as

\[
\dot{\hat{\varepsilon}} = -\gamma \text{sgn}(b_m) \mu \zeta_1,
\]

\[
\dot{\hat{\theta}} = \gamma \text{sgn}(b_m) \mu \zeta_1 \gamma \text{sgn}(b_m) \mu \zeta_1 + \gamma \omega \zeta_1.
\]

with \( \gamma > 0 \) and \( \gamma \) specified in the proof of Theorem 3.1. By noting that \( \omega^T \theta = b_m v_1 + \omega^T \theta \) and

\[
\dot{\chi}_2 = \hat{\chi}_2 + \omega + \omega^T \theta + \epsilon_2
\]

it is straightforward to verify that the resulting system, called the error system, is

\[
\dot{\chi} = A(z, \theta) + W_e(z, \theta) + W_\epsilon(z, \theta) + b_m \alpha \zeta_1, \tag{34}
\]

where (35) is shown at the bottom of the page, and

\[
W_e = \left[ 1 + \frac{\partial \alpha_1}{\partial y}, \ldots, 1 - \frac{\partial \alpha_{\rho-1}}{\partial y} \right]^T
\]

\[
W_\epsilon = W_e(z, \theta) + \zeta_1 \frac{\partial \alpha_\rho}{\partial \hat{\xi}} \tag{36}
\]

and \( \rho \triangleq q + m + 1 \). We see that \( \epsilon \) is absent from (34).

Theorem 3.1 (Tuning Functions Scheme): All the signals in the closed-loop adaptive system consisting of the plant (I), the filters (3k(6), the observer (15), the update laws (31) and (32), and the control law (30) are globally uniformly bounded for all \( t \geq 0 \), and global asymptotic tracking is achieved: \( \lim_{t \to \infty} [y(t) - y_*(t)] = 0 \).

Proof: See [12].

IV. STRENGTHENED OBSERVER AND CONTROLLER

Observer: We strengthen the Marino-Tomei observer by adding the stability enhancing, nonlinear term \( \kappa_2 \omega^2 l(y - \hat{\chi}_1) \) in (15)

\[
\dot{\hat{\chi}} = A\hat{\chi} + K_0(y - \hat{\chi}_1) + \kappa_2 \omega^2 l(y - \hat{\chi}_1) + l(\omega + \omega^T \hat{\theta}) \tag{37}
\]

so that the observer error system becomes

\[
\dot{\varepsilon} = (A_e - \kappa_2 \omega^2 l e^T) \varepsilon + l \omega^T \hat{\theta}. \tag{38}
\]

Lemma 4.1: Suppose in (38) that \( \omega \) and \( \hat{\theta} \) are piecewise continuous on \([0, t_f]) \). If \( \theta \in L_{\infty}[0, t_f] \), then \( \varepsilon \in L_{\infty}[0, t_f] \).

Proof: Since \( \varepsilon_i^T (x^T - A_e)^{-1} \varepsilon = 1/2 (s + c_\rho) \) is SPR, then there exist \( P_\varepsilon = P_\varepsilon^T > 0 \) and \( q_\varepsilon > 0 \) such that

\[
A_\varepsilon^T P_\varepsilon + P_\varepsilon A_\varepsilon \leq -q_\varepsilon I, \quad P_\varepsilon I = c_\varepsilon. \tag{39}
\]

Therefore, along the solutions of (38) we have

\[
\frac{d}{dt} [\varepsilon_i^T P_\varepsilon] \leq -q_\varepsilon |\varepsilon_i|^2 - 2\kappa_2 |\omega|^2 \varepsilon x^T P_\varepsilon e^T \varepsilon

+ 2\varepsilon^T P_\varepsilon \varepsilon \varepsilon = -q_\varepsilon |\varepsilon_i|^2 - 2\kappa_2 |\omega|^2 \varepsilon_i^2 + 2\varepsilon_i \omega^T \hat{\theta}

\leq -q_\varepsilon |\varepsilon_i|^2 + \frac{1}{2q_\varepsilon} |\varepsilon_i|^2 \tag{40}
\]
which implies that \( c \in \mathcal{L}_\infty[0, t_f] \), whenever \( \ddot{\theta} \in \mathcal{L}_\infty[0, t_f] \).

