# Homework 8

Problem 1.

For the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u_x(0) &= 0\,, \end{aligned}$$

find the input reference signal $u^r(1,t)$ so that the output $u(0,t)$ obeys the reference signal

$$u^r(0,t) = t^3\,.$$

Problem 2.

For the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u(0) &= 0\,, \end{aligned}$$

(note the Dirichlet boundary condition at $x = 0$), find the input reference signal $u^r(1,t)$ so that the output $u_x(0,t)$ obeys the reference signal

$$u_x^r(0,t) = \sin \omega t\,.$$

Problem 3.

For the Euler-Bernoulli beam

$$\begin{aligned} &u_{tt} + u_{xxxx} = 0 \\ &u_{xx}(0) = u_{xxx}(0) = 0\,, \end{aligned}$$

show that

$$u^r(x,t) = \frac{\sinh\left(\sqrt{\omega}x\right) + \sin\left(\sqrt{\omega}x\right)}{2\sqrt{\omega}} \sin(\omega t)$$

is a solution to the system. You can do this by following the procedure in class and "building" the solution, as well as by simply differentiating $u^r(x,t)$ to show that it verifies the PDE and the boundary conditions. I will of course be more impressed by the former.

This result shows that you can produce the output trajectory

$$\begin{aligned} u^r(0,t) &= 0 \\ u_x^r(0,t) &= \sin(\omega t) \end{aligned}$$

with the controls

$$\begin{aligned} u^r(1,t) &= \frac{\sinh\sqrt{\omega} + \sin\sqrt{\omega}}{2\sqrt{\omega}} \sin(\omega t) \\ u_x^r(1,t) &= \frac{\cosh\sqrt{\omega} + \cos\sqrt{\omega}}{2} \sin(\omega t)\,. \end{aligned}$$