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Research Note

Three-axis attitude control design for a spacecraft based on Lyapunov stability criteria

M. Bagheri[a,*], M. Kabganian[a] and R. Nadafi[b]

a. *Department of Mechanical Engineering, Amirkabir University of Technology, Tehran, P.O. Box 158754413 Iran.*
b. *Space Science and Technology Institute, Amirkabir University of Technology, Tehran, P.O Box 158754413, Iran.*

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Abstract. A three-axis attitude control design, based on Lyapunov stability criteria, to stabilize spacecraft and orient it to its desired altitude, is presented in this paper. This attitude control system is assumed to have four reaction wheels with optimal arrangement. The reaction wheels are located in a square pyramidal configuration. Control system inputs are the attitude parameter in the quaternion form and the angular velocity of the spacecraft and reaction wheels. The controller output is the torque required to eliminate error. In this study, actuators (reaction wheels) are modeled and the required torque for the attitude maneuver is converted to the voltage of actuators. Armature voltage and current are limited to 12 volts and 3 amps, respectively. Also, each wheel has an angular velocity limit of 370 rad/sec. Numerical simulations indicate that the spacecraft reaches the desired attitude after 34 seconds, which shows the reliability of the mentioned configuration, with respect to actuator failure. The results show that in cases of failure of one reaction wheel, the spacecraft can reach the desired attitude, but needs more time. Moreover, results demonstrated controller robustness against parameter variation and disturbances. It is robust against up to a 350% change in the spacecraft moment of inertia, and robust against a disturbance of up to 0.0094 N.m, which is equal to 38% in comparison with the allowable reaction wheel capacity.

1. Introduction

The Attitude Determination and Control System (ADCS) is an important subsystem of a spacecraft, which plays an important role in spacecraft missions. The controller algorithm is an essential part of the ADCS subsystem that commands the actuators. Many spacecraft have been placed in Low Earth Orbits (LEO) in recent years. Since these spacecraft are close to earth, the effect of orbital disturbances is enormous [1,2]. Using attitude controllers for stabilization and tracking desired trajectories has increased in the second half of the twentieth century. The first studies were on the passive controller for stabilization [3]. Recently, Wang and Xu studied the equilibrium attitude and stability of a rigid spacecraft on a stationary orbit around a uniformly-rotating asteroid [4]. But, in order to achieve better performance and accuracy, active attitude control is used [5]. In recent years, much research has been executed into designing adaptive control and non-linear control [6,7].

The Reaction Wheel (RW) is the most common actuator for rotational control of spacecraft, since it has a simple structure. Also, for accurate attitude control systems and moderately fast maneuvers, RWs are well suited, because they allow continuous and smooth

*. *Corresponding author. Tel.: +98-21 6454 3473, Fax: +98-21 6641 9736*
E-mail addresses: M.bagheri@aut.ac.ir (M. Bagheri); kabgan@aut.ac.ir (M. Kabganian); rezanadafi@aut.ac.ir (R. Nadafi)

control with comparatively low disturbing torques and are used to provide torques about three body axes.

Active control of spacecraft attitude has been executed by various researchers since the 1960's. RWs consist of a DC motor with a flywheel assembled on its axis to provide a higher moment of inertia. A control system of a three RW configuration, which is parallel to the principal axes of the body, is not complex, but, if one of them fails, the control system is unable to track the desired trajectories. Actuator failure is known to be the main cause of many space mission failures.

In some cases, the system has a failure in one of its actuators. Since these systems are under-actuated, their behavior should be analyzed [8]. In recent years, much research has been executed on the stability of under-actuated systems. Kasai et al. analyzed arbitrary rest-to-rest attitude maneuvers for a satellite using two Single-Gimbal Control Moment Gyros (2SGCMGs), which is a kind of under-actuated problem [9]. External disturbance torque was usually considered, and surveyed the robustness of the control system. Wang et al. analyzed the attitude controllability of an under-actuated spacecraft using one or two thrusters [10]. In this paper, it is assumed that the satellite is in the LEO and, as mentioned above, the orbital disturbances are enormous. The effects of these disturbances on the attitude of a spacecraft have been surveyed.

It is assumed that each reaction wheel has a torque limit and an angular momentum limit. Then, there is an additional problem with regard to saturation [11], and TIAN developed a controller which included the variable input saturation limit [12].

For these reasons, a square pyramidal configuration is used for RWs to increase the reliability and avoidance of saturation. In this configuration, the RWs are not placed along the body axes and their rotational axes are inclined to the $x_B - y_B$ plane by an angle, β (Figure 1).

