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STABLE DESIGN OF ATTITUDE CONTROL FOR A SPACECRAFT

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The three-axis attitude control design based on Lyapunov stability criteria to stabilizing the spacecraft and orients it to desired attitude is presented in this paper. This attitude control system is assumed to have four reaction wheels with optimal arrangement. The reaction wheels are located in square pyramidal configuration. Control system inputs are attitude parameter in the quaternion form and the angular velocity of spacecraft and reaction wheels. The controller output is the torque required to eliminate error. In this study, actuators (reaction wheels) are modeled and required torque for attitude maneuver is converted to voltage of actuators. Armature voltage and armature current is limited to 12 volts and 3 amps respectively. Also, each wheel has an angular velocity limit to 370 rad/sec. Numerical simulations indicate that the spacecraft reaches desired attitude after 34 seconds and show the reliability of mentioned configuration with respect to actuator failure. The results show that in case of failure of one reaction wheel, the spacecraft can reach desired attitude but needs more time. Moreover, results demonstrated the controller robustness against parameter variation and disturbances. It is robust against with up to 350% change in spacecraft moment of inertia and robust against disturbance up to 0.0094 N.m that is equal 38% in comparison with the allowable reaction wheel capacity.

INTRODUCTION

Attitude determination and control system (ADCS) is an important subsystem of a spacecraft which plays an important role in the spacecraft missions. The controller algorithm is essential part of ADCS subsystem that commands the actuators. Many spacecrafts are placed in low earth orbits (LEO) in recent years. Since these spacecrafts are close to earth, effect of orbital disturbances is enormous. [1,2] Using attitude controllers for stabilization and tracking desired trajectory has increased in the second half of the twentieth century. The first studies were on the passive controller for stabilization. [3] But in order to achieve better performance and accuracy, active attitude control is used. [4]

Active control of spacecraft attitude has been addressed by various researchers since the sixties of the 20th century. The reaction wheel (RW) is the most common actuator for rotational control of spacecrafts since it has a simple structure. Also, for accurate attitude control systems and moderately fast maneuvers, RWs are well suited because they allow continuous and smooth control with comparatively low disturbing torques and used to provide torques about three body axes.

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RWs consist of a DC motor with a flywheel assembled on its axis to provide a higher moment of inertia. Control system of three RWs configuration which are parallel to the principal axes of body is not complex, but if one of them fails, the control system is unable to track the desired trajectories. The actuator failures are known to be the main cause to many space mission failures. It is assumed that each reaction wheel has a torque limit, and also has an angular momentum limit. Then, there is an additional problem with regard to saturation. [5]

For these reasons, a square pyramidal configuration is used for RWs to increase the reliability and avoidance of saturation. In this configuration, the RWs are not placed along the body axes and their rotational axes are inclined to the $x_B - y_B$ plane by an angle β, Figure 1.

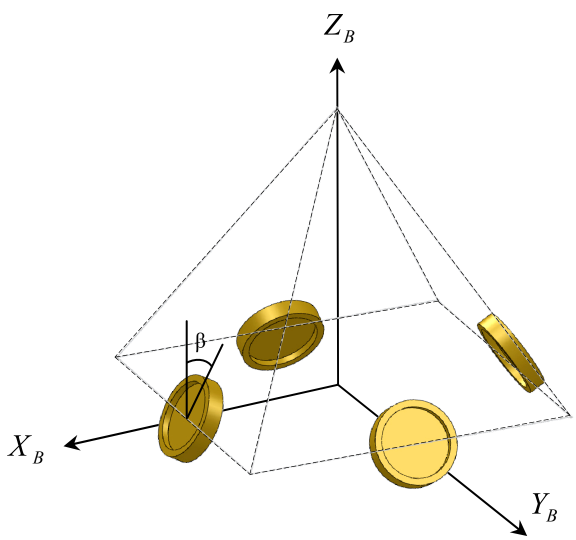


Figure 1. Square Pyramidal Configuration of RWs

In the present paper, a three axis attitude control based on Lyapunov criteria is designed and as mentioned before, RWs are arranged in a square pyramidal configuration. In order to find applied control torque for each RWs, the norm of torque vector should be minimized. The scope of this paper is modeling and simulation of closed loop system that show robustness against external disturbances while maintaining sufficient consistency in parameter variation. The main contribution of this paper is designing a control law and developing a closed loop modeling.

