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ANALYTICAL AND EXPERIMENTAL PREDICTOR-BASED TIME DELAY CONTROL OF BAXTER ROBOT

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ABSTRACT

In this paper, a predictor-based controller for a 7-DOF Baxter manipulator is formulated to compensate a time-invariant input delay during a pick-and-place task. Robot manipulators are extensively employed because of their reliable, fast, and precise motions although they are subject to large time delays like many engineering systems. The time delay may lead to the lack of high precision required and even catastrophic instability. Using common control approaches on such delay systems can cause poor control performance, and uncompensated input delays can produce hazards when used in engineering applications. Therefore, destabilizing time delays need to be regarded in designing control law. First, delay-free dynamic equations are derived using the Lagrangian method. Then, we formulate a predictor-based controller for a 7-DOF Baxter manipulator, in the presence of input delay, in order to track desirable trajectories. Finally, the results are experimentally evaluated.

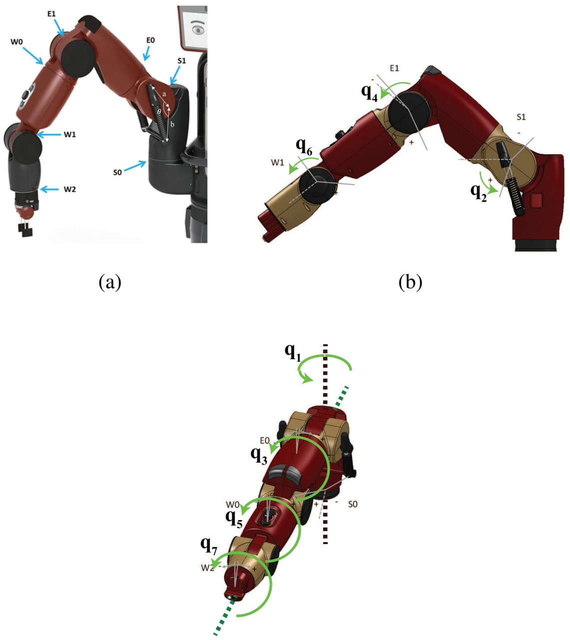
1 Introduction

Robot manipulators are extensively employed because of their reliable, fast, and precise motions [1] although they are subject to large time delays like many engineering systems. Motivation of studying delay, as a common dynamic phenomenon, is induced by applications in control of traffic systems [2], teleoperators [3], and robot manipulators [4, 5], to name only a few.

Time delay may lead to the lack of high precision required and even catastrophic instability. The large input delays emanate from communication delays in sensor-actuator networks, or from the time-consuming computational burden of multi-agent networks. Note that the robustness margin to small input delays can be optimized by applying finite-dimensional feedback laws when the input delay is short relative to the plant's time scales. However, a large input delay needs to be compensated through controller design [6].

Tackling the challenge of control problems with input delays has always been significantly important. In 1959, Smith presented the compensator known as the Smith predictor [7]. However, the Smith predictor may fail to achieve closed-loop stability when the plant is unstable, even though a nominal controller was designed to stabilize the delay-free system [6]. Many efforts were carried out for linear systems subject to the input delays [8–16]. Tsubakino *et al.* [17] proposed a predictor-based state feedback controller for multi-input linear time-invariant (LTI) systems with different time delays in each input channel using the modified backstepping transformation due to the differences among delays.

In addition to the recent developments of predictor-based control laws for nonlinear systems with input delays [18–28], Bekiaris-Liberis and Krstić [29] addressed the problem of stabilization of multi-input nonlinear systems with distinct arbitrary large input delays, and developed a nonlinear version of



(a) (b) (c)

FIGURE 1. (a) The 7-DOF Baxter manipulator; The joints' configuration: (b) sagittal view; (c) top view

the predictor-based control law.

Motivated by the harmful consequences of input delays on the stability and performance of such control systems, we formulate and implement a predictor-feedback controller for the compensation of large input delays of the 7-DOF Baxter manipulator (a multi-input highly nonlinear system). We assume that all input channels induce the same delay, since all commands are sent to the joints simultaneously.

The paper is organized as follows. We begin with the mathematical modeling of the system in Section II, along with deriving equations in order to formulate the predictor-feedback control law in the presence of time delay. In Section III, we present the global asymptotic stability of the closed-loop system using predictor-feedback control law and necessary assumptions. Finally, Section IV is devoted to the results of experiments (pick-and-place task) in order to reveal the significance of the predictor.