In the present paper, a three axis attitude control based on Lyapunov criteria is designed and, as mentioned before, RWs are arranged in a square pyramidal configuration. In order to find applied control torque for each RW, the norm of the torque vector should be minimized. The scope of this paper is the modeling and simulation of a closed loop system (Figure 2) that show robustness against external disturbances, while maintaining sufficient consistency in parameter variation. The main contribution of this paper is in designing a control law and developing a closed loop modeling.

2. Equation of motion for spacecraft attitude

The attitude motion of a spacecraft containing a rigid body and four RWs is formulated. In this study, the

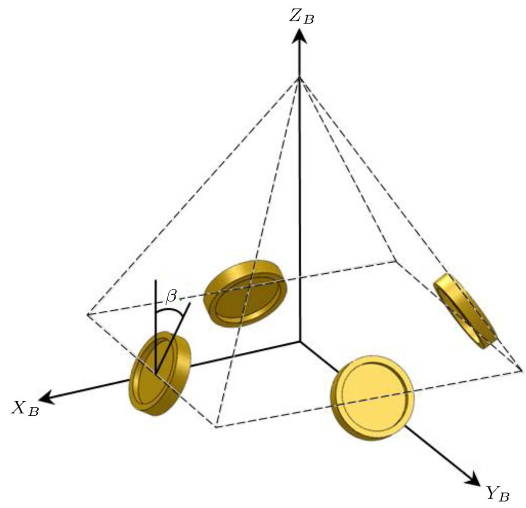


Figure 1. Square pyramidal configuration of RWs.

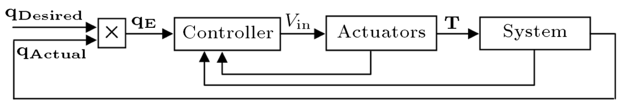


Figure 2. Closed loop modeling.

coupling between the orbital and the attitude motion and the disturbance torque are ignored. Note that in the present paper, all bold letters represent vectors, and scalar variables will be denoted as an italicized variable. Also, the names of matrices are bold and matrices are denoted with a dash under the matrix name.

According to the law of conservation of angular momentum, the differential equation of motion for a rigid spacecraft can be written as follows:

$$\mathbf{T} = \dot{\mathbf{h}}_{\mathbf{tI}} = \dot{\mathbf{h}}_{\mathbf{t}} + \mathbf{w} \times \mathbf{h}_{\mathbf{t}}, \tag{1}$$

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}, \tag{2}$$

where $\dot{\mathbf{h}}_{\mathbf{tI}}$, $\mathbf{h}_{\mathbf{t}}$ and $\dot{\mathbf{h}}_{\mathbf{t}}$ are time derivative of the spacecraft angular momentum in the inertia frame, spacecraft angular momentum and time derivative of spacecraft angular momentum in the Spacecraft Body Frame (SBF), respectively. The terms, ω_x, ω_y and ω_z, are angular velocities about the body coordinate axes.

According to Eq. (3), the total angular momentum of the system consists of the angular momentums of all components fixed about the spacecraft rotation axis, $\mathbf{h}_{\mathbf{s}}$, plus the angular momentum of the rotational component about the same axis, $\mathbf{h}_{\mathbf{w}}$, that are in the SBF.

$$\mathbf{h}_{\mathbf{t}} = \mathbf{h}_{\mathbf{s}} + \mathbf{h}_{\mathbf{w}}. \tag{3}$$

Therefore, Eq. (1) can be written as follows:

$$\mathbf{T} = \dot{\mathbf{h}}_{\mathbf{s}} + \boldsymbol{\omega} \times \mathbf{h}_{\mathbf{s}} + \dot{\mathbf{h}}_{\mathbf{w}} + \boldsymbol{\omega} \times \mathbf{h}_{\mathbf{w}}. \tag{4}$$

$\mathbf{T}$ is the sum of all external moments applied to the spacecraft. It consists of two terms: the control torque generated by the reaction control jets and torques due to external torques such as aerodynamic disturbance torques and gravity gradients, which all have been neglected as mentioned. $\mathbf{I_s}$ is moment of inertia of the all spacecraft components that are fixed, then:

$$\mathbf{h_s} = \mathbf{I_s}\boldsymbol{\omega}, \tag{5}$$

$$\dot{\mathbf{h}}_\mathbf{s} = \mathbf{I_s}\dot{\boldsymbol{\omega}}. \tag{6}$$

The transformation matrix, $\underline{\mathbf{C}}_\mathbf{w}^\mathbf{b}$, that transforms the wheel momentum from an individual wheel axis to spacecraft body axes can be defined as follows:

$$\underline{\mathbf{C}}_\mathbf{w}^\mathbf{b} = \begin{bmatrix}\mathbf{a_1} & \mathbf{a_2} & \mathbf{a_3} & \mathbf{a_4}\end{bmatrix}, \tag{7}$$

where $\mathbf{a_i}$ is the reaction wheel spin unit vector. In Eq. (4), $\mathbf{h_w}$ is the angular momentum of the reaction wheel, which is in the SBF as mentioned before, and is defined as follows:

$$\mathbf{h_w} = \underline{\mathbf{C}}_\mathbf{w}^\mathbf{b} \left[I_{\text{RW1}}\ \omega_{\text{RW1}};\ I_{\text{RW2}}\ \omega_{\text{RW2}};\ I_{\text{RW3}}\ \omega_{\text{RW3}};\ I_{\text{RW4}}\ \omega_{\text{RW4}}\right], \tag{8}$$

where I_{RW1} and $\omega_{\text{RW}i}$ are the inertia moment of disk in its rotation direction and the angular velocity of the disk of the ith RW, respectively. Also, the time derivative of the angular momentum of reaction is:

$$\dot{\mathbf{h}}_\mathbf{w} = \underline{\mathbf{C}}_\mathbf{w}^\mathbf{b} \left[I_{\text{RW1}}\ \dot{\omega}_{\text{RW1}};\ I_{\text{RW2}}\ \dot{\omega}_{\text{RW2}};\ I_{\text{RW3}}\ \dot{\omega}_{\text{RW3}};\ I_{\text{RW4}}\ \dot{\omega}_{\text{RW4}}\right]. \tag{9}$$

Finally, the equation of motion for spacecraft attitude without any external torque can be written as follows:

$$\mathbf{I_s}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{I_s}\boldsymbol{\omega} - \dot{\mathbf{h}}_\mathbf{w} - \boldsymbol{\omega} \times \mathbf{h_w}. \tag{10}$$

2.1. Actuator modeling

In general, an electric motor is coupled with the load through various mechanical transmission systems. Its angular rate can be varied by applying a torque to the motor about its spin axis. As the wheel accelerates, it applies an equal and opposite reaction torque on the spacecraft that is used to control its attitude [13]. A schematic diagram of the mentioned actuator is shown in Figure 3.

According to Figure 3, for a DC motor, the following relation, based on Kirchoff's law, can be written as follows:

$$V_{\text{in}} - V_b = R_R I + L_R\frac{dI}{dt}, \tag{11}$$

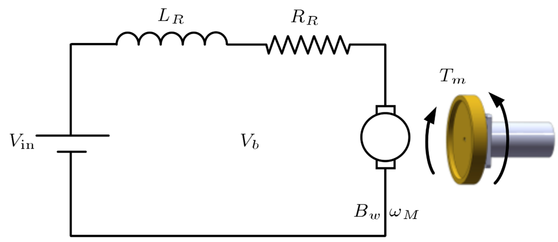


Figure 3. Schematic diagram of DC motor [13].

where V_{in}, R_R and L_R are the voltage input to the electrical motor, electrical resistance and armature inductance of the DC motor, respectively. Also, V_b is defined as:

$$V_b = K_b\omega_M, \tag{12}$$

where K_b is the motor back electromagnetic force constant, and ω_M is the angular velocity of the wheel relative to the spacecraft body, defined as follows:

$$\omega_M = \omega_{\text{RW}} - \omega. \tag{13}$$

Also, the output motor torque based on Newton's law is:

$$I_{\text{RW}}\ \dot{\omega}_{\text{M}} = T_m - B_w\omega_M, \tag{14}$$

where B_w is viscous friction and T_m is electromagnetic torque, which is relative to the armature current, I, by:

$$T_m = K_m I, \tag{15}$$

where K_m is motor torque constant. In regard to Eqs. (16) and (17), the voltage of RW, V_{in}, (controller output) can be converted to the rate of rotor angular momentum, $\dot{h}_w$:

$$V_{\text{in}} - K_b(\omega_{\text{RW}} - \omega) = R_R I + L_R\frac{dI}{dt}, \tag{16}$$

$$I_{\text{RW}}(\dot{\omega}_{\text{RW}} - \dot{\omega}) = K_m I - B_w(\omega_{\text{RW}} - \omega), \tag{17}$$

where R_R, L_R, B_w, K_b and K_m are the usual parameters of the electrical motor as mentioned. A SIMULINK model of the actuator is shown in Figure 4.