**Assumption 4.1:** In addition to \( \text{sgn} \, b_m \), a positive constant \( \rho_m \) is known such that \( |b_m| \geq \rho_m \).

**Controller:** We only spell out the differences from the tuning functions design. The stabilizing functions \( \alpha_i \) are designed to render the state \( [z_1, \ldots, z_i]^T \) bounded whenever \( \dot{z}_i \) is bounded and \( \dot{z}_i \) is either bounded or square-integrable. This is achieved with the nonnegative nonlinear damping functions \( s_i \):

\[
s_i = d + \rho \left( \frac{1}{b_m} \right)^2 \tag{41}
\]

\[
s_2 = d \left( \frac{\partial \alpha_1}{\partial y} \right)^2 + \rho \left( \frac{\partial \alpha_1}{\partial y} \right)^2 \tag{42}
\]

\[
s_i = d \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 + \rho \left( \frac{\partial \alpha_{i-1}}{\partial y} \right)^2 \tag{43}
\]

which consist of three terms, each counteracting the effects of disturbances \( c_2, \ddot{\theta}, \dot{\theta} \). The nonlinear damping functions appear in the modified stabilizing functions \( \alpha_i \):

\[
\alpha_i = -\frac{\text{sgn} \, b_m}{\rho_m} \left( c + s_i \right) z_i + \frac{1}{b_m} \alpha_i \tag{44}
\]

\[
\alpha_i = -\left( c + s_i \right) z_i + \alpha_i + \frac{\partial \alpha_{i-1}}{\partial \bar{x}} \kappa_0 |\omega|^2 (y - \dot{\chi}_i) \tag{45}
\]

The term \( \left( \frac{\partial \alpha_{i-1}}{\partial \bar{x}} \kappa_0 |\omega|^2 (y - \dot{\chi}_i) \right) \) in (45) accommodates for the strengthening in the observer.

It is straightforward to verify that the resulting error system is

\[
\dot{z} = A'_c (z, t)z + W_c (z, t) c_2 + W_\phi (z, t)^T \ddot{\theta} + D (z, t) \dot{\theta}_i \tag{46}
\]

where

\[
A'_c (z, t) = \begin{bmatrix}
-\frac{b_m}{\rho_m} \left( c + s_i \right) & b_m & 0 & \cdots & 0 \\
-b_m & -(c + s_2) & 1 & \cdots & 0 \\
0 & -1 & \ddots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & 1 \\
0 & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & -1 - (c + s_3)
\end{bmatrix} \tag{47}
\]

\[
W_c (z, t) = \begin{bmatrix}
\frac{1}{\partial \alpha_1} \\
\vdots \\
-\frac{\partial \alpha_{i-1}}{\partial y} \\
\frac{\omega^T + \frac{\alpha_i}{b_m} c_i^T}{\eta_T} \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \kappa_0 |\omega|^2 (y - \dot{\chi}_i) \\
\vdots \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \omega^T + z_i c_i^T \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \omega^T \\
\vdots \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \omega^T \\
0
\end{bmatrix} \tag{48}
\]

\[
W_\phi (z, t)^T = \begin{bmatrix}
-\frac{\partial \alpha_1}{\partial \bar{x}} \omega^T \\
\vdots \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \omega^T \\
0
\end{bmatrix} \tag{49}
\]

\[D (z, t) = \begin{bmatrix}
-\frac{\partial \alpha_1}{\partial \bar{x}} \\
\vdots \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \\
-\frac{\partial \alpha_{i-1}}{\partial \bar{x}} \\
0
\end{bmatrix} \tag{50}
\]

