Homework 1

<u>Problem 1.</u>

Consider the heat equation

$$w_t = w_{xx}$$

for $x \in (0,1)$ with the initial condition $w_0(x) = w(x,0)$ and boundary conditions

$$w_x(0) = 0$$
$$w_x(1) = -\frac{1}{2}w(1).$$

Show that

$$\|w(t)\| \le e^{-\frac{t}{4}}\|w_0\|.$$

<u>Problem 2.</u>

Consider the Burgers equation

$$w_t = w_{xx} - ww_x$$

for $x \in (0,1)$ with the initial condition $w_0(x) = w(x,0)$ and boundary conditions

$$w(0) = 0$$
$$w_x(1) = -\frac{1}{6}\left(w(1) + w^3(1)\right).$$

Show that

$$\|w(t)\| \le e^{-\frac{t}{4}}\|w_0\|.$$

Hint: complete the squares.