2.2. Square pyramidal arrangement of the reaction wheels

A square pyramidal arrangement is used because of the advantages of this configuration as mentioned. The torques produced along the three body axes are T_{cx}, T_{cy} and T_{cz}. In order to compute the components of $\mathbf{T_c}$ along the three body axes, the following can be written:

$$\mathbf{T_c} = \underline{\mathbf{C}}_\mathbf{w}^\mathbf{b}\dot{\mathbf{h}}_\mathbf{w}. \tag{18}$$

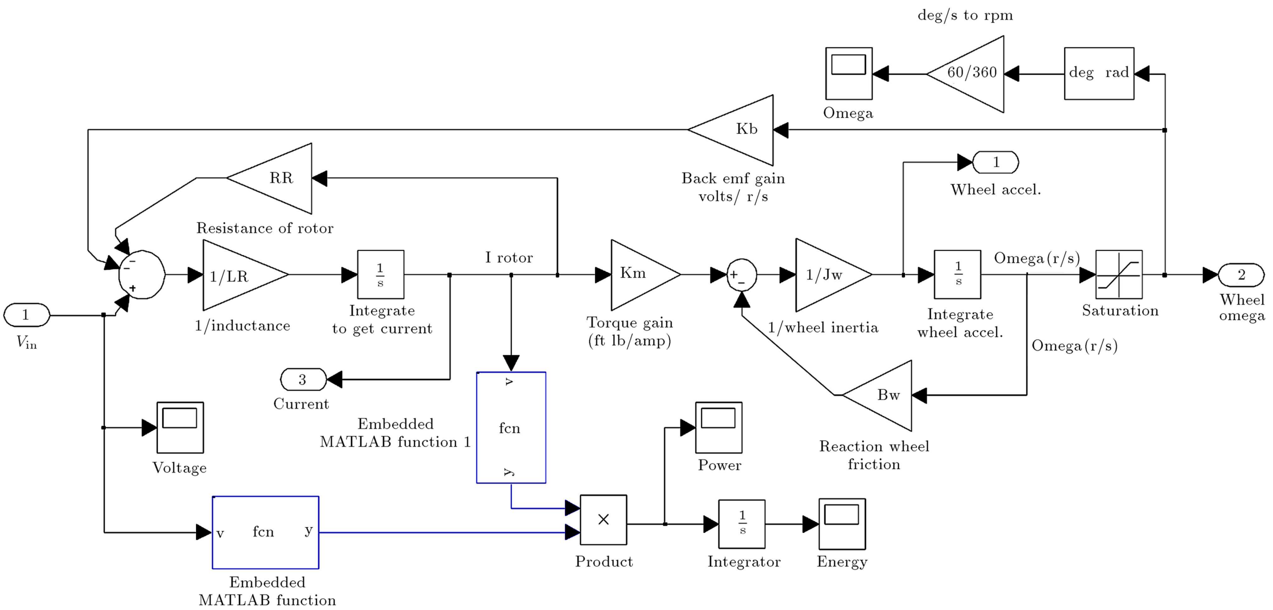


Figure 4. SIMULINK model for DC motor.

Therefore:

$$\begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} = \begin{bmatrix} \sin\beta & 0 & -\sin\beta & 0 \\ 0 & \sin\beta & 0 & -\sin\beta \\ \cos\beta & \cos\beta & \cos\beta & \cos\beta \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}. \quad (19)$$

In Eq. (19), β is shown in Figure 1 and the torques generated by the RW aligned to their axis are called T_i, which are defined as follows:

$$T_i = I_{\mathrm{RW}i}\ \dot{\omega}_{\mathrm{RW}i}. \quad (20)$$

Now, in order to calculate the components, T_i, which are the control torques to be applied by each one of the four wheels, since matrix $[A_w]$ is not square, cannot be inverted. To find the vector components of T_i, minimize the norm of the RW torque vector by defining the Hamiltonian as follows:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \frac{1}{2\sin\beta} \begin{bmatrix} 1 & 0 & \frac{1}{\sin(2\beta)} & \frac{\sin\beta}{2} \\ 0 & 1 & \frac{1}{\sin(2\beta)} & -\frac{\sin\beta}{2} \\ -1 & 0 & \frac{1}{\sin(2\beta)} & \frac{\sin\beta}{2} \\ 0 & -1 & \frac{1}{\sin(2\beta)} & -\frac{\sin\beta}{2} \end{bmatrix} \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \\ 0 \end{bmatrix}. \quad (21)$$

The control torque vector, $\mathbf{T_c}$, is computed by the control law and then, according to Eq. (21), T_i is obtained. By using Eq. (20), the rate of change of the angular momentum of each RW ($\dot{\omega}_{\mathrm{RW}i}$) will be calculated. Finally, by solving the differential Eqs. (16) and (17), the required voltage input to RWs (V_{in}) is computed.

