Homework 4

Problem 1.

For the plant

$$u_t = u_{xx} + bu_x + \lambda u \tag{1}$$

$$u_x(0) = -\frac{b}{2}u(0) \tag{2}$$

design the Neumann stabilizing controller ($u_x(1)$ actuated).

Hint: by transforming the plant to a system without b-term, reduce the problem to Problem 1 from Homework 3.

Problem 2.

For the plant

$$u_t = u_{xx} + 3e^{2x}u(0) \tag{3}$$

$$u_x(0) = 0 \tag{4}$$

design the Dirichlet stabilizing controller.

Hint: use the Laplace-domain formula derived in class.

Problem 3.

Design the observer for the following system:

$$u_t = u_{xx} \tag{5}$$

$$u_x(0) = -qu(0) \tag{6}$$

$$u(1) = U(t) \tag{7}$$

with only $u_x(1)$ available for measurement.

Follow these steps:
1) Write down the observer for this system.

2) Use the transformation

$$\tilde{u}(x) = \tilde{w}(x) - \int_x^1 p(x,y)\tilde{w}(y)\,dy \tag{8}$$

to map the error system into the target system

$$
\begin{aligned}
\tilde{w}_t &= \tilde{w}_{xx} && (9)\\
\tilde{w}_x(0) &= 0 && (10)\\
\tilde{w}(1) &= 0\,. && (11)
\end{aligned}
$$

Show that $p(x,y)$ satisfies the PDE

$$
\begin{aligned}
p_{xx}(x,y) &= p_{yy}(x,y) && (12)\\
p_x(0,y) &= -qp(0,y) && (13)\\
p(x,x) &= -q\,. && (14)
\end{aligned}
$$

and that the observer gains are given by $p_{10} = 0$ and $p_1(x) = p(x,1)$.

3) Solve the PDE for $p(x,y)$ (look for the solution in the form $p(x,y) = \phi(y-x)$). Find $p_1(x)$.

Problem 4.

Find the frequency domain representation of the plant

$$
\begin{aligned}
u_t &= u_{xx} && (15)\\
u_x(0) &= -qu(0) && (16)\\
u(1) &= U(t) && (17)
\end{aligned}
$$

with $u(0)$ measured and $u(1)$ actuated, i.e. find $G(s)$ such that $u(0,s) = G(s)U(s)$.