MAE 282

Adaptive Control — Prof. M. Krstic

Homework 2

Consider the model of aircraft wing rock

$$\begin{aligned}\dot{x}_1 &= x_2 & (1)\\ \dot{x}_2 &= -\theta_1 x_1 + \theta_2 x_2 - \theta_3 x_1^2 x_2 + x_3 & (2)\\ \dot{x}_3 &= -x_3 + u\,, & (3)\end{aligned}$$

where x_1 denotes the roll angle, x_2 denotes the roll rate, x_3 denotes the aileron deflection angle, and u denotes the control input.

1. Design the control law $x_3 = \alpha_3(x_1, x_2, \theta)$ that stabilizes the system

$$\begin{aligned}\dot{x}_1 &= x_2 & (4)\\ \dot{x}_2 &= \varphi^T(x_1, x_2)\theta + x_3\,, & (5)\end{aligned}$$

   where

$$\varphi(x_1, x_2) = \begin{bmatrix} -x_1 \\ x_2 \\ -x_1^2 x_2 \end{bmatrix}, \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}. \qquad (6)$$

   Pick α_2 to achieve

$$\begin{aligned}\dot{x}_1 &= x_2 & (7)\\ \dot{x}_2 &= -x_1 - 2x_2\,. & (8)\end{aligned}$$

2. Find a non-adaptive controller $u = \alpha_3(x_1, x_2, x_3, \theta)$ for the system (1)–(2) by using the Lyapunov function

$$V_3 = V_2(x_1, x_2) + \frac{1}{2}\zeta_3^2\,, \qquad \zeta_3 = x_3 - \alpha_2(x_1, x_2, \theta)\,, \qquad (9)$$

   where

$$V_2 = \bar{x}_2^T P \bar{x}_2 = 3x_1^2 + 2x_1 x_2 + x_2^2\,, \quad \bar{x}_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad P = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}. \qquad (10)$$

3. Design a tuning function based adaptive controller and parameter update law using the Lyapunov function

$$V_a = V_2(x_1, x_2) + \frac{1}{2}z_3^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}\,, \qquad (11)$$

   where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter estimation error, $\hat{\theta}$ is the parameter estimate, Γ is a positive definite symmetric matrix, and

$$z_3 = x_3 - \alpha_3(x_1, x_2, x_3, \hat{\theta})\,. \qquad (12)$$

4. (Lyapunov/tuning functions.) Implement your tuning functions adaptive controller from Problem 3 numerically and perform simulations of the closed-loop system. Compare with the simulations of the open-loop system. Is your three-dimensional estimator vector $\hat{\theta}(t)$ converging to the true value θ?

5. (Gradient with filtering.) For the parametric model

$$\dot{x}_2 \quad = \quad \varphi^T(x_1, x_2)\theta + x_3\,, \tag{13}$$

design a gradient-based (swapping-based) parameter estimator with regressor filtering. Perform simulations of the closed-loop adaptive system with the certainty-equivalence control law designed in Problem 2 and employing the gradient-based estimator. Compare the performance of this adaptive controller with the the tuning functions adaptive controller from Problem 4.

6. (Gradient without filtering.) For the parametric model

$$\dot{x}_2 \quad = \quad -\theta_1 x_1 + \theta_2 x_2 - \theta_3 x_1^2 x_2 + x_3 \tag{14}$$

design a gradient-based parameter estimator without regressor filtering. Compare the convergence properties of this estimator with the estimator from Problem 5 for the open-loop system. Then implement a certainty-equivalence adaptive controller employing this gradient estimator without regressor filtering and compare it with the performance of the adaptive controllers from Problems 4 and 5.

7. (Least-squares with filtering.) For the parametric model (13) design a least-squares based parameter estimator with regressor filtering. Compare the convergence properties of this estimator with the estimator from Problem 5 for the open-loop system. Then implement a certainty-equivalence adaptive controller employing this least-squares estimator and compare it with the performance of the adaptive controllers from Problems 4 and 5.

8. (Least-squares without filtering.) For the parametric model (14) design a least-squares based parameter estimator without regressor filtering. Compare the convergence properties of this estimator with the estimators from Problems 6 and 7 for the open-loop system. Then implement a certainty-equivalence adaptive controller employing this least-squares estimator without regressor filtering and compare it with the performance of the adaptive controllers from Problems 4, 6, and 7.