2 Mathematical Modeling

The redundant manipulator, which is being studied here, has 7-DOF as shown in Fig. 1. The manipulator's Denavit-Hartenberg parameters are shown in Table 1 provided by the manufacturer.

The mass, Coriolis, and gravitational (stiffness) matrices are symbolically derived using the Euler-Lagrange equation. We employed the symbolic toolbox in MATLAB to obtain the coupled nonlinear dynamic equations as follows [30–32],

TABLE 1. Baxter's Denavit-Hartenberg Parameters

Link	a_i	d_i	α_i	θ_i
1	0.069	0.27035	$-\pi/2$	θ_1
2	0	0	$\pi/2$	$\theta_2+\pi/2$
3	0.069	0.36435	$-\pi/2$	θ_3
4	0	0	$\pi/2$	θ_4
5	0.010	0.37429	$-\pi/2$	θ_5
6	0	0	$\pi/2$	θ_6
7	0	0.3945	0	θ_7

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+\phi(q)=\tau \tag{1}$$

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=U, \quad U=\tau-\phi(q) \tag{2}$$

where, $q,\dot{q},\ddot{q}\in\mathbb{R}^7$ are angles, angular velocities and accelerations of joints, respectively, and $\tau\in\mathbb{R}^7$ indicates the vector of joints' driving torques. Also, $M(q)\in\mathbb{R}^{7\times7}$ and $C(q,\dot{q})\in\mathbb{R}^{7\times7}$, and $\phi(q)\in\mathbb{R}^7$ are the mass, Coriolis, and gravitational matrices, respectively; which are symbolically derived using the Euler-Lagrange equation.

The multi-input nonlinear system (2) can be rewritten as 14^{th}-order Ordinary Differential Equations (ODE) with the following general state-space form,

$$\dot{X}=f(X,U) \tag{3}$$

where $X=\left[q_1,\cdots,q_7,\dot{q}_1,\cdots,\dot{q}_7\right]^T\in\mathbb{R}^{14}$.

3 Designing the Predictor-Based Controller

Dealing with unmolded dynamics including joints' friction and backlash, along with strong dynamic interconnections would cause a complicated problem of designing robust and computationally efficient control schemes to avoid the large destabilizing time delays. Therefore, in this section, we formulate a predictor-based controller for a multi-input nonlinear system in the presence of input delay.

In order to demonstrate the generality of our approach, consider the following general multi-input nonlinear system with constant input delay,

$$\dot{X}(t)=f\left(X(t),U_1(t-D),\cdots,U_m(t-D)\right) \tag{4}$$

where, $X \in \mathbb{R}^n$ is the vector of states, $U_1, \cdots, U_m \in \mathbb{R}$ are the control inputs, D is input delay, and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a locally Lipschitz vector field. We assume that a feedback law $U_i(t) = \kappa_i(X)$ is known such that the functions $\kappa_i : \mathbb{R}^n \to \mathbb{R}$ globally asymptotically stabilize the delay-free system. On the other hand, $\dot{X}(t) = f(X(t), \kappa(X(t)))$ is globally asymptotically stable in the absence of delay. Therefore, in delay system the control law needs to be as follows,

$$U_i(t-D) = \kappa_i(X(t)) \tag{5}$$

which can alternatively be written as,

$$U_i(t) = \kappa_i(X(t+D)) = \kappa_i(P(t)) \tag{6}$$

where $P(t)$ is the D-time units ahead predictor of $X(t)$. The predictor feedback law for the system (4) is given by,

$$P(t) = X(t) + \int_{t-D}^{t} f(P(\theta), U_1(\theta), U_2(\theta), \cdots, U_m(\theta))\, d\theta \tag{7}$$

with initial conditions for the integral (7),

$$P(\theta) = X(0) + \int_{-D}^{\theta} f(P(s),, U_1(s), U_2(s), \cdots, U_m(s))\, ds \tag{8}$$

where $\theta \in [-D, 0]$.

