Preface

The subject of this monograph is global stabilization and optimal control of nonlinear systems with uncertain models. The book offers a unified view of (I) *deterministic disturbance attenuation*, (II) *stochastic control*, and (III) *adaptive control* for nonlinear systems. In order to provide the big picture, we take the approach of presenting the fundamentals, rather than giving a comprehensive coverage for the most general classes of systems.

The efforts in nonlinear control over the last few years have led to the level of generality in which conceptual solutions exist for all systems affine in control and the "disturbance," and constructive designs exist for a specific (but broad in terms of physical relevance) class of nonlinear systems. The intention of the book is to communicate this message to a large audience of researchers, students, engineers, and mathematicians, in the areas of robust and adaptive nonlinear control, nonlinear $\mathcal{H}_\infty$, stochastic nonlinear control (including risk-sensitive) and other related areas of control and dynamical systems theory.

While the book covers a broad area to which many people have contributed over the years to bring it to the current status, the results in the book are based on the authors' research in the last three years and includes dissertation work by the second author with the first author, her PhD advisor.

Parts I and II deal with systems described, respectively, by the differential equations

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)d + g_2(x)u \\ dx &= f(x)dt + g_1(x)dw + g_2(x)udt \end{aligned}$$

where u is control, $d(t)$ is a deterministic disturbance, and $w(t)$ is an independent Wiener process with $E\left\{dwdw^{\mathrm{T}}\right\} = \Sigma(t)\Sigma(t)^{\mathrm{T}}dt$. The most general problems solved for these systems are when $d(t)$ and $\Sigma(t)$ are functions whose bound is not known to the control designer or when $g_1(0) \neq 0$, which prevents the existence of an equilibrium at the origin in the presence of the disturbance.

In these cases, controllers are designed which guarantee that[1]

$$
\begin{aligned}
|x(t)| &< \alpha\left(\sup_{[0,t]} |d(\tau)|\right) \\
P\left\{|x(t)| < \alpha\left(\sup_{[0,t]} \left|\Sigma(\tau)\Sigma(\tau)^{\mathrm{T}}\right|\right)\right\} &= 1
\end{aligned}
$$

for all $t \geq 0$, and

$$
\begin{aligned}
\int_0^\infty \left[l(x) + \gamma_2\left(|u|\right)\right] dt &\leq \int_0^\infty \gamma_1\left(|d|\right) dt \\
E\left\{\int_0^\infty \left[l(x) + \gamma_2\left(|u|\right)\right] dt\right\} &\leq \int_0^\infty \gamma_1\left(\left|\Sigma\Sigma^{\mathrm{T}}\right|\right) dt
\end{aligned}
$$

where $\alpha, \gamma_1, \gamma_2$ are some continuous and strictly increasing functions vanishing at zero and l is a positive definite function. The book shows that the respective problems are equivalent and provides constructive solutions based on control Lyapunov functions. The focus is on inverse (rather than direct) optimality because it avoids Hamilton-Jacobi-Isaacs pde's and still results in Kalman-type stability margins.

Part III deals with parametric uncertainties. For example, for the system

$$
\dot{x} = f(x) + g_1(x)\theta + g_2(x)u\,,
$$

where θ is a constant unknown vector, it develops controllers which guarantee global stability, regulation of x to zero, and optimality with respect to a cost functional that involves integral penalty on x and u, and terminal penalty on the parameter estimation error $\theta - \hat{\theta}$. Since this type of optimality leaves open the question of asymptotic behavior of the parameter estimate $\hat{\theta}$, **Chapter 7** gives an in-depth geometric study of convergence via invariant manifold theory, a first study of this kind in the adaptive literature which shows that not every adaptive controller converges to a stabilizing frozen controller.

Chapter 8, the last chapter of the book, addresses a very different type of adaptive optimal control problem, the one in which the equilibrium is unknown and the objective is to optimize steady state performance. In this problem the system is not required to be affine in either the control or the parameter but the results are local and achieved via averaging and singular perturbations.

Chapters 1–6 require little background beyond basic linear systems theory (including stochastic systems). Lyapunov theory is included in the book, although not to substitute for a pedagogical text like, e.g., Khalil [58], but to help build more advanced stability concepts in the presence of deterministic

[1]To reduce notation, in the preface we assume that the initial condition x_0 is zero and make additional simplifications of statements; for a precise description of the results the reader is referred to the corresponding chapters.

and stochastic disturbances. Chapters 7 and 8 require the knowledge of invariant manifold theory, averaging, singular perturbations and some real analysis. Due to its length and a uniform presentation of robust/stochastic/adaptive nonlinear control, the book can be used for a second level course on nonlinear control.

Acknowledgments. The authors thank Petar Kokotovic who was the main source of feedback on the material during its research phase. We have also greatly benefited from interaction with Andrzej Banaszuk, Tamer Basar, Randy Freeman, Bill Helton, Clas Jacobson, Mrdjan Jankovic, Ioannis Kanellakopoulos, Zigang Pan, Laurent Praly, Rodolphe Sepulchre, Bob Skelton, Eduardo Sontag, Jing Sun, and Takis Tsiotras. We thank Zhonghua Li who helped with typing Chapters 2 and 5, and Hsin-hsiung Wang who helped with Chapter 8.

We gratefully acknowledge the support that we have received from National Science Foundation, Air Force Office of Scientific Research, United Technologies Research Center, and Ford Motor Company.

MIROSLAV KRSTIĆ
HUA DENG

San Diego, California, January 1998

Contents