ENME 606
NONLINEAR CONTROL DESIGN — Prof. M. Krstic

Homework 6

Due April 29, 1996

1. Extend Lemma 2.8 in Krstic et al. to systems of the form

$$\begin{aligned} \dot{x} &= f(x,\xi)\,, \quad f(0,0)=0 \\ \dot{\xi} &= u\,, \end{aligned}$$

where $f(x,\xi)$ is continuously differentiable. Assume that $\alpha(x)$ and $V(x)$ are available such that $\frac{\partial V}{\partial x} f(x,\alpha(x)) \leq -W(x)$, where $W(x)$ is a positive definite function.
Hint: Note that, by the mean-value theorem, the continuous differentiability of $f(x,\xi)$ implies that there exists a continuous function $\psi(x,\xi)$ such that

$$f(x,\xi) - f(x,\alpha(x)) = z\psi(x,\xi)\,,$$

where $z = \xi - \alpha(x)$.

2. Apply the result from Problem 1 to stabilize the system

$$\begin{aligned} \dot{x} &= x^2 - \xi - \xi^2 \\ \dot{\xi} &= u\,. \end{aligned}$$

3. Apply the results from Problems 1 and 2 to stabilize the system

$$\begin{aligned} \dot{x} &= x^2 - \xi_1 - \xi_1^2 \\ \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= u\,. \end{aligned}$$

4. Design a (robust) controller based on nonlinear damping and backstepping which achieves input-to-state stability with respect to the perturbation Δ in the system

$$\begin{aligned} \dot{x} &= \xi + x + x^3\Delta \\ \dot{\xi} &= u\,. \end{aligned}$$

Can you derive a *linear* controller that globally asymptotically stabilizes the equilibrium $x = \xi = 0$ in the case where Δ is *constant* and *nonpositive*?