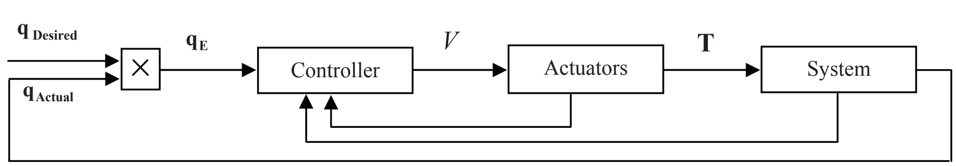


Figure 2. Closed Loop Modeling

EQUATION OF MOTION FOR SPACECRAFT ATTITUDE

Attitude motion of spacecraft that contains a rigid body is formulated. In this study, the coupling between the orbital and the attitude motion and disturbance torque are ignored. According to the law of conservation of angular momentum to three-dimensional, the differential equation of motion for a rigid spacecraft can be written as fallows

$$\mathbf{T} = \dot{\mathbf{h}}_{\mathbf{tI}} = \dot{\mathbf{h}}_{\mathbf{t}} + \boldsymbol{\omega} \times \mathbf{h}_{\mathbf{t}} \tag{1}$$

$$\boldsymbol{\omega} = [\omega_x \quad \omega_y \quad \omega_z] \tag{2}$$

Where $\mathbf{h_t}$ and $\dot{\mathbf{h}}_{\mathbf{t}}$ are spacecraft angular momentum and differentiation of angular momentum in the spacecraft body frame (SBF), respectively, and the subscript " I " indicates a derivative in the inertial frame. The terms ω_x, ω_y and ω_z are angular velocities about the body coordinate axes.

According to Eq. (3), the total angular momentum of system consists of the angular momentums of all component are fixed about the spacecraft rotation axis, $\mathbf{h_s}$, plus the angular momentum of the rotational component about the same axis, $\mathbf{h_{wb}}$, that are in the SBF.

$$\mathbf{h_t} = \mathbf{h_s} + \mathbf{h_{wb}} \tag{3}$$

Therefore, Eq. (1) can be written as follows

$$\mathbf{T} = \mathbf{I_s}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I_s}\boldsymbol{\omega} + \dot{\mathbf{h}}_{\mathbf{wb}} + \boldsymbol{\omega} \times \mathbf{h_{wb}} \tag{4}$$

$\mathbf{T}$ is the sum of all external moments applied to the spacecraft. It consists of two terms: the control torque generated by the reaction control jets and torques due to external torque such as aerodynamic disturbance torques and gravity gradient that all have been neglected as mentioned. $\mathbf{I_s}$ is moment of inertia of the all spacecraft components that are fixed.

Can be defined a transformation matrix $\mathbf{C_w^b}$ that transforms the wheel momentum from individual wheel axis to spacecraft body axes. In this study, is used four RWs, then the transformation from wheel frame to SBF becomes

$$\mathbf{C_w^b} = [\mathbf{a_1} \quad \mathbf{a_2} \quad \mathbf{a_3} \quad \mathbf{a_4}] \tag{5}$$

Where $\mathbf{a_i}$ is reaction wheel spin column vector. In Eq. (4) $\mathbf{h_{wb}}$ is angular momentum of reaction wheel that is in the SBF as mentioned before, is defined as following

$$\mathbf{h_{wb}} = \mathbf{C_w^b}\mathbf{h_w} \tag{6}$$

Where $\mathbf{h_w}$ is angular momentum of reaction wheel in wheel frame. The rate of change of momentum for a reaction wheel is

$$\dot{\mathbf{h}}_{\mathbf{wi}} = -\mathbf{T_i} \tag{7}$$

The control torque is defined as follows

$$\mathbf{T_c} = \mathbf{C_w^b}\mathbf{T_i} \tag{8}$$

Finally, the spacecraft rate of change of momentum under the influence of a RW torques is

$$\mathbf{I_s}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}\times\mathbf{I_s}\boldsymbol{\omega} + \mathbf{T_{wb}} \tag{9}$$

In Eq. (9) $\mathbf{T_{wb}}$ is the torque applied to the spacecraft by the RW array in SBF and defined as

$$\mathbf{T_{wb}} = \mathbf{T_c} - \boldsymbol{\omega}\times\mathbf{C_w^b}\mathbf{h_w} \tag{10}$$

Actuator Modeling

In general, the electric motor is coupled with the load through various mechanical transmission systems. Its angular rate can be varied by applying a torque to the motor about its spin axis. As the wheel accelerates it applies an equal and opposite reaction torque on the spacecraft that is used to control its attitude. A schematic diagram of mentioned actuator is shown in following figure.

