**AMES 282**
ADAPTIVE CONTROL — Prof. M. Krstic

# MIDTERM EXAM

Open books and notes. Total points: 30. **Due: February 23, 1999**

1. (7 points)
   Consider the nonlinear plant
   $$\dot{x} = f(x)^T\theta$$
   where $f(x)$ is a know $C^1$ matrix-valued function and $\theta$ is an unknown vector of constant parameters. In addition, consider the "nonlinear observer"
   $$\dot{\hat{x}} = (A_0 - \lambda f(x)^T f(x)P)(\hat{x} - x) + f(x)^T\hat{\theta}$$
   and the parameter update law
   $$\dot{\hat{\theta}} = \Gamma f(x)P\epsilon$$
   where $\lambda > 0$, $\Gamma = \Gamma^T > 0$, $A_0$ is a matrix that satisfies
   $$PA_0 + A_0^T P = -I$$
   for some $P = P^T > 0$, and $\epsilon$ is the observer error
   $$\epsilon = x - \hat{x}\,.$$
   Show that $\hat{\theta}, \epsilon \in L_\infty$ and $\epsilon, \dot{\hat{\theta}} \in L_2$.

2. (8 points)
   Consider the same plant as in Problem 1 but with "nonlinear filters"
   $$\begin{aligned}\dot{\Omega}_0 &= (A_0 - \lambda f(x)^T f(x)P)(\Omega_0 - x)\\ \dot{\Omega}^T &= (A_0 - \lambda f(x)^T f(x)P)\Omega^T + f(x)^T\end{aligned}$$
   and with a least squares update law
   $$\begin{aligned}\dot{\hat{\theta}} &= \Gamma\Omega\epsilon\\ \dot{\Gamma} &= -\Gamma\Omega\Omega^T\Gamma\,, \qquad \Gamma(0) = \Gamma(0)^T > 0\end{aligned}$$
   where $\lambda, A_0, P$ are as in Problem 1, and the estimation error is
   $$\epsilon = x + \Omega_0 - \Omega^T\hat{\theta}\,.$$
   Show that $\hat{\theta}, \epsilon, \dot{\hat{\theta}} \in L_\infty$ and $\epsilon, \dot{\hat{\theta}} \in L_2$.

3. (7 points)
   Consider the system

$$y = P(s)\,[a \sin \omega t + b \cos \omega t - u]$$

   where $y$ is the measured output of the system, $u$ is the control input, $P(s)$ is a known SPR transfer function, $\omega$ is a known positive constant, and $a$ and $b$ are unknown constants. Your task is to design a controller that ensures that $y(t) \to 0$ (while, of course, keeping $y(t)$ bounded). This is a problem of adaptive cancellation of a sinusoidal disturbance of unknown amplitude and phase. Your control design must be accompanied by a proof that $y(t) \to 0$.

4. (8 points)
   Problem 7.5, Parts (a) and (b), in *Ioannou & Sun.*
   Take all filter poles and desired closed-loop poles to be at $s = -2$.