Note that $P(t)$ is calculated based on its past values, however a solution $P(t)$ to (7) does not always exist since the control applied after $t = 0$ has no effect on the plant over the time interval $[0, D]$; system (4) can consequently exhibit finite escape over that interval. Therefore, in order to ensure the global existence of the predictor state, we assume that, for all initial conditions and all locally bounded input signals, the system's solutions exist for all time. This property is called forward completeness.

Definition 1. A system is *forward-complete* if it has bounded solutions (and a suitable continuous gain function) for any bounded input.

Forward-complete systems include all linear systems both stable and unstable, as well as various nonlinear systems that have linearly bounded nonlinearities, such as systems containing trigonometric nonlinearities, as a result of rotational motions (e.g. robotic manipulators). Therefore, the following assumption for our system is made,

Assumption 1. *The system $\dot{X} = f(X, U_1, \cdots, U_m)$ is forward complete with respect to $U = [U_1, \cdots, U_m]^T$.*

Assumption 1 guarantees that the system does not escape in finite time and, in particular, before the input reaches the system at $t = D$. It ensures that for every initial condition and every locally bounded input signal the corresponding solution is defined for all $t \geq 0$.

Forward complete systems yield global stability when predictor feedback is applied to them. In order to have the global asymptotic stability of closed-loop system $\dot{X}(t) = f(X(t), \kappa(X(t)))$, at the expense of not having a Lyapunov functional available, it can be shown that the closed-loop system should be Input-to-State Stable (ISS). Note that we use the common definition of class $\mathscr{K}$, $\mathscr{K}_\infty$ and $\mathscr{KL}$ functions from [33].

Definition 2. A system is *ISS* if there exist $\gamma \in \mathscr{K}, \beta \in \mathscr{KL}$ such that for all initial conditions $x(t_0)$, u, and $t \geq 0$

$$|x(t)| \leq \beta(|x_0|, t - t_0) + \gamma(\|u\|_\infty) \tag{9}$$

where $\|u\|_\infty$ is the L_∞ norm of u. An equivalent condition for ISS is the existence of a smooth (continuously differentiable) $V : \mathbb{R}^n \to \mathbb{R}$ (ISS-Lyapunov function) such that $\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$ for some $\alpha_1, \alpha_2 \in \mathscr{K}_\infty$ and satisfying for all x, u:

$$\frac{\partial V}{\partial x} f(x, u) \leq -\alpha(|x|) + \sigma(|u|) \tag{10}$$

where $\sigma \in \mathscr{K}$ and $\alpha \in \mathscr{K}_\infty$

The Input-to-State Stability of our system can be proved using the following theorem [34].

Theorem 1. *A system is ISS provided that:*

1. *it admits a quasi ISS Lyapunov function:*

$$\dot{V}_1 \leq -\gamma(|y|) + \sigma_1(|u|) \tag{11}$$

2. *it admits an Input-Output to State Stable (IOSS) Lyapunov function:*

$$\dot{V}_2 \leq -\alpha(|x|) + \sigma_2(|u|) + \lambda(|y|) \tag{12}$$

3. $\lim\sup_{r \to \infty} \lambda(r)/\gamma(r) < +\infty$

Note that in (2), $M(q)$ is a periodic function. In particular, $M(q)$ is a uniformly bounded matrix and $\exists \alpha_1, \alpha_2$ such that $\alpha_1 I \leq H(q) \leq \alpha_2 I$. Therefore, we have the following inequalities,

$$|H(q)| \leq c_1 \tag{13}$$

$$|\dot{H}| \leq c_2 |\dot{q}| \tag{14}$$

$$|C(q, \dot{q})| \leq c_3 |\dot{q}| \tag{15}$$

In order to control the system (2), U is chosen as

$$U = -K_P(q-q_d) - K_D\dot{q} \tag{16}$$

where, K_P and K_D are symmetric positive definite matrices, q_d is the vector of desired trajectories, and q_d indicates a generic function of time to be regarded as input of the closed-loop system. Considering the following Lyapunov function, we can establish that our system holds the first assumption of Theorem 1,

$$V_1(q,\dot{q}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \frac{1}{2}q^T K_P q \tag{17}$$

Note that V_1 is not a proper ISS-Lyapunov function for the system since $\dot{V}_1$ is only negative semi-definite [34].