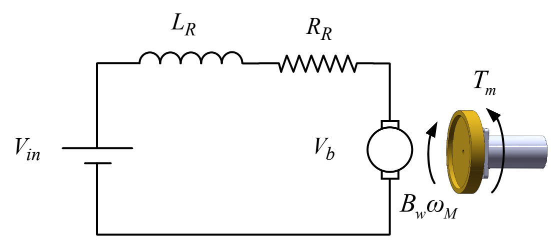


Figure 3. Schematic Diagram of DC Motor

According to Figure 3, for a DC motor the following relation based on Kirchoff's law can be written as following

$$V_{in} - V_b = R_R\,I + L_R\frac{dI}{dt} \tag{11}$$

Where V_{in}, R_R and L_R are is the voltage input to the electrical motor, electrical resistance and armature inductance of the DC motor, respectively. V_b is defined as

$$V_b = K_b\omega_M \tag{12}$$

Where ω_M is angular velocity of wheel relative to spacecraft body and defined as follows

$$\omega_M = \omega_{RW} - \omega_s \tag{13}$$

Also, output motor torque based on the Newton's law is

$$\dot{h}_w = T_m - B_w\omega_M \tag{14}$$

Where B_w is viscous friction and T_m is electromagnetic torque that is relative to the armature current, I, by

$$T_m = K_m I \tag{15}$$

In regard to Eq. (16) and Eq. (17), the voltage of RW, V, (controller output) can be converted to rate of rotor angular momentum, $\dot{h}_w$.

$$V - K_b\left(\omega_{RW} - \omega\right) = R_R\, I + L_R \frac{dI}{dt} \tag{16}$$

$$\dot{h}_w = I_{RW}\left(\dot{\omega}_{RW} - \dot{\omega}\right) = K_m\, I - B_w\left(\omega_{RW} - \omega\right) \tag{17}$$

Where R_R, L_R, B_w, K_b and K_m are usual parameters of the electrical motor as mentioned. [6]

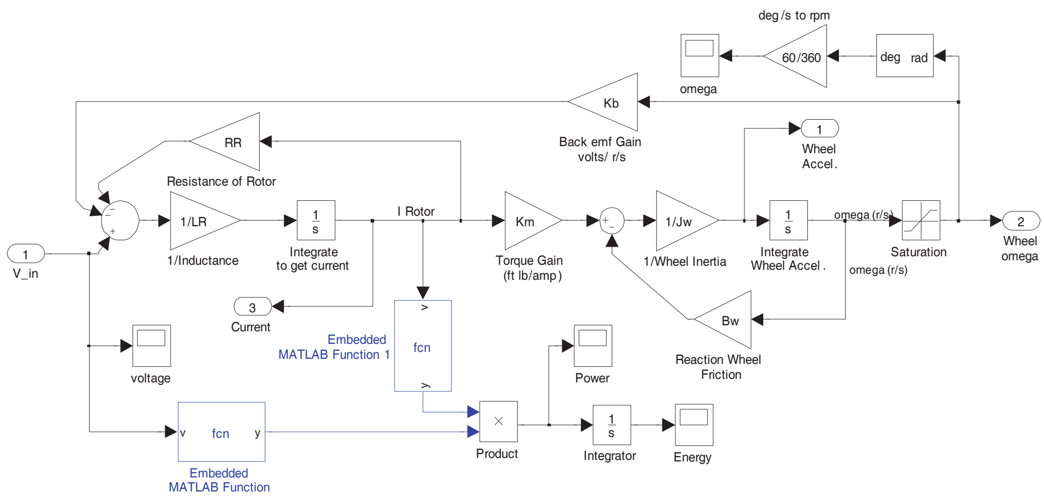


Figure 4. SIMULINK Model for Dc Motor

Square Pyramidal Arrangement of the Reaction Wheels

Square pyramidal arrangement is used because of the advantages of this configuration as mentioned. The torques produced along the three body axes are T_{cx}, T_{cy} and T_{cz}. In order to compute the components of torque vector along the three body axis, the following can be written

$$\begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{bmatrix} = \begin{bmatrix} sin\beta & 0 & -sin\beta & 0 \\ 0 & sin\beta & 0 & -sin\beta \\ cos\beta & cos\beta & cos\beta & cos\beta \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \left[A_w\right]\left[T\right] \tag{18}$$