3. The control design based on stability criteria Lyapunov

In order to design a pointing attitude control and guarantee the attitude stability of the spacecraft, consider the following candidate Lyapunov function:

$$V = \frac{1}{2}\boldsymbol{\omega}^{\mathbf{T}}\underline{\mathbf{I}}_{\mathbf{s}}\boldsymbol{\omega} + \eta\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}} + \eta(1 - q_{4E})^2, \quad (22)$$

where $\mathbf{q_E}$ is the error attitude quaternion, η is a positive number, and V is a candidate Lyapunov function, which is positive definite and radially unbounded. In Eq. (34), the error is defined as follows [14]:

$$[\mathbf{q_E}; q_{4E}] = \begin{bmatrix} q_{4T} & q_{3T} & -q_{2T} & q_{1T} \\ -q_{3T} & q_{4T} & q_{1T} & q_{2T} \\ q_{2T} & -q_{1T} & q_{4T} & q_{3T} \\ -q_{1T} & -q_{2T} & -q_{3T} & q_{4T} \end{bmatrix} \begin{bmatrix} -q_{1S} \\ -q_{2S} \\ -q_{3S} \\ q_{4S} \end{bmatrix}, \quad (23)$$

$$\mathbf{q_E} = [q_{1E};\ q_{2E};\ q_{3E}], \quad (24)$$

where q_{iT} and q_{iS} are components of the target attitude quaternion and the component of the spacecraft attitude in quaternion form, respectively. The first time derivative of V is given by:

$$\dot{V} = \boldsymbol{\omega}^{\mathbf{T}}\underline{\mathbf{I}}_{\mathbf{s}}\dot{\omega} + \eta\dot{\mathbf{q}}_{\mathbf{E}}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}} + \eta\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{E}} - 2\eta(1 - q_{4E})\dot{q}_{4E}. \quad (25)$$

As $\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}$ is a 1×3 vector and $\dot{\mathbf{q}}_{\mathbf{E}}$ is a 3×1 vector, then $\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{E}}$ is a scalar and can be shown:

$$\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{E}} = \left(\dot{\mathbf{q}}_{\mathbf{E}}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}}\right)^T = \dot{\mathbf{q}}_{\mathbf{E}}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}}. \quad (26)$$

According to Eq. (26), Eq. (25) can be simplified as follows:

$$\dot{V} = \boldsymbol{\omega}^{\mathbf{T}}\mathbf{I}_{\mathbf{s}}\dot{\boldsymbol{\omega}} + 2\eta\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\dot{\mathbf{q}}_{\mathbf{E}} - 2\eta(1 - q_{4E})\dot{q}_{4E}. \quad (27)$$

The time derivative of the quaternion, by knowing the angular velocity of the spacecraft, is calculated as follows [15]:

$$\dot{\mathbf{q}}_{\mathbf{E}} = \frac{1}{2}\underline{\boldsymbol{\omega}}\mathbf{q}_{\mathbf{E}} + \frac{1}{2}q_4\boldsymbol{\omega}, \tag{28}$$

$$\dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^{\mathbf{T}}\mathbf{q}_E, \tag{29}$$

where $\underline{\boldsymbol{\omega}}$ and $\mathbf{q}$ are defined as:

$$\underline{\boldsymbol{\omega}} = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix}. \tag{30}$$

Substituting Eqs. (28) and (29) into Eq. (27), results in Eq. (31):

$$\begin{aligned}\dot{V} =& \boldsymbol{\omega}^{\mathbf{T}}\mathbf{I_s}\dot{\boldsymbol{\omega}} + 2\eta\left(\frac{1}{2}\underline{\boldsymbol{\omega}}\mathbf{q}_{\mathbf{E}} + \frac{1}{2}q_{4E}\boldsymbol{\omega}\right)^T\mathbf{q}_{\mathbf{E}} \\ &+ 2\eta(1-q_{4E})\frac{1}{2}\boldsymbol{\omega}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}}.\end{aligned} \tag{31}$$

Eq. (31) can be written in an elegant form as follows:

$$\begin{aligned}\dot{V} =& \boldsymbol{\omega}^{\mathbf{T}}\mathbf{I_s}\dot{\boldsymbol{\omega}} + \eta(\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\underline{\boldsymbol{\omega}}^{\mathbf{T}} + q_{4E}\boldsymbol{\omega}^{\mathbf{T}})\mathbf{q}_{\mathbf{E}} \\ &+ \eta(1-q_{4E})\boldsymbol{\omega}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}}.\end{aligned} \tag{32}$$

