ENME 606
NONLINEAR CONTROL DESIGN — Prof. M. Krstic

MIDTERM EXAM — April 10, 1996

NAME: _____

Open books and notes.

Total points: 25.

Time 5:00–6:15.
(3 minutes/point)

Problem 1. (4 pts)

Consider the system

$$\begin{aligned} \dot{x} &= -x^3 + y \\ \dot{y} &= -y^3 \, . \end{aligned}$$

Using Lemma 5.6, prove that this system is globally asymptotically stable.

Problem 2. (5 pts)

Provide a direct Lyapunov proof that the system from Problem 1 is globally asymptotically stable. Hint: Use a Lyapunov function of the form

$$V = \frac{1}{2m}x^{2m} + \frac{1}{2}y^2\,,$$

where m is an integer that you have to determine. To solve this problem, you will need to use Young's inequality $ab \leq \frac{1}{4}a^4 + \frac{3}{4}b^{4/3}$.

Problem 3. (4 pts)

Using LaSalle's theorem and the Lyapunov function $V = \frac{1}{2}x^2 + \frac{1}{4}y^4$, prove the global asymptotic stability of the system

$$\begin{aligned} \dot{x} &= -x^3 + y^3 \\ \dot{y} &= -x\,. \end{aligned}$$

Problem 4. (2 pts)

Consider the scalar system

$$\dot{x} \;=\; -x + \frac{2}{1+t} x \cos x \,.$$

By viewing the second term as a perturbation $g(t,x)$ and using Corollary 5.4 and Lemma 5.8, prove that the equilibrium $x = 0$ of the system is globally exponentially stable.

Problem 5. (10 pts total)

Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 x_2 - x_2^2 + u \\ \dot{x}_2 &= x_1 x_2 - x_2^2 + u \\ \dot{x}_3 &= x_1 + x_1 x_2 - x_2^2 - (x_3 - x_1)^3 + u \\ y &= x_1 - x_2 \, . \end{aligned}$$

(a) (3 pts) Show that the system has relative degree two.

(b) (4 pts) Transform the system into the normal form. Hint: For the zero dynamics use $\eta = x_1 - x_3$.

(c) (1 pt) Show that the origin of the zero dynamics is globally asymptotically stable.

(d) (2 pts) Design a feedback law that achieves global asymptotic stability.