In Eq. (18) the torques generated by the RW align to their axis are called T_i as mentioned. Now, in order to calculate the components T_i, which are the control torques to be applied by each one of the four wheels since the matrix $\left[A_w\right]$ is not square, cannot be inverted. To find the vector components of T_i, minimize the norm of RW torque vector by defining the Hamiltonian as following

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \frac{1}{2\sin\beta} \begin{bmatrix} 1 & 0 & \frac{1}{\sin(2\beta)} & \frac{\sin\beta}{2} \\ 0 & 1 & \frac{1}{\sin(2\beta)} & -\frac{\sin\beta}{2} \\ -1 & 0 & \frac{1}{\sin(2\beta)} & \frac{\sin\beta}{2} \\ 0 & -1 & \frac{1}{\sin(2\beta)} & -\frac{\sin\beta}{2} \end{bmatrix} \begin{bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \\ 0 \end{bmatrix} \tag{19}$$

The control vector T_c is computed by the control law.

THE CONTROL DESIGN BASED ON STABILITY CRITERIA LYAPUNOV

Knowing the angular velocity of the spacecraft with respect to the inertial reference frame, the time derivative of the quaternion calculated as follows [7]

$$\dot{\mathbf{q}} = \frac{1}{2}\underline{\boldsymbol{\omega}}\,\mathbf{q} + \frac{1}{2}q_4\boldsymbol{\omega} \tag{20}$$

$$\dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^{\mathbf{T}}\mathbf{q} \tag{21}$$

Where $\underline{\boldsymbol{\omega}}$ and $\mathbf{q}$ are defined as

$$\underline{\boldsymbol{\omega}} = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} \tag{22}$$

$$\mathbf{q} = [q_1 \quad q_2 \quad q_3] \tag{23}$$

Attitude control law is expressed in the following Eq. (24)

$$\mathbf{T_c} = -\eta\mathbf{p} - \xi\boldsymbol{\omega} + \underline{\boldsymbol{\omega}} \times \mathbf{h_{wb}} \tag{24}$$

Where η and ξ are positive numbers and $\mathbf{p}$ is error attitude quaternion. One advantage of this control law is that $\boldsymbol{\omega}$ and $\mathbf{h_w}$ are available vectors. Therefore, based on Eq. (24), Eq. (9) and Eq. (10), closed-loop dynamic model can be written as follows

$$\mathbf{I_s}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I_s}\boldsymbol{\omega} = -\eta\,\mathbf{p} - \xi\,\boldsymbol{\omega} \tag{25}$$

In order to guarantee the attitude stability of the spacecraft, consider the following the candidate Lyapunov function

$$V = \frac{1}{2}\boldsymbol{\omega}^{\mathbf{T}}\mathbf{I_s}\boldsymbol{\omega} + \eta\,\mathbf{p}^{\mathbf{T}}\mathbf{p} + \eta\left(1 - p_4\right)^2 \tag{26}$$

Where η is a positive number as mentioned before and V is a candidate Lyapunov function that is positive definite and radially unbounded. In Eq. (24) the error is defined as follows

$$\overline{\mathbf{p}} = (\overline{\mathbf{q}}_\mathbf{S})^{-1}\underline{\mathbf{q}}_\mathbf{T} = \begin{bmatrix} q_{T4} & q_{T3} & -q_{T2} & q_{T1} \\ -q_{T3} & q_{T4} & q_{T1} & q_{T2} \\ q_{T2} & -q_{T1} & q_{T4} & q_{T3} \\ -q_{T1} & -q_{T2} & -q_{T3} & q_{T4} \end{bmatrix} \begin{bmatrix} -q_{S1} \\ -q_{S2} \\ -q_{S3} \\ q_{S4} \end{bmatrix} \tag{27}$$

$$\overline{\mathbf{p}} = [\mathbf{p} \quad p_4] \tag{28}$$

Where $\overline{\mathbf{q}}_\mathbf{S}$ is attitude of spacecraft in quaternion form and $\underline{\mathbf{q}}_\mathbf{T}$ is target attitude quaternion. The first time derivative of V is given by

$$\dot{V} = \boldsymbol{\omega}^\mathbf{T}\mathbf{I}_\mathbf{s}\dot{\boldsymbol{\omega}} + \eta\,\dot{\mathbf{p}}^\mathbf{T}\mathbf{p} + \eta\,\mathbf{p}^\mathbf{T}\dot{\mathbf{p}} - 2\eta(1-p_4)\,\dot{p}_4 \tag{29}$$

