MAE 282
ADAPTIVE CONTROL Prof. M. Krstic

Homework 3

Due June 5, 2001

1. Consider the 'unicycle' model of a slipping automobile wheel depicted in Figure 1. The tire

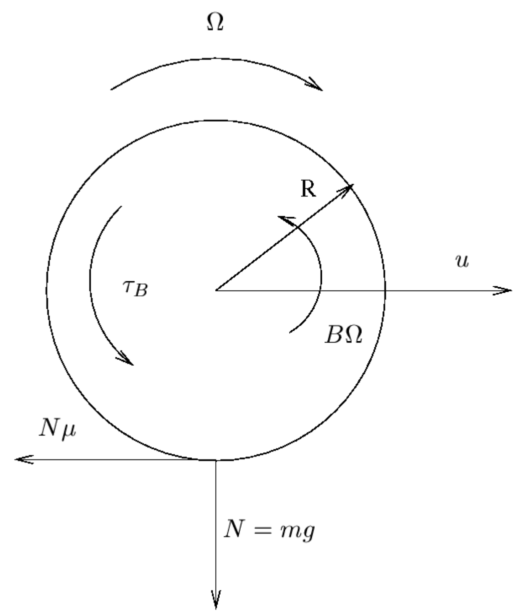

Figure 1: The wheel forces.

dynamics are described by the following two equations

$$m\dot{u} = -N\mu(\lambda) \tag{1}$$
$$I\dot{\Omega} = -B\Omega + NR\mu(\lambda) - \tau_{\mathrm{B}}\,, \tag{2}$$

where u is the linear velocity and Ω is the angular velocity of the wheel, m is the mass and $N = mg$ is the weight of the vehicle, R is the radius of the wheel, I is the moment of inertia of the wheel, $B\Omega$ is the bearing friction torque, τ_{B} is the breaking torque, $\mu(\lambda)$ is the friction force coefficient, and the wheel *slip* is defined as

$$\lambda(u, \Omega) = \frac{u - R\Omega}{u} \tag{3}$$

for the case of braking when $R\Omega \leq u$. The friction force coefficient $\mu(\lambda)$ is shown in Figure 2, from which it is seen that there exists an optimum μ^* at λ^*. Since $\dot{u}$ is measurable via an accelerometer (they are already in use for airbags), the following simple controller

$$\tau_B = -\frac{cIu}{R}(\lambda - \lambda_0) - B\Omega - \frac{I\Omega}{u}\dot{u} - mR\dot{u}\,, \qquad c > 0 \tag{4}$$

is implementable, and it is easy to see that it yields

$$\frac{1}{c}\dot{\lambda} = -\lambda + \lambda_0\,. \tag{5}$$

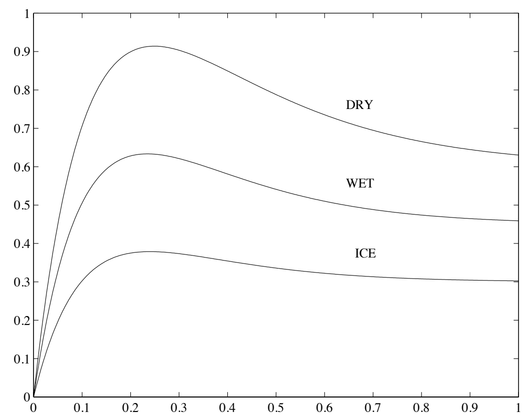

Figure 2: The friction force coefficient $\mu(\lambda)$.

Treating $y = -\dot{u}$ as the output, design an extremum seeking scheme that estimates λ_0 to maximize $\mu(\lambda)$ in steady state and present simulation results. Use

$$\mu(\lambda) = 2\mu^* \frac{\lambda^* \lambda}{\lambda^{*2} + \lambda^2}. \tag{6}$$

with $\lambda^* = 0.25$ and $\mu^* = 0.6$. Choose the vehicle/wheel parameters as $m = 400kg$, $B = 0.01$, $R = 0.3m$. Let the initial conditions be: $u(0) = 120km/hr = 33.33m/sec$ for linear velocity, and $\Omega(0) = 111.1rad/sec$ for angular velocity, which makes $\lambda(0) = 0$. How much time does it take for your "vehicle" to stop? How about the distance?

2. Design $u(x, \hat{\theta})$ and $\dot{\hat{\theta}}$ to globally stabilize the system

$$\dot{x} = \theta x^4 + (1 + x)u$$

while keeping u bounded. (There is more than one solution to this problem.)

3. Applying backstepping, design an adaptive controller that globally stabilizes the system

$$\begin{aligned} \dot{x} &= \theta x^4 + (1 + x)\xi, \qquad \theta > 0 \\ \dot{\xi} &= u. \end{aligned}$$

4. Design an adaptive controller that makes the state variable y globally asymptotically track the reference signal $y_r(t)$ in the system

$$\begin{aligned} \dot{y} &= \xi + y^2\theta \\ \dot{\xi} &= u. \end{aligned}$$

5. Design a tuning functions controller for the linear system

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta x_1 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u. \end{aligned}$$

Is the resulting control law linear or nonlinear in x?