$$\dot{V}_1 = -\dot{q}^T K_D \dot{q} + \dot{q}^T K_P q_d \leq -\alpha_1|\dot{q}|^2 + \alpha_2|q_d|^2 \tag{18}$$

where α_1 and α_2 are sufficiently small and large constants, respectively. Defining the system output y as $\dot{q}$, V_1 becomes a quasi ISS-Lyapunov function. We hence define the following Lyapunov function,

$$V_2(q,\dot{q}) = \frac{1}{2}\dot{q}^T H(q)\dot{q} + \frac{1}{2}q^T K_P q + \varepsilon\frac{q^T H(q)\dot{q}}{\sqrt{1+q^T q}} \tag{19}$$

which is a positive definite function for a sufficiently small value of ε. The derivative of V_2 with respect to time is [34],

$$\begin{aligned}
\dot{V}_2(q,\dot{q}) = &- \dot{q}^T\left(K_D - \varepsilon\frac{H}{\sqrt{1+q^T q}}\right)\dot{q} - \varepsilon\frac{q^T K_P q}{\sqrt{1+q^T q}} \\
&+ \dot{q}^T K_P q_d + \varepsilon\frac{q^T K_P q_d}{\sqrt{1+q^T q}} + \varepsilon\frac{q^T\left(\dot{H}-C-K_D\right)\dot{q}}{\sqrt{1+q^T q}} \\
&- \varepsilon\frac{\left(q^T\dot{q}\right)\left(q^T H\dot{q}\right)}{(1+q^T q)^{3/2}} \\
&\leq \frac{1}{2}\dot{q}^T K_D\dot{q} - \frac{1}{2}\varepsilon\frac{q^T K_P q}{\sqrt{1+q^T q}} \\
&+M_1\left(|q_d| + |q_d|^2\right) + M_2|\dot{q}|^2
\end{aligned} \tag{20}$$

where $\varepsilon > 0$ is sufficiently small and $M_1, M_2 > 0$ are sufficiently large. The last inequality follows due to bounded H, $\dot{H}$, and C as in (13) - (15), and exploiting the fact that $\forall v, w \in \mathbb{R} \quad -v^2 + vw \leq -v^2/2 + 2w^2$. Consequently, the following is held:

$$\frac{1}{2}\dot{q}^T K_D\dot{q} - \frac{1}{2}\varepsilon\frac{q^T K_P q}{\sqrt{1+q^T q}} \leq -\varepsilon_1\frac{|\dot{q}|^2 + |q|^2}{\sqrt{1+|\dot{q}|^2+|q|^2}} \tag{21}$$

Defining $\alpha(r) = r^2/\sqrt{1+r^2}$ and $x = [q^T \ \dot{q}^T]^T$, we observe that V_2 is an IOSS Lyapunov function [34],

$$\dot{V}_2 \leq -\alpha(|x|) + M_1|q_d|^2 + M_2|y|^2 \tag{22}$$

Finally, according to (18) and (22), $\lambda(r)/\gamma(r)$ is a constant, and the assumption 3 of Theorem 1 is satisfied,

$$\lim\sup_{r\to\infty}\frac{M_2 r^2}{\alpha_2 r^2} < \infty \tag{23}$$

Therefore, using Theorem 1, the system (2) is ISS indicating that the following assumption for our system is held,

Assumption 2. *The system $\dot{X} = f(X, \omega_1 + \kappa_1(X), \cdots, \omega_m + \kappa_m(X))$ is Input-to-State Stable (ISS) with respect to $\omega = [\omega_1, \cdots, \omega_m]^T$.*

The stability proof of the closed-loop system with input delay is based on an equivalent representation of plant (4), using transport PDEs (first-order hyperbolic PDEs) for the actuator states, as well as an equivalent PDE representation of the predictor states (7). Therefore, we present the equivalent representations for the plant and the predictor states using transport PDE.

3.1 Equivalent Representation of the Plant Using Transport PDEs for the Actuator States

Delay, as a common dynamic phenomenon, can be represented by a partial differential equation (PDE) of transport type, which evolves in one spatial dimension, with one derivative in space and one derivative in time.