As $\mathbf{p}^T\dot{\mathbf{p}}$ is a scalar, can be shown

$$\mathbf{p}^\mathbf{T}\dot{\mathbf{p}} = \left(\dot{\mathbf{p}}^\mathbf{T}\mathbf{p}\right)^\mathbf{T} = \dot{\mathbf{p}}^\mathbf{T}\mathbf{p} \tag{30}$$

According to Eq. (30), Eq. (29) can be simplified as follows

$$\dot{V} = \boldsymbol{\omega}^\mathbf{T}\mathbf{I}_\mathbf{s}\dot{\boldsymbol{\omega}} + 2\eta\,\dot{\mathbf{p}}^\mathbf{T}\mathbf{p} - 2\eta(1-p_4)\,\dot{p}_4 \tag{31}$$

The substituting of Eq. (20) into Eq. (31), results in Eq. (32).

$$\dot{V} = \boldsymbol{\omega}^\mathbf{T}\mathbf{I}_\mathbf{s}\dot{\boldsymbol{\omega}} + 2\eta\left(\frac{1}{2}\underline{\boldsymbol{\omega}}\,\mathbf{p} + \frac{1}{2}p_4\,\boldsymbol{\omega}\right)^\mathrm{T}\mathbf{p} + 2\eta\,(1-p_4)\frac{1}{2}\boldsymbol{\omega}^\mathbf{T}\mathbf{p} \tag{32}$$

Eq. (32) can be written in the elegant form as following

$$\dot{V} = \boldsymbol{\omega}^\mathbf{T}\mathbf{I}_\mathbf{s}\dot{\boldsymbol{\omega}} + \eta\left(\mathbf{p}^\mathbf{T}\underline{\boldsymbol{\omega}}^\mathbf{T} + p_4\,\boldsymbol{\omega}^\mathbf{T}\right)\mathbf{p} + \eta(1-p_4)\boldsymbol{\omega}^\mathbf{T}\mathbf{p} \tag{33}$$

Can be shown $\mathbf{p}^\mathbf{T}\underline{\boldsymbol{\omega}}^\mathbf{T}\mathbf{p} = 0$. Then, Eq. (33) can be simplified as

$$\dot{V} = \boldsymbol{\omega}^T\left(\mathbf{I}_\mathbf{s}\dot{\boldsymbol{\omega}} + \eta\,\mathbf{p}\right) \tag{34}$$

According to Eq. (25), Eq. (34) can be simplified as follows

$$\dot{V} = \boldsymbol{\omega}^\mathbf{T}\left(-\boldsymbol{\omega}\times\mathbf{I}_\mathbf{s}\boldsymbol{\omega} - \xi\,\boldsymbol{\omega}\right) \le 0 \tag{35}$$

Note that $\boldsymbol{\omega}^\mathbf{T}(\boldsymbol{\omega}\times\mathbf{I}\boldsymbol{\omega}) = 0$. Finally,

$$\dot{V} = -\boldsymbol{\omega}^\mathbf{T}\xi\,\boldsymbol{\omega} \le 0 \tag{36}$$

As can be seen, the first time derivative of Lyapunov function is negative semi-definite. For investigation attitude stability of the spacecraft using Lyapunov stability theory. [8]

The convergence of the spacecraft's attitude is proven using Eq. (36).

$$\lim_{t \to \infty} \boldsymbol{\omega} = 0 \tag{37}$$

According to Eq. (25), the closed loop differential equation can be written as follows

$$\mathbf{I_s}\dot{\boldsymbol{\omega}} = -\eta\, \mathbf{p} - \xi\, \boldsymbol{\omega} - \boldsymbol{\omega} \times \mathbf{I_s}\boldsymbol{\omega} \tag{38}$$

Using relations (37) and (38) can be expressed

$$\lim_{t \to \infty} \mathbf{p} = 0 \tag{39}$$

Eq. (39) shows that the error reaches zero and attitude of the spacecraft converges to the desired attitude. Lyapunov function satisfies the requirements of Barbashin-Krasovskii Theorem.[1] Then, the global asymptotic stability of the individual spacecraft attitude controllers is proven.

RESULTS

Moment of inertia matrix is expressed in spacecraft body coordinate system. In order to avoid saturation, the angle of the square arrangement is chosen by trial and error and is considered equal to 58 degrees. All four wheels have the same moment of inertia. The flywheels mass and its shape are optimized to obtain a high inertia/mass ratio. The system parameters and initial conditions are given in Table 1.