Since $\underline{\boldsymbol{\omega}}$ is a skew symmetric matrix, $\mathbf{q}_{\mathbf{E}}^{\mathbf{T}}\underline{\boldsymbol{\omega}}^{\mathbf{T}}\mathbf{q}_{\mathbf{E}} = 0$ can then be shown, and Eq. (32) can be simplified as:

$$\dot{V} = \boldsymbol{\omega}^{\mathbf{T}}(\underline{\mathbf{I}}_{\mathbf{s}}\dot{\boldsymbol{\omega}} + \eta\mathbf{q}_{\mathbf{E}}). \tag{33}$$

The attitude control law is expressed in the following Eq. (34):

$$\mathbf{T_c} = -\eta\mathbf{q}_{\mathbf{E}} - \xi\boldsymbol{\omega} + \underline{\boldsymbol{\omega}} \times \mathbf{h_w}, \tag{34}$$

where ξ is a positive number. One advantage of this control law is that $\boldsymbol{\omega}$ and $\mathbf{h_w}$ are available vectors. Therefore, based on Eqs. (34) and (18), the closed-loop dynamic model (Eq. (10)) can be written as follows:

$$\underline{\mathbf{I}}_{\mathbf{s}}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \underline{\mathbf{I}}_{\mathbf{s}}\boldsymbol{\omega} = -\eta\mathbf{q}_{\mathbf{E}} - \xi\boldsymbol{\omega}. \tag{35}$$

According to Eq. (35), Eq. (33) can be simplified as follows:

$$\dot{V} = \boldsymbol{\omega}^{\mathbf{T}}(-\boldsymbol{\omega} \times \underline{\mathbf{I}}_{\mathbf{s}}\boldsymbol{\omega} - \xi\boldsymbol{\omega}) \leq 0. \tag{36}$$

Note that $\boldsymbol{\omega}^{\mathbf{T}}(\boldsymbol{\omega} \times \underline{\mathbf{I}}_{\mathbf{s}}\boldsymbol{\omega}) = 0$. Finally:

$$\dot{V} = -\boldsymbol{\omega}^{\mathbf{T}}\xi\boldsymbol{\omega} \leq 0. \tag{37}$$

As can be seen, the first time derivative of the Lyapunov function is negative semi-definite. The convergence of the spacecraft's attitude is proven using Eq. (37).

$$\lim_{t\to\infty} \boldsymbol{\omega} = 0. \tag{38}$$

According to Eq. (35), the closed loop differential equation can be written as follows:

$$\underline{\mathbf{I}}_{\mathbf{s}}\dot{\boldsymbol{\omega}} = -\eta\mathbf{q}_{\mathbf{E}} - \xi\boldsymbol{\omega} - \boldsymbol{\omega} \times \underline{\mathbf{I}}_{\mathbf{s}}\boldsymbol{\omega}, \tag{39}$$

which, using Eqs. (38) and (39), can be expressed as:

$$\lim_{t\to\infty} \mathbf{q}_{\mathbf{E}} = 0. \tag{40}$$

Eq. (40) shows that the error reaches zero and the attitude of the spacecraft converges to that desired. The Lyapunov function satisfies the requirements of the Barbashin-Krasovskii theorem. Then, the global asymptotic stability of the individual spacecraft attitude controller is proven.

4. Results

The moment of inertia matrix is expressed in the spacecraft body coordinate system. In order to avoid saturation, the angle of the square arrangement is chosen by trial and error, and is considered equal to 58 degrees. All four wheels have the same moment of inertia. The flywheel mass and shape are optimized to obtain a high inertia/mass ratio. The system parameters and initial conditions are given in Table 1.

Consider the initial conditions and desired attitude as follows:

$$\omega_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix},$$

$$\varphi_0 = 0.0563 \quad \text{rad},$$

$$\theta_0 = 0.0778 \quad \text{rad},$$

Table 1. Simulation parameters.

Parameter	Value	Unit
$\underline{\mathbf{I}}_{\mathbf{s}}$	$\begin{bmatrix} 0.3380 & 0.0013 & -0.00012 \\ 0.0013 & 0.3389 & -0.0034 \\ -0.00012 & -0.0034 & 0.03278 \end{bmatrix}$	kg.m^2
$I_{wx} = I_{wy}$	0.00027	kg.m^2
$I_{\mathrm{RW}} \equiv I_{wz}$	0.00054	kg.m^2
ω^*_{sat}	370	rad/s
V^*_{sat}	12	V
I^*_{sat}	3	A

* The subscript "sat" refers to the saturation value for motors.

$$\psi_0 = 0.0755 \quad \text{rad},$$

$$\varphi_{\text{desired}} = 0.34 \quad \text{rad},$$

$$\theta_{\text{desired}} = 0.27 \quad \text{rad},$$

$$\psi_{\text{desired}} = 0.16 \quad \text{rad}.$$

The following figure shows the attitude motion of the spacecraft. As mentioned above the initial conditions, the attitude controller generates torque commands to orient the spacecraft to the desired attitude, and the actuators will provide it.

