9.2 The error system is

$$\begin{aligned} \dot{\tilde{X}} &= A\tilde{X} - \mathrm{e}^{AD} L\tilde{u}(0) \\ \tilde{u}_t &= \tilde{u}_x - C\mathrm{e}^{Ax} L\tilde{u}(0) \\ \tilde{u}(D) &= C\tilde{X} \,. \end{aligned}$$

Consider the transformation

$$\tilde{w}(x) = \tilde{u}(x) - C\mathrm{e}^{A(x-D)}\tilde{X} \,.$$

First of all, we see that $\tilde{w}(D) = \tilde{u}(D) - C\tilde{X} = 0$. Let us calculate

$$\begin{aligned} \tilde{w}_t - \tilde{w}_x &= \tilde{u}_x - C\mathrm{e}^{Ax} L\tilde{u}(0) - C\mathrm{e}^{A(x-D)}(A\tilde{X} - \mathrm{e}^{AD} L\tilde{u}(0)) \\ &\quad - \tilde{u}_x + C\mathrm{e}^{A(x-D)} A\tilde{X} \\ &= 0 \,. \end{aligned}$$

Finally,

$$\begin{aligned} \dot{\tilde{X}} &= A\tilde{X} - \mathrm{e}^{AD} L\tilde{u}(0) \\ &= A\tilde{X} - \mathrm{e}^{AD} L(\tilde{w}(0) + C\mathrm{e}^{-AD}\tilde{X}) \\ &= (A - \mathrm{e}^{AD} LC\mathrm{e}^{-AD})\tilde{X} - \mathrm{e}^{AD} L\tilde{w}(0) \,. \end{aligned}$$

9.3 Let us take the Laplace transform of the $\hat{u}$–system:

$$\begin{aligned} s\hat{u}(x,s) &= \hat{u}'(x,s) + C\mathrm{e}^{Ax} L(Y(s) - \hat{Y}(s)) \\ \hat{u}(0,s) &= \hat{Y}(s) \,. \end{aligned}$$

The solution of this ODE is

$$\hat{u}(x,s) = \hat{Y}(s)\mathrm{e}^{sx} - \int_0^x \mathrm{e}^{s(x-\xi)} C\mathrm{e}^{A\xi} L(Y(s) - \hat{Y}(s))\, d\xi \,.$$

Since $\hat{u}(D,s) = C\hat{X}(s)$, we get

$$\hat{Y}(s) = C\hat{X}(s)\mathrm{e}^{-sD} + \int_0^D \mathrm{e}^{-s\xi} C\mathrm{e}^{A\xi} L(Y(s) - \hat{Y}(s))\, d\xi \,.$$

Taking the Laplace transform we obtain

$$\hat{Y}(t) = C\hat{X}(t-D) + \int_0^D C\mathrm{e}^{A\xi} L(Y(t-\xi) - \hat{Y}(t-\xi))\, d\xi \,.$$

Finally, after a change of variables $\theta = t - \xi$ we have

$$\hat{Y}(t) = C\hat{X}(t-D) + C\int_{t-D}^t \mathrm{e}^{A(t-\theta)} L(Y(\theta) - \hat{Y}(\theta))\, d\theta \,.$$