Table 1. Simulation Parameters

Parameter	Value	Unit
$\mathbf{I}$	$\begin{bmatrix} 0.3380 & 0.0013 & -0.00012 \\ 0.0013 & 0.3389 & -0.0034 \\ -0.00012 & -0.0034 & 0.03278 \end{bmatrix}$	$Kg.m^2$
$I_{wx} = I_{wy}$	0.00027	$Kg.m^2$
I_{wz}	0.00054	$Kg.m^2$
ω_{sat}	370	rad/s
V_{sat}	12	V
I_{sat}	3	A

Consider the initial conditions and desired attitude as following

$$\boldsymbol{\omega}_\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\varphi_0 = 0.0563\ rad\ , \quad \theta_0 = 0.0778\ rad\ , \quad \psi_0 = 0.0755\ rad$$

$$\varphi_{desired} = 0.34\ rad\ , \quad \theta_{desired} = 0.27\ rad\ , \quad \psi_{desired} = 0.16\ rad$$

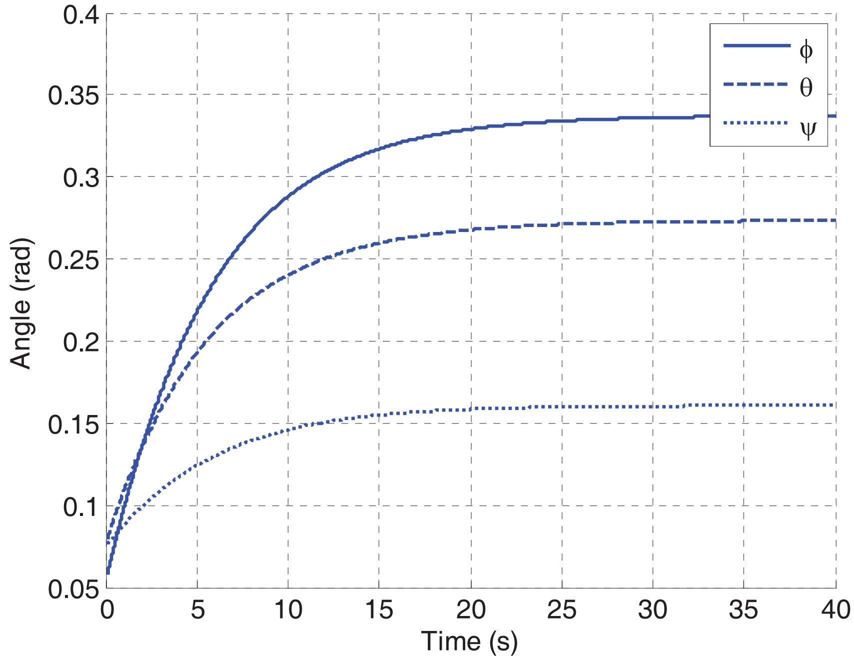
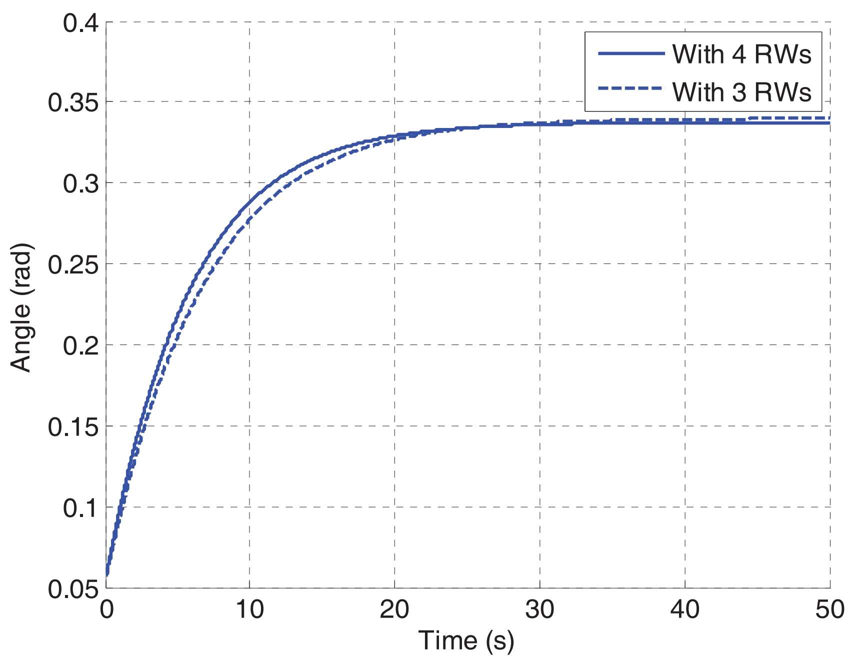
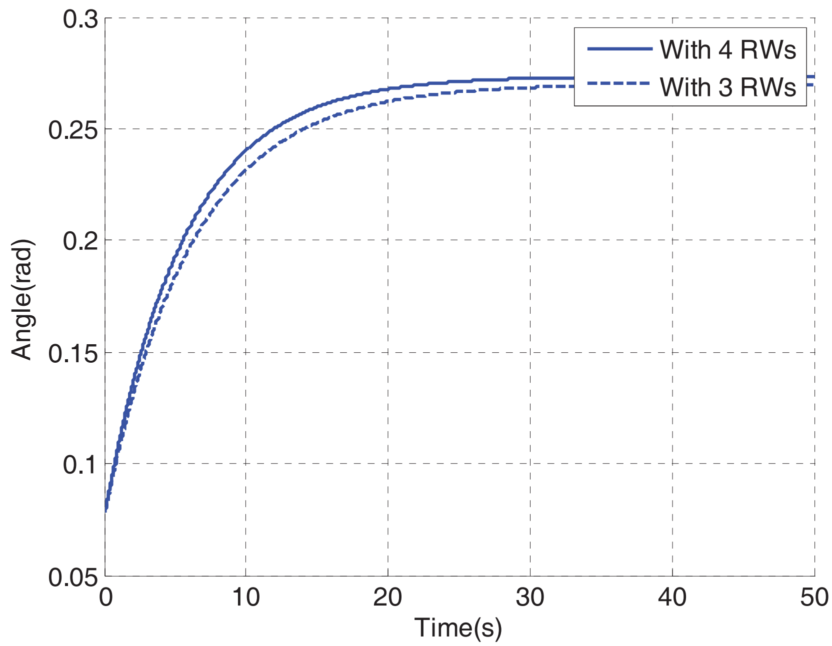


