Homework 4

Problem 1.

For the plant

\[ u_t = u_{xx} + bu_x + \lambda u \]  
\[ u_x(0) = -\frac{b}{2}u(0) \]

design the Neumann stabilizing controller \((u_x(1) \text{ actuated})\).

Hint: by transforming the plant to a system without \(b\)-term, reduce the problem to Problem 1 from Homework 3.

Problem 2.

For the plant

\[ u_t = u_{xx} + 3e^{2x}u(0) \]  
\[ u_x(0) = 0 \]

design the Dirichlet stabilizing controller.

Hint: use the Laplace-domain formula derived in class.

Problem 3.

Design the observer for the following system:

\[ u_t = u_{xx} \]  
\[ u_x(0) = -qu(0) \]  
\[ u(1) = U(t) \]

with only \(u_x(1)\) available for measurement.

Follow these steps:
1) Write down the observer for this system.
2) Use the transformation

\[ \tilde{u}(x) = \tilde{w}(x) - \int_x^1 p(x, y)\tilde{w}(y) \, dy \]  

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to map the error system into the target system

\[ \ddot{w}_t = \ddot{w}_{xx} \quad (9) \]
\[ \ddot{w}_x(0) = 0 \quad (10) \]
\[ \ddot{w}(1) = 0. \quad (11) \]

Show that \( p(x, y) \) satisfies the PDE

\[ p_{xx}(x, y) = p_{yy}(x, y) \quad (12) \]
\[ p_x(0, y) = -qp(0, y) \quad (13) \]
\[ p(x, x) = -q. \quad (14) \]

and that the observer gains are given by \( p_{10} = 0 \) and \( p_1(x) = p(x, 1) \).

3) Solve the PDE for \( p(x, y) \) (look for the solution in the form \( p(x, y) = \phi(y - x) \)). Find \( p_1(x) \).

Problem 4.

Find the frequency domain representation of the plant

\[ u_t = u_{xx} \quad (15) \]
\[ u_x(0) = -qu(0) \quad (16) \]
\[ u(1) = U(t) \quad (17) \]

with \( u(0) \) measured and \( u(1) \) actuated, i.e. find \( G(s) \) such that \( u(0, s) = G(s)U(s) \).