As can be seen in Figure 5, the controller performs well and approximately after 35 seconds the spacecraft achieves the desired attitude.

In order to check controller robustness against actuator failure, we assumed a failure occurred in one of the actuators and the controller attempts to put the spacecraft with 3 RWs into the desired attitude.

According to Figure 6, the controller handles uncertain actuator failures effectively and the spacecraft just needs some extra time to achieve the desired attitude.

Also, in order to check controller robustness against disturbance torque, the maximum applied disturbance torque is 18% of the maximum torque-producing capacity of RW.

According to Figure 7, the controller handles uncertain actuator failures effectively and the spacecraft just needs some extra time to achieve the desired attitude.

Finally, controller robustness against parameter uncertainty will be checked. In this case, the moment of inertia of the spacecraft has increased to 150% its actual value.

As can be seen in Figure 8, the spacecraft oriented to the desired attitude and stabilized, but required some extra time.

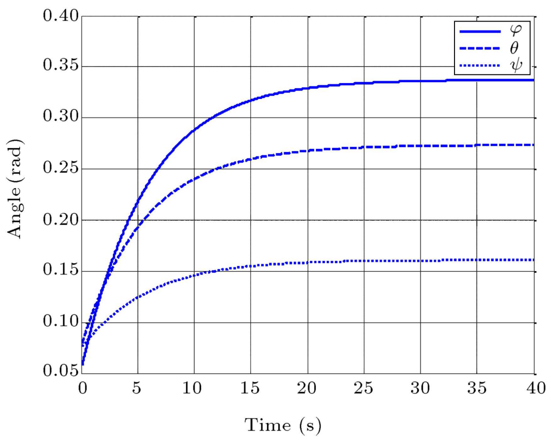
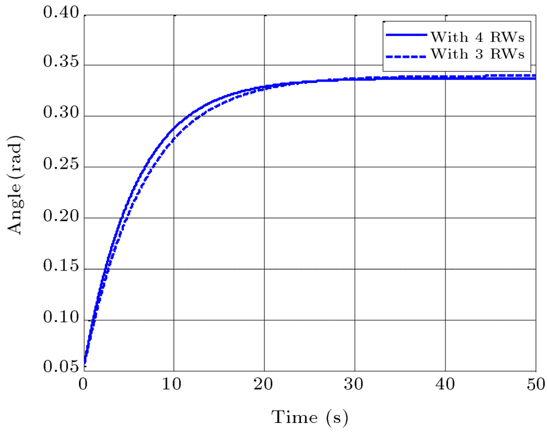
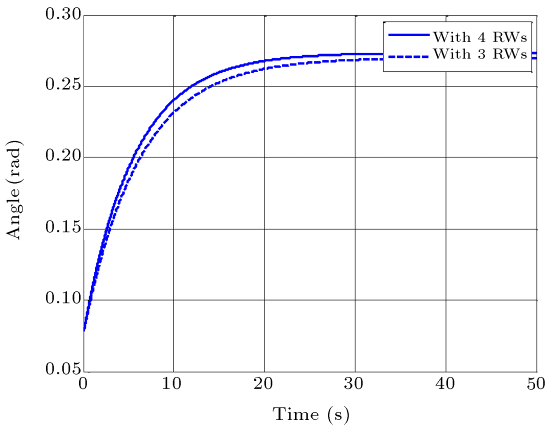


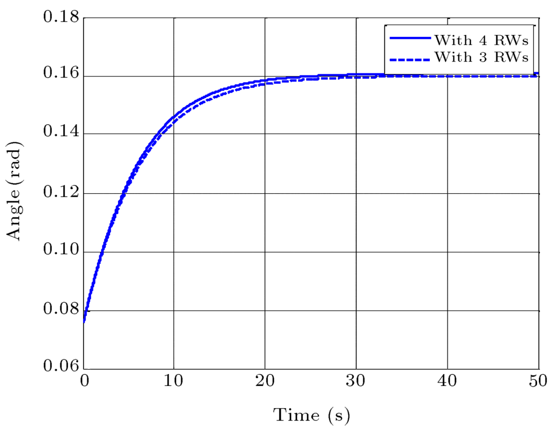
Figure 5. Attitude time response.