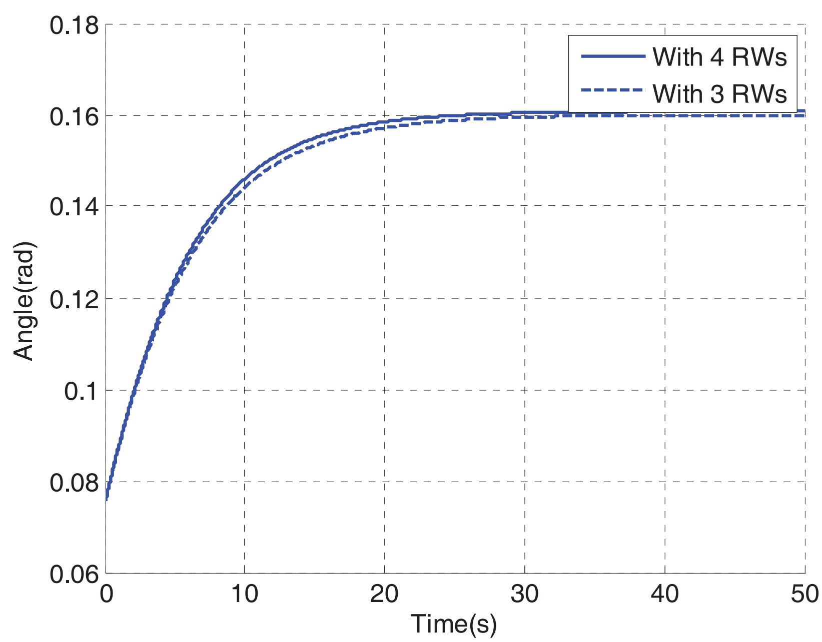
Figure 5. Attitude Time Response



(a)



(b)



(c)

Figure 6. Comparing Between Controller with 4 RWs and 3 RWs, (a) φ , (b) θ and (c) ψ