ODEs with delays are interconnected systems of ODEs and transport PDEs. A control system with ODE plant in the presence of input delay, has a cascade PDE-ODE structure, where the control signal enters through a boundary condition of the PDE [6]. System (4) can be written equivalently as the following PDE system,

$$\dot{X}(t) = f(X(t), u_1(0,t), \cdots, u_m(0,t)) \tag{24}$$

$$\partial_t u_i(x,t) = \partial_x u_i(x,t), \quad x \in (0,D), \quad i = 1,\cdots,m \tag{25}$$

$$u_i(D,t) = U_i(t) \qquad i = 1,\cdots,m \tag{26}$$

Note that the solutions to (25) and (26) are given by

$$u_i(x,t) = U_i(t+x-D), \ \ x\in[0,D] \qquad i = 1,\cdots,m \tag{27}$$

3.2 Transport PDE Representation of the Predictor States

The predictor states $P(\theta)$ (8) can be written as

$$p(x,t) = X(t) + \int_0^x f\left(p(y,t), u_1(y,t), \cdots, u_m(y,t)\right) dy \quad (28)$$

where $x \in [0,D]$. Based on (28), the functions $p(x,t)$ satisfying the following ODE in x:

$$\partial_x p(x,t) = f\left(p(x,t), u_1(x,t), \cdots, u_m(x,t)\right) \quad x \in [0,D] \quad (29)$$

with initial solution,

$$p(0,t) = X(t) \quad (30)$$

We intend to reveal that the solution to (29) and (30) is

$$p(x,t) = X(t+x), \quad x \in [0,D] \quad (31)$$

Note that the function $X(t+x)$ satisfies the ODE in x (29) due to the (4):

$$X'(t+x) = f\left(X(t+x), U_1(t+x-D), \cdots, U_m(t+x-D)\right), \quad (32)$$

for all $t \geq 0$ and $0 \leq x \leq D$. It is resulted from the uniqueness of solutions to the ODE (4). Therefore, by defining

$$p(D,t) = P(t) \quad (33)$$

and using the fact that p is a function of one variable, namely $x+t$, one can conclude that

$$P(t+x-D) = p(x,t), \quad x \in [0,D] \quad (34)$$

Doing the change of variables $x = \theta + D - t$ in (28), and utilizing (27), (30), and (34) results in

$$P(\theta) = X(t) + \int_{t-D}^{\theta} f\left(P(s), U_1(s), \cdots, U_m(s)\right) ds \quad (35)$$

where $t - D \leq \theta \leq t$. Finally, with this representation, the following holds

$$U_i(t) = u_i(D,t) = \kappa_i(p(D,t)), \quad i = 1, \cdots, m \quad (36)$$

We define the backstepping transformation of u_i and its inverse backstepping based on the following lemmas:

Lemma 1. *The direct backstepping transformations of u_i, $i = 1, \cdots, m$, are defined by*

$$w_i(x,t) = u_i(x,t) - \kappa_i(p(x,t)) \quad (37)$$

with $x \in [0,D]$, $p(x,t)$ is defined in (28)*, and transform system* (24)-(26) *to the following "target system":*

$$\dot{X}(t) = f\Big(X(t), w_1(0,t) + \kappa_1(X(t)), w_m(0,t) + \kappa_m(X(t))\Big) \quad (38)$$

$$\partial_t w_i(x,t) = \partial_x w_i(x,t), \quad x \in (0,D) \quad (39)$$

$$w_i(D,t) = 0 \quad (40)$$

Lemma 2. *The inverse backstepping transformations of* (37) *are defined by*

$$u_i(x,t) = w_i(x,t) + \kappa_i(\pi(x,t)), \quad i = 1, \cdots, m \quad (41)$$

with $x \in [0,D]$ and $\pi(x,t)$ is defined as follows

$$\pi(x,t) = X(t) + \int_0^x f\Big(\pi(y,t)\ ,\ w_1 + \kappa_1(\pi(y,t)), \cdots, w_m + \kappa_m(\pi(y,t))\Big) dy \quad (42)$$