\[1 \text{ Our identifiers will guarantee } |b_m(t)| \geq \rho_m, \forall t \geq 0.\]
Proof: Combining Lemmas 4.2 and 5.1, it is straightforward to show that all the signals are globally uniformly bounded. To prove the tracking, we first rewrite (46) as
\[ \dot{z} = A_z(z, t) \Sigma + W_z(z, t) \Sigma + W(\Sigma, t) T \dot{\Sigma} + D(z, t) \dot{\Sigma} \]  
(57)
where \( A_z \) is obtained from \( A \) by replacing \( b_m \) by \( b_m \), and \( W_0 \) is obtained from \( W_0 \) by replacing \( b_m \) by \( b_m \). Let us define \( M(z, t) = W_z(z, t) (T^T/\|T\|^2) \) and consider \( \dot{\Sigma} = z - M \) as the solutions of (57) and (38)
\[ \dot{\Sigma} = A_\Sigma \Sigma + [W_0 z^T \Sigma - \Sigma M - \Sigma (A_0 - \kappa_0 \Sigma )] e_T + A_2 M ] e + D \dot{\Sigma}, \]  
(58)
where the bracketed expression and \( D \) are bounded. It is now straightforward to derive
\[ \frac{d}{dt} \left( \| \Sigma \| \right)^2 \leq -c_{\Sigma} \| \Sigma \|^2 + \frac{1}{\sigma} \| W_0 z^T \Sigma - \Sigma M - \Sigma (A_0 - \kappa_0 \Sigma ) \|^2 e_T + A_2 M ] e \]  
(59)
where \( c_{\Sigma} \) is a suitable constant and \( \sigma > 0 \) is a constant. Since \( z \in \mathbb{L}_2 \), it follows by [1, Theorem IV.1.9] that \( \Sigma \in \mathbb{L}_2 \). Thus \( z \in \mathbb{L}_2 \). From (57) we can see that \( \dot{z} \in \mathbb{L}_\infty \), which, along with \( z \in \mathbb{L}_\infty \), is piecewise continuous on \( (0, t_f) \), then (68) and (69) guarantee that \( \theta, \Sigma, \dot{\Sigma} \in \mathbb{L}_\infty (0, t_f) \).

**Lemma 6.1:** If \( \Omega \) is piecewise continuous on \( (0, t_f) \), then (68) and (69)

Proof (Outline): It is readily shown that
\[ \frac{d}{dt} \left( \frac{1}{2} \| \Omega \|^2 \right) \leq -c_{\Omega} \| \Omega \|^2 + \frac{1}{4\kappa_0} \| \Theta \|^2 \]  
(70)
which proves that \( \Omega \) is bounded. For both the gradient and the least-squares update laws it can be shown that
\[ \frac{d}{dt} \left( \frac{1}{2} \| \Theta \|^2 + \frac{1}{\sigma} \| \Theta e_T \|^2 \right) \leq -\frac{1}{\sigma} \frac{c^2}{1 + \kappa_0 \| \Theta \|^2} \]  
(71)
where \( P_I \) is positive definite. The conclusions of the lemma are immediate from (71) and (70).

**Theorem 6.1 (Swapping-Based Scheme):** All the signals in the closed-loop adaptive system consisting of the plant (1), the filters (9)-(10), the observer (37), the identifier filters (64), either the gradient (68) or the least-squares (69) update law, and the control law (30) are globally uniformly bounded for all \( \kappa \geq 0 \), and global asymptotic tracking is achieved as \( t \to \infty \).