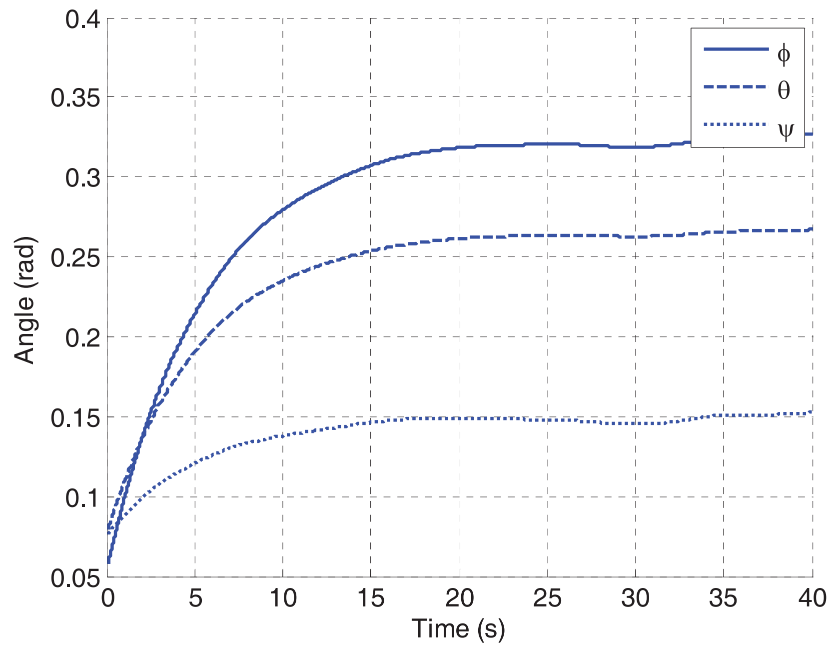


Figure 7. Attitude Time Response in the Presence of Disturbance

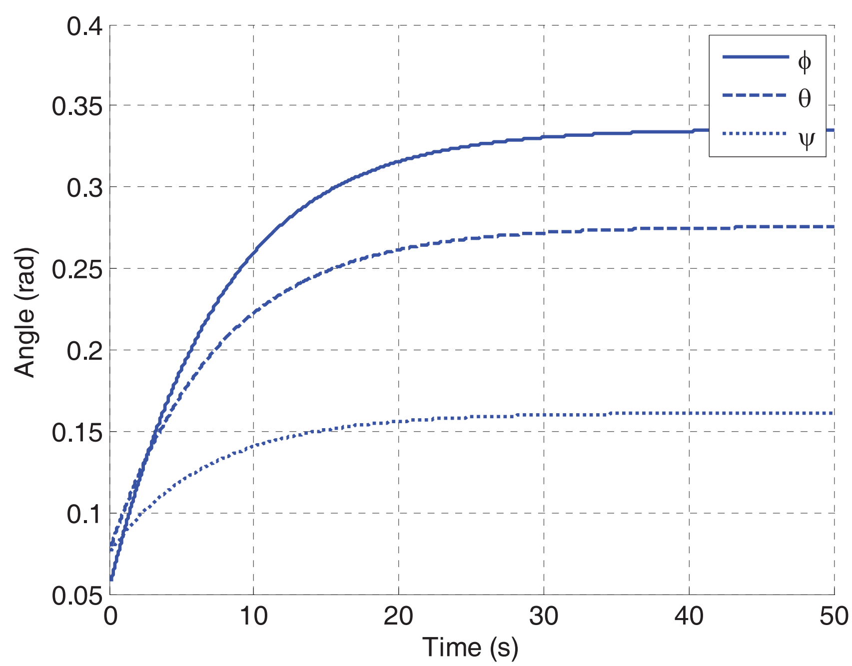


Figure 8. Attitude Time Response with uncertainty

CONCLUSION

In this study, designed a three-axis attitude control to stabilizing and orient the spacecraft to desired attitude. Numerical simulations indicate that this controller is robust against parameter variation and disturbances and show the reliability of mentioned configuration.

As can be seen in Figure 5, spacecraft reaches desired attitude and is stabilized after 34 second. In Figure 6 the spacecraft attitude time response between normal condition and the case one of the reaction wheels fails is compared. It is clear that spacecraft reaches desired attitude in more time (40 second).

In Figure 6 the effect of disturbance on the system is studied and results show that the spacecraft can reach desired attitude in the presence of disturbance 0.0034 N.m (18% in comparison with the allowable reaction wheel capacity). This controller has robustness against with up to 0.018 N.m (43%). Also, in Figure 7 can be seen, the spacecraft reaches desired attitude and stabilize with 150% change in spacecraft moment of inertia. It is robust against with up to 350%.

Our future plans are to develop and implement mentioned controller for novel 3DOF ADCS simulator was designed and manufactured in the *system dynamics and control research laboratory* of the mechanical engineering department of Amirkabir University of Technology. [9]

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