Therefore,

$$\partial_x \pi(x,t) = f\left(\pi(x,t), \kappa_1(x,t) + w_1(x,t), \cdots, \kappa_m(x,t) + w_m(x,t)\right) \quad (43)$$

where $x \in [0,D]$, with the following initial solution,

$$\pi(0,t) = X(t) \quad (44)$$

Also, based on assumption 2, we can mention the following lemma,

Lemma 3. *There exist a class $\mathcal{KL}$ function β_1 such that the following holds*

$$\bar{\Xi}(t) \leq \beta_1\left(\bar{\Xi}(0), t\right), \qquad \text{for all } t \geq 0 \quad (45)$$

where

$$\bar{\Xi}(t) = |X(t)| + \sum_{i=1}^{m} \|w_i(.,t)\|_\infty \quad (46)$$

The following lemmas help the proof of closed-loop system stability [29],

Lemma 4. *There exist class $\mathscr{K}_\infty$ functions ρ such that*

$$\|p(.,t)\|_\infty \leq \rho(\Xi(t)) \tag{47}$$

where

$$\Xi(t) = |X(t)| + \sum_{i=1}^{m} \|u_i(.,t)\|_\infty \tag{48}$$

Lemma 5. *There exist class $\mathscr{K}_\infty$ functions $\bar{\rho}$ such that*

$$\|\pi(.,t)\|_\infty \leq \bar{\rho}(\bar{\Xi}(t)) \tag{49}$$

where $\bar{\Xi}$ is defined in (46).

Lemma 6. *There exist class $\mathscr{K}_\infty$ functions ρ and $\bar{\rho}$ such that*

$$\bar{\Xi}(t) \leq \rho(\Xi(t)) \tag{50}$$

$$\Xi(t) \leq \bar{\rho}(\bar{\Xi}(t)) \tag{51}$$

Finally, based on the mentioned technical lemmas 1-6, the following theory can be proved [29].

Theorem 2. *Consider the closed-loop system consisting of the plant* (24) - (26) *and the control laws* (36)*. Under Assumptions 1 and 2 there exists a β function belongs to class $\mathscr{KL}$ such that for all initial conditions $X_0 \in \mathbb{R}^n$ and $u_{i_0} \in C[0,D]$, $i = 1,\cdots,m$, which are compatible with the feedback laws, that is, they satisfy $u_{i_0}(D) = \kappa_i(p(D,0))$, $i = 1,\cdots,m$, the closed-loop system has a unique solution $X(t) \in C^1[0,\infty)$ and $u_i(x,t) \in C\left([0,D]\times[0,\infty)\right)$, $i = 1,\cdots,m$, and the following holds for all $t \geq 0$,*

$$\Xi(t) \leq \beta\left(\Xi(0),t\right), \qquad \text{for all } t \geq 0 \tag{52}$$

where $\Xi(t)$ defined in (48).

Proof of Theorem 2: By combining (51) with (45), we get that $\Xi(t) \leq \bar{\rho}\left(\beta_1(\bar{\Xi}(0),t)\right)$, for all $t \geq 0$, and hence, with (51) we arrive at (52) with $\beta(s,t) = \bar{\rho}\left(\beta_1(\rho(s),t)\right)$. The proof of existence and uniqueness of a solution $X(t) \in C^1[0,\infty)$ and $u_i(x,t) \in C([0,D]\times[0,\infty))$, $i = 1,\cdots,m$, is shown as follows. Using relation (29) for $t = 0$, the compatibility of the initial conditions $u_{i_0}, i = 1,\cdots,m$, with the feedback laws (36) guarantee that $p(x,0) \in C^1[0,D]$. Hence, using relations (28), (30), and the fact that $u_{i_0} \in C[0,D]$, $i = 1,\cdots,m$, it also follows from (37) that $w_{i_0} \in C[0,D]$, $i = 1,\cdots,m$. The solutions to (39) and (40) are given for all $i = 1,\cdots,m$ by

$$w_i(x,t) = \begin{cases} w_{i_0}(t+x), & 0 \leq x+t \leq D \\ 0, & x+t \geq D \end{cases} \tag{53}$$

There is a unique solution for (53) due to the uniqueness of the solution to (39) and (40). Hence, the compatibility of the initial conditions u_{i_0}, $i = 1,...,m$, with the feedback laws (36) guarantee the existence of a unique solution $w_i(x,t) \in C\left([0,D]\times[0,\infty)\right)$, $i = 1,\cdots,m$. Also, it follows that $X(t) \in C^1[0,\infty)$ from the target system (38). The fact that $\pi(x,t) = X(t+x)$, for all $t \geq 0$ and $x \in [0,D]$, along with the inverse backstepping transformations (41) guarantee that $u_i(x,t) \in C\left([0,D]\times[0,\infty)\right)$, $i = 1,\cdots,m$. The proof is completed [29].