Proof: With Lemmas 4.2 and 6.1, we establish the global uniform boundedness of all the signals. To prove the tracking, we first show that \( \Sigma \) is square-integrable. Let us consider \( \Psi = \Sigma - \Omega T \dot{\theta} \) which satisfies
\[ \dot{\Psi} = -(c_{\Sigma} + \kappa_0 \| \Sigma \|^2) \Psi - \Omega T \dot{\theta} \]  
(72)
It can easily be seen that
\[ \frac{d}{dt} \left( \frac{1}{2} \| \Psi \|^2 \right) \leq -\frac{c_{\Sigma}}{2} \| \Psi \|^2 + \frac{1}{2c_{\Sigma}} \| \Omega T \dot{\theta} \|^2. \]  
(73)
Since \( \Omega T \dot{\theta} \in \mathbb{L}_2 \), then by [1, Theorem IV.1.9], \( \Psi \in \mathbb{L}_2 \). In view of the fact that \( \epsilon = \epsilon_1 + \psi \), this proves that \( \epsilon_1 \in \mathbb{L}_2 \). From (62) it follows that \( \epsilon = \epsilon_1 + \epsilon \), which proves that \( \epsilon \in \mathbb{L}_2 \). Following the same arguments as in the proof of Theorem 5.1, (cf. (57)-(59)), we show that \( z \in \mathbb{L}_2 \). The tracking is deduced via Barbalat’s lemma. The flexibility to incorporate any of the standard parameter update laws in the swapping-based scheme is achieved at the expense of additional filters for the identifier.

**VII. TRANSIENT PERFORMANCE ANALYSIS**

We first derive \( \mathbb{L}_\infty \)-bounds for estimation-based schemes. To simplify the analysis, we let \( \Gamma = \gamma I \), as well as \( c_0 = c = \kappa_0 = 0 \). For the same reason, we implement \( \chi_1 (0) = \gamma (0) \) to get \( \epsilon_1 (0) = 0 \).

**Theorem 7.1 (Observer-Based Scheme):** In the adaptive system (1), (3), (4), (55), (50), the following inequality holds:
\[ \| z \| \leq \frac{1}{\sqrt{c}} (M \| \theta (0) \| + N \| e (0) \|) + \| z (0) \| e^{-\gamma t/2} \]  
(74)
where \( M \) and \( N \) are nonincreasing functions of \( c, d, \kappa, g \).

