1. Consider the ‘unicycle’ model of a slipping automobile wheel depicted in Figure 1. The tire dynamics are described by the following two equations

\[
m \dot{u} = -N \mu(\lambda) \tag{1}
\]

\[
I \dot{\Omega} = -B \Omega + N R \mu(\lambda) - \tau_B, \tag{2}
\]

where \( u \) is the linear velocity and \( \Omega \) is the angular velocity of the wheel, \( m \) is the mass and \( N = mg \) is the weight of the vehicle, \( R \) is the radius of the wheel, \( I \) is the moment of inertia of the wheel, \( B \Omega \) is the bearing friction torque, \( \tau_B \) is the breaking torque, \( \mu(\lambda) \) is the friction force coefficient, and the wheel slip is defined as

\[
\lambda(u, \Omega) = \frac{u - R \dot{\Omega}}{u} \tag{3}
\]

for the case of braking when \( R \dot{\Omega} \leq u \). The friction force coefficient \( \mu(\lambda) \) is shown in Figure 2, from which it is seen that there exists an optimum \( \mu^* \) at \( \lambda^* \). Since \( \dot{u} \) is measurable via an accelerometer (they are already in use for airbags), the following simple controller

\[
\tau_B = -\frac{c I u}{R} (\lambda - \lambda_0) - B \Omega - \frac{I \Omega}{u} \dot{u} - m R \dot{u}, \quad c > 0 \tag{4}
\]

is implementable, and it is easy to see that it yields

\[
\frac{1}{c} \dot{\lambda} = -\lambda + \lambda_0. \tag{5}
\]
Figure 2: The friction force coefficient $\mu(\lambda)$.

Treating $y = -\dot{u}$ as the output, design an extremum seeking scheme that estimates $\lambda_0$ to maximize $\mu(\lambda)$ in steady state and present simulation results. Use

$$\mu(\lambda) = 2\mu^* \frac{\lambda^*\lambda}{\lambda^*^2 + \lambda^2}. \quad (6)$$

with $\lambda^* = 0.25$ and $\mu^* = 0.6$. Choose the vehicle/wheel parameters as $m = 400kg$, $B = 0.01$, $R = 0.3m$. Let the initial conditions be: $u(0) = 120km/hr = 33.33m/sec$ for linear velocity, and $\Omega(0) = 111.1rad/sec$ for angular velocity, which makes $\lambda(0) = 0$. How much time does it take for your “vehicle” to stop? How about the distance?