By using (27), Theorem 2 can be written in the form of standard delay notation as follows [29],

Corollary 1 (Theorem 2 in Standard Delay Notation)**.** *Consider the closed-loop system consisting of the plant* (4) *and the control laws* (6)-(7)*. Under Assumptions 1 and 2, the following holds for all $t \geq 0$,*

$$\Omega(t) \leq \beta\left(\Omega(0),t\right) \tag{54}$$

where

$$\Omega(t) = |X(t)| + \sum_{i=1}^{n} \sup_{t-D\leq\theta\leq t} |U_i(\theta)| \tag{55}$$

As Assumptions 1 and 2 are held for our system, we can employ Theorem 2 and Corollary 1, and formulate a predictor-based controller for our system to compensate any large input delays.

Experimental Results

We implement the predictor-based controller for Baxter robot through a pick-and-place task, whereas there is a similar input delay in all input channels. We formulate the control scheme for the highly interconnected nonlinear system and then experimentally validate.

First, in order to reveal the significant destabilizing effect of delay for the robot (Fig. 2), we intentionally apply 0.05s input delay and then control the manipulator without any predictor. The join-space trajectories are demonstrated in Fig. 4.

Shown in Fig. 4 reveals that, for a small input delay, the closed-loop system becomes unstable. It is worth mentioning that we operate the robot using joint torque control mode, as an advanced control scheme. By operating the robot in the torque control mode, access is granted to the lowest control levels. On

FIGURE 2. The failing robot without the predictor

FIGURE 3. A stable obstacle-avoidance pick-and-place task using the predictor-based controller

the other hand, we cannot capture more and accurate data, since Baxter is moving stochastically leading to the catastrophic malfunction. The AVI files of the experiments are accessible through our Dynamic Systems and Control Laboratory (DSCL) website.

Then, we intentionally apply 0.5s input delay and control the system to compensate the destabilizing delay, as shown in Fig. 3. As all the assumptions are held for the system, we take the advantage of the predictor-based controller using Theorem 2 and Corollary 1, in order to globally asymptotically stabilize the robot. The results are shown in Fig. 5.

As can be seen, the robot does not have any motion at the first 0.5s, since there is no control input to the system. Note that the gravity compensation torques need to be applied on the Baxter manipulator in order to oppose the effect of gravitational force; note that this is a basic mode which is, by default, active for the onset of the robot operation. Then, the robot begins tracking the desired join-space trajectories just after 0.5s. As can be observed, the joints are tracking, almost perfectly, the desirable trajectories. As shown in Fig. 5, the negligible error mainly roots on the inaccuracy of sensors and actuators.

