Homework 8

Problem 1.

For the heat equation
\[ u_t = u_{xx} \]
\[ u_x(0) = 0, \]
find the input reference signal \( u^r(1, t) \) so that the output \( u(0, t) \) obeys the reference signal
\[ u^r(0, t) = t^3. \]

Problem 2.

For the heat equation
\[ u_t = u_{xx} \]
\[ u(0) = 0, \]
(note the Dirichlet boundary condition at \( x = 0 \)), find the input reference signal \( u^r(1, t) \) so that the output \( u_x(0, t) \) obeys the reference signal
\[ u^r_x(0, t) = \sin \omega t. \]

Problem 3.

For the Euler-Bernoulli beam
\[ u_{tt} + u_{xxxx} = 0 \]
\[ u_{xx}(0) = u_{xxx}(0) = 0, \]
show that
\[ u^r(x, t) = \frac{\sinh(\sqrt{\omega}x) + \sin(\sqrt{\omega}x)}{2\sqrt{\omega}} \sin(\omega t) \]
is a solution to the system. You can do this by following the procedure in class and “building” the solution, as well as by simply differentiating \( u^r(x, t) \) to show that it verifies the PDE and the boundary conditions. I will of course be more impressed by the former.

This result shows that you can produce the output trajectory
\[ u^r(0, t) = 0 \]
\[ u^r_x(0, t) = \sin(\omega t) \]
with the controls
\[ u^r(1, t) = \frac{\sinh \sqrt{\omega} + \sin \sqrt{\omega}}{2\sqrt{\omega}} \sin(\omega t) \]
\[ u^r_x(1, t) = \frac{\cosh \sqrt{\omega} + \cos \sqrt{\omega}}{2} \sin(\omega t). \]