AMES 282
Adaptive Control — Prof. M. Krstic

Homework 2

Due May 21, 2001

1. Ioannou & Sun, Problem 6.13.
   Do only parts (a), (b), and (e). In part (e), simulate both the designs from (a) and (b) and compare.

2. Ioannou & Sun, Problem 6.16.
   Important result: shows that, if the plant is relative degree one and minimum phase, you only need a static output-feedback controller with an adaptive gain which is "pumped-up" to a sufficient level by the nondecreasing update law $\dot{\theta} = y_p^2$. Additional hint: use Lyapunov function
   $$V = x_1^{\mathrm{T}} P x_1 + \frac{1}{2} y_p^2 + \frac{|k_p|}{2} (\theta - M)^2 \,,$$
   where M is an arbitrarily large constant to be selected.

3. In the relative-degree-one minimum-phase plant
   $$W_{\mathrm{p}}(s) = \frac{Z_{\mathrm{p}}(s)}{R_{\mathrm{p}}(s)}$$
   the numerator $Z_{\mathrm{p}}(s)$ is known and the reference model $W_{\mathrm{m}}(s) = \dfrac{Z_{\mathrm{m}}(s)}{R_{\mathrm{m}}(s)}$ is chosen such that $Z_{\mathrm{m}}(s) = Z_{\mathrm{p}}(s)$. (All the above polynomials are monic.)

   (a) Assume that $R_{\mathrm{p}}(s)$ is also known and show that the nonadaptive MRC problem can be solved with a controller of the form
   $$u(s) = r(s) + \frac{Z_{\mathrm{c}}(s)}{R_{\mathrm{c}}(s)} y(s) \,.$$
   Derive the matching conditions for the coefficients of Z_{c} and R_{c}.

   (b) When a single coefficient of $R_{\mathrm{p}}(s)$ is unknown design an adaptive (MRAC) controller with only one adjustable parameter. Give an update law and discuss stability.

4. Consider the system
   $$y = P(s) \left[a \sin \omega t + b \cos \omega t - u \right]$$
   where y is the measured output of the system, u is the control input, $P(s)$ is a known SPR transfer function, ω is a known positive constant, and a and b are unknown constants. Your task is to design a controller that ensures that $y(t) \to 0$ (while, of course, keeping $y(t)$ bounded). This is a problem of adaptive cancellation of a sinusoidal disturbance of unknown amplitude and phase. Your control design must be accompanied by a proof that $y(t) \to 0$.

5. Ioannou & Sun, Problem 7.5, Parts (a) and (b).
   Take all filter poles and desired closed-loop poles to be at $s = -2$.

6. Consider the nonlinear plant

$$\dot{x} = f(x)^T \theta$$

where $f(x)$ is a know C^1 matrix-valued function and θ is an unknown vector of constant parameters. In addition, consider the "nonlinear observer"

$$\dot{\hat{x}} = (A_0 - \lambda f(x)^T f(x) P)(\hat{x} - x) + f(x)^T \hat{\theta}$$

and the parameter update law

$$\dot{\hat{\theta}} = \Gamma f(x) P \epsilon$$

where $\lambda > 0$, $\Gamma = \Gamma^T > 0$, A_0 is a matrix that satisfies

$$P A_0 + A_0^T P = -I$$

for some $P = P^T > 0$, and ϵ is the observer error

$$\epsilon = x - \hat{x}\,.$$

Show that $\hat{\theta}, \epsilon \in L_\infty$ and $\epsilon, \dot{\hat{\theta}} \in L_2$.

7. Consider the same plant as in Problem 1 but with "nonlinear filters"

$$\begin{aligned} \dot{\Omega}_0 &= (A_0 - \lambda f(x)^T f(x) P)(\Omega_0 - x) \\ \dot{\Omega}^T &= (A_0 - \lambda f(x)^T f(x) P)\Omega^T + f(x)^T \end{aligned}$$

and with a least squares update law

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma \Omega \epsilon \\ \dot{\Gamma} &= -\Gamma \Omega \Omega^T \Gamma\,, \qquad\qquad \Gamma(0) = \Gamma(0)^T > 0 \end{aligned}$$

where λ, A_0, P are as in Problem 1, and the estimation error is

$$\epsilon = x + \Omega_0 - \Omega^T \hat{\theta}\,.$$

Show that $\hat{\theta}, \epsilon, \dot{\hat{\theta}} \in L_\infty$ and $\epsilon, \dot{\hat{\theta}} \in L_2$.