AMES 282
ADAPTIVE CONTROL Prof. M. Krstic

Homework 5

1. Ioannou & Sun, Problem 6.13.
   Do only parts (a), (b), and (e). In part (e), simulate both the designs from (a) and (b) and compare.

2. Ioannou & Sun, Problem 6.16.
   Important result: shows that, if the plant is relative degree one and minimum phase, you only need a static output-feedback controller with an adaptive gain which is "pumped-up" to a sufficient level by the nondecreasing update law $\dot{\theta} = y_p^2$. Additional hint: use Lyapunov function
   $$V = x_1^{\mathrm{T}} P x_1 + \frac{1}{2} y_p^2 + \frac{|k_p|}{2}(\theta - M)^2 \,,$$
   where M is an arbitrarily large constant to be selected.

3. In the relative-degree-one minimum-phase plant
   $$W_{\mathrm{p}}(s) = \frac{Z_{\mathrm{p}}(s)}{R_{\mathrm{p}}(s)}$$
   the numerator $Z_{\mathrm{p}}(s)$ is known and the reference model $W_{\mathrm{m}}(s) = \dfrac{Z_{\mathrm{m}}(s)}{R_{\mathrm{m}}(s)}$ is chosen such that $Z_{\mathrm{m}}(s) = Z_{\mathrm{p}}(s)$. (All the above polynomials are monic.)

   (a) Assume that $R_{\mathrm{p}}(s)$ is also known and show that the nonadaptive MRC problem can be solved with a controller of the form
   $$u(s) = r(s) + \frac{Z_{\mathrm{c}}(s)}{R_{\mathrm{c}}(s)} y(s) \,.$$
   Derive the matching conditions for the coefficients of Z_{c} and R_{c}.

   (b) When a single coefficient of $R_{\mathrm{p}}(s)$ is unknown design an adaptive (MRAC) controller with only one adjustable parameter. Give an update law and discuss stability.

4. Consider the system
   $$y = P(s)\left[a \sin \omega t + b \cos \omega t - u\right]$$
   where y is the measured output of the system, u is the control input,
   $$P(s) = \frac{1}{s+1} \,,$$
   ω is a known positive constant, and a and b are unknown constants. Your task is to design a controller that ensures that $y(t) \to 0$ (while, of course, keeping $y(t)$ bounded). This is a problem of adaptive cancellation of a sinusoidal disturbance of unknown amplitude and phase. Your control design must be accompanied by a proof that $y(t) \to 0$.