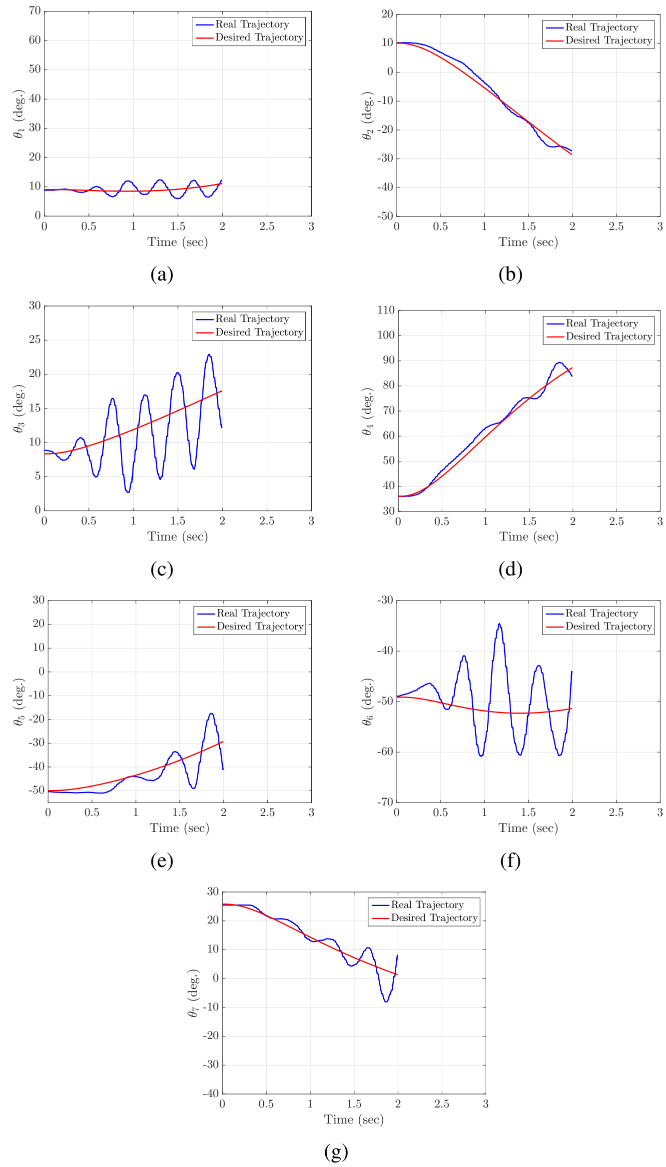


FIGURE 4. Inability to track the desired trajectories without any predictor in the presence of 0.05s input delay

Conclusion

Through this paper, we presented the formulation and implementation of the predictor-based controller for a highly interconnected nonlinear system, Baxter manipulator as a case study, with the time-invariant input delay. The results reveal the significance of predictor used in the feedback control law subject to the destabilizing time delay. We established that our system is forward complete and Input-to-State Stable, and then formulated the predictor-based controller utilizing Theorem 2. The results demonstrate that the manipulator becomes stable and, as expected, tracks the desired joint-space trajectories despite the

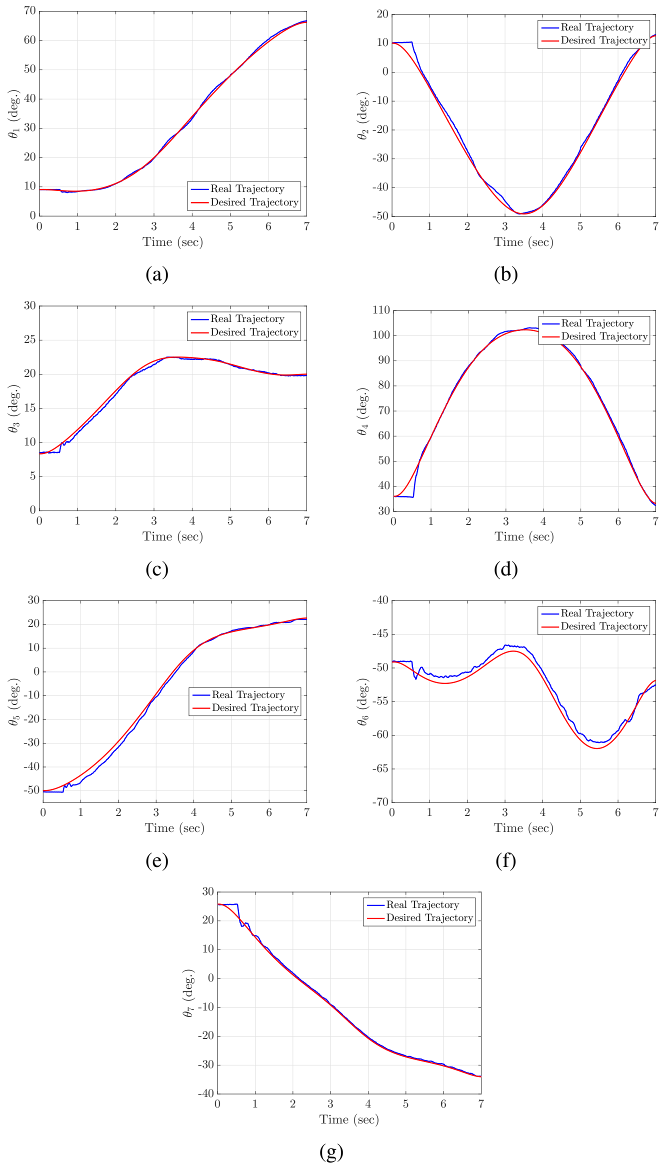


FIGURE 5. Tracking the desired trajectories using predictor-based controller in the presence of 0.5s input delay

destabilizing input delay. The negligible tracking error mainly roots on sensors and actuators inaccuracy.

We are currently focusing our efforts on the creation of a nonlinear adaptive time-delay controller for highly interconnected multi-agent systems; with application to high-DOF robots.

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