(a)



(b)



(c)

Figure 6. Comparing between controller with 4 RWs and 3 RWs, (a) φ, (b) θ and (c) ψ.

5. Conclusion

In this study, we designed a three-axis attitude controller to stabilize and orient a spacecraft to the desired attitude. Numerical simulations indicate that this controller is robust against parameter variation and disturbances, and show the reliability of the mentioned configuration.

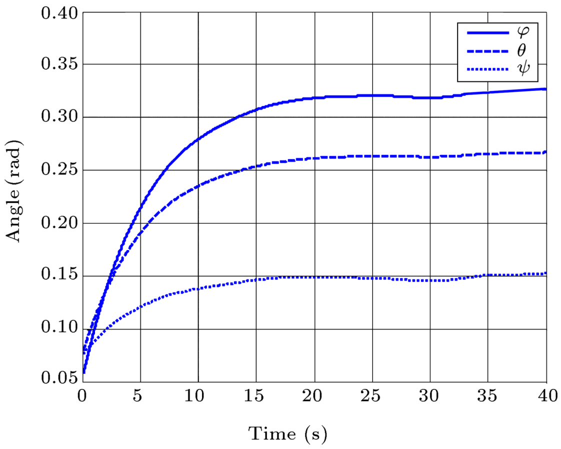


Figure 7. Attitude time response in the presence of disturbance.

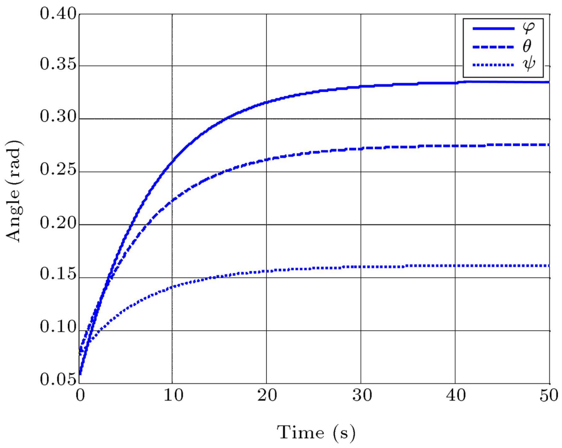


Figure 8. Attitude time response with uncertainty.

As can be seen in Figure 5, the spacecraft reaches the desired attitude and is stabilized after 34 seconds. In Figure 6, the spacecraft attitude time response between normal conditions and a case in which one of the reaction wheels fails is compared. It is clear that the spacecraft reaches the desired attitude with more time (40 seconds).

In Figure 7, the effect of disturbance on the system is studied and results show that the spacecraft can reach the desired attitude in the presence of disturbance 0.0034 N.m (18% in comparison with the allowable reaction wheel capacity). This controller has robustness against up to 0.018 N.m (43%). Also, in Figure 8, it can be seen that the spacecraft reaches the desired attitude and stabilizes with 150% change in the spacecraft moment of inertia. It is robust against up to 350%.

Our future plans are to develop and implement the mentioned controller for the novel 3DOF ADCS simulator, which was designed and manufactured in the *System Dynamics and Control Research Laboratory* of the Mechanical Engineering Department at Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran.

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Biographies

Mostafa Bagheri was born in 1988. He received BS and MS degrees in Mechanical Engineering in 2010 and 2013, respectively, from Amirkabir University of Technology (Tehran Polytechnic), Iran, where he is currently research assistant in the System Dynamics and Control Research Laboratory at the Mechanical Engineering Department. His main research interests include: areas of attitude and control of space systems, adaptive control, orbital mechanic, trajectory optimization, and engineering mathematics.

Mansour Kabganian received a BS degree from Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, in 1975, an MS degree from the University of Tarbiat Modarres, Tehran, in 1988, and a PhD degree from the University of Ottawa, Canada, in 1995, all in Mechanical Engineering. He is currently working in the Department of Mechanical Engineering at Amirkabir University of Technology, Tehran, as Professor and Head of the System Dynamics and Control Research Laboratory. He is also Head of the Space Dynamics and Control Research Center, which he established in 2009. His main research interests include space systems, adaptive control, stability, and identification of dynamical systems such as spacecraft, vehicles and microrobotics.

Reza Nadafi received BS and MS degrees in Mechanical Engineering in 2005 and 2007, respectively, from Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, where he is currently faculty member of the Space Dynamics and Control Research Group. His main research interests include: guidance and control of spacecraft, trajectory design, attitude control, adaptive control, stability and microsystems.