Proof: First, we note from (61) that
\[ \frac{d}{dt} \left( \frac{1}{2} \| \epsilon \|^2 \right) \leq -\frac{c}{2} \| \epsilon \|^2 + \frac{1}{2c} \| \epsilon_\tau \|^2 + \frac{1}{2c} \| \Theta \| \dot{\theta} \| \]  
(75)
Combining (50) an (75) we compute
\[ \frac{d}{dt} \left( \frac{1}{2} \| \epsilon \|^2 + \frac{c^2}{4kg} \right) \leq -\frac{c}{2} \| \epsilon \|^2 + \frac{1}{4} \| \epsilon \| \dot{\theta} \| \]  
(76)
and, since the second term in the parentheses is zero, it is straightforward to obtain
\[
\|z(t)\| \leq \sqrt{\frac{\rho}{2c} \left( \frac{1}{d} \|z_2\|^2 + \frac{1}{\kappa} \right)} + \frac{\gamma}{c} \|\hat{\theta}\|_\infty + \frac{1}{2c} \|\hat{\theta}\|_\infty
\]
(77)
where we have used \( c_1(t) = 0 \). Now we determine bounds on \( \|\varepsilon_2\|_\infty, \|\hat{\theta}\|_\infty \) and \( |\xi_1|_\infty \). First, from (62) we have
\[
\frac{d}{dt} (|\xi_1|_h^2) = - |\xi_1|^2
\]
(78)
which implies that \( |\xi_1|_h^2 \leq \left| \frac{1}{\Delta(P)} \right| |\xi_1(0)|_h^2 \). Since \( c_1(0) = 0 \), then \( \xi_1(t) = T \varepsilon(0) = [0, I_{n-1}] \varepsilon(0) \) and it follows that:
\[
|\xi_1|_h^2 \leq \left| \frac{1}{\Delta(P)} \right| |\varepsilon(0)|_h^2.
\]
(79)
Along the solutions of (61), (62), and (55) we have
\[
\frac{d}{dt} \left( |\xi_1|_h^2 + \frac{1}{2} |\xi_1|_h^2 + \frac{1}{2} |\hat{\theta}|^2 \right) \leq - c_1^2 - \frac{\kappa}{\gamma} |\hat{\theta}|^2
\]
(80)
which yields
\[
|\hat{\theta}|_\infty \leq \left| \frac{\hat{\theta}(0)}{c} \right| + \frac{\gamma}{c} |\xi_1(0)|_h^2
\]
(81)
(\|\xi_1\|_\infty \leq \frac{\gamma}{c} \left( |\xi_1(0)|_h^2 + \frac{\gamma}{c} |\xi_1(0)|_h^2 \right).
\]
(82)
To obtain a bound on \( \|\varepsilon_2\|_\infty \), from (60) we recall that \( c_2 = \frac{n(1 - \lambda\varepsilon_1 + \xi_1)}{c} \), which, by virtue of (79) and (82), shows that
\[
\|\varepsilon_2\|_\infty \leq \frac{2(1 - \lambda\varepsilon_1 + \xi_1)}{c} |\hat{\theta}(0)|^2
\]
(83)
Substituting (79), (81), and (83) into (77), we arrive at (74) with
\[
M = \sqrt{\frac{\rho}{2c} \left( \frac{1}{c} + \frac{\gamma^2}{c} \right) + \frac{(n-1)^2 \lambda^2}{c^2} \left( \frac{1}{\alpha} + \frac{\gamma^2}{2c} \right)}
\]
(84)
\[
N = \sqrt{\frac{\rho \lambda(P)}{2c} \left( \frac{\gamma M^2}{c^2} + \frac{1}{\lambda(P)} \frac{1}{d} + \frac{\gamma^2}{2c^2} \right)}
\]
(85)
Now we consider the swapping-based scheme. For simplicity, we initialize \( \bar{\xi}(0) = -c_1(0) = 0 \) and \( \Omega(0) = 0 \) to set \( \bar{\xi}(0), \Omega(0) \) to zero. Theorem 7.2 (Swapping-Based Scheme): In the adaptive system
\[
(1), (3)-(6), (37), (63)-(64), (68), (80),
\]
(30), the following inequality holds:
\[
|\xi(t)| \leq \frac{1}{\sqrt{c}} \left( M |\bar{\xi}(0)| + N |\varepsilon(0)| \right) + |\xi(t)| \leq c_1 \varepsilon(t)
\]
(86)
where \( M \) and \( N \) are nonincreasing functions of \( c, d, \kappa, \gamma \).

Proof: We derive an \( L_\infty \)-bound on \( z(0) \) rewritten as
\[
|\xi(t)| \leq \sqrt{\frac{\rho}{4c} \left( \frac{1}{d} |\varepsilon_2|_h^2 + \frac{1}{\kappa} |\hat{\theta}|_\infty^2 + \frac{1}{g} |\hat{\theta}|_\infty^2 \right)} + |\xi(0)| \leq c_1 \varepsilon(t)
\]
(87)
It remains to determine bounds on \( \|\xi_2\|_\infty, \|\hat{\theta}\|_\infty \) and \( |\xi_1|_\infty \). First, from (71), using \( c(0) = 0 \) and \( |\xi(0)|_h^2 \leq \frac{\chi(P)}{\alpha} |\varepsilon(0)|_h^2 \), we get
\[
|\hat{\theta}|_\infty^2 \leq \frac{\gamma}{c} |\xi_1(0)|_h^2 + \frac{2 \chi(P)}{c^2} |\varepsilon(0)|_h^2.
\]
(88)
Noting that (61) yields
\[
\frac{d}{dt} \left( \frac{1}{2} |\varepsilon_2|_h^2 \right) \leq - c_1^2 - \frac{1}{4c} |\hat{\theta}|^2 + \frac{1}{2c^2} \left. \right| \varepsilon_2(t) \right|_h^2
\]
(89)
we obtain
\[
|\xi_1|_h^2 \leq \frac{1}{2c} |\xi_1(0)|_h^2 + \frac{1}{c^2} |\xi_1|_h^2
\]
(90)
Recalling that \( c_2 = (n-1)\lambda\varepsilon_1 + \xi_1 \), using (79) and (90), we get
\[
|\xi_1|_h^2 \leq \frac{(n-1)^2 \lambda^2}{c^2} |\hat{\theta}|_\infty^2 + 2 \left( \frac{1 + (n-1)^2 \lambda^2}{c^2} \right)
\]
(91)
(\frac{\chi(P)}{\lambda(P)} |\varepsilon(0)|_h^2).
\]
(92)
By substituting (88) and (90) into (92) and also (94) into (93), and then the two results, along with (88) into (87), we arrive at (86) with
\[
M = \sqrt{\frac{\rho}{4c} \left( \frac{1 + \gamma^2}{c^2} + \frac{(n-1)^2 \lambda^2}{c^2} \left( \frac{1}{\alpha} + \frac{\gamma^2}{2c^2} \right) \right)}
\]
(93)
\[
N = \sqrt{\frac{\rho \lambda(P)}{2} \left( \frac{\gamma M^2}{c^2} + \frac{1}{\lambda(P)} \frac{1}{d} + \frac{\gamma^2}{2c^2} \right)}
\]
(94)
\[
|\|\xi_2\|_\infty^2 \leq \frac{2 \gamma^2 |\Omega|_\infty^2 |\|\xi_2\|_\infty^2 + |\varepsilon_2|_h^2.
\]
(95)
To obtain a bound on \( \|\xi_2\|_\infty \), along (67) and (62) we consider
\[
\frac{d}{dt} \left( \frac{1}{2} |\varepsilon_2|_h^2 + \frac{1}{c} |\xi_1|_h^2 \right) \leq - c_1^2 - \frac{1}{4c} |\hat{\theta}|^2 \leq 0
\]
(96)
which yields
\[
|\|\xi_2\|_\infty^2 \leq \frac{\gamma}{c} |\xi_1(0)|_h^2.
\]
(97)
Although the initial states \( z_2(0), \cdots, z_p(0) \) may depend on \( c \) and \( d \), this dependence can be removed by setting \( z(0) = 0 \) with the standard trajectory or reference model initialization explained in [10]. From (74) and (86) it is evident that the \( L_\infty \)-performance bounds in both the observer-based scheme and the swapping-based scheme can be made as small as desired by initializing \( z(0) = 0 \) and increasing \( c \).

To obtain a similar \( L_2 \)-bound for the tuning functions scheme, the design has to be augmented with nonlinear damping terms. Unlike for the estimation-base schemes, for the tuning functions scheme one can also derive an \( L_2 \)-bound
\[
|\|z|_2 \leq \frac{1}{\sqrt{2c}} \left( |\xi(0)|_h^2 + \gamma \varepsilon(0)|_h^2 \right) + \frac{1}{c} |\varepsilon(0)|_h^2 - 1
\]
(98)
\[
\frac{1}{c} |\varepsilon(0)|_h^2 - 1
\]
(99)
VIII. CONCLUSIONS

The adaptive schemes proposed in this paper advance the state-of-the-art of adaptive nonlinear output-feedback control in several directions. They remove the main drawbacks of the original Marino–Tomei design. Only the minimal number of parameters is updated, and any standard update law can be incorporated in the swapping-based scheme. The estimation-based approach can now be used for adaptive nonlinear output-feedback control without any growth restrictions. The modifications made in the Marino–Tomei controller make it possible to systematically improve the transient performance by increasing certain design parameters.

REFERENCES


Abstract—Within this brief paper, a stable indirect adaptive controller is presented for a class of interconnected nonlinear systems. The feedback and adaptation mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. In addition, each subsystem is able to adaptively compensate for disturbances and interconnections with unknown bounds. The adaptive scheme is illustrated through the longitudinal control of a string of vehicles within an automated highway system (AHS).