MAE 282 Prof. M. Krstic

ADAPTIVE CONTROL Spring 2001

FINAL EXAM

Take home. Open books and notes.

Total points: 60

Due Friday, June 8, 2001, at **4pm in Krstic's office**.

Late submissions will not be accepted. **Collaboration not allowed.**

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1. [30 pts] In this problem you will learn how to design adaptive pole placement controllers in discrete time. Consider the plant

$$y = \frac{1}{z^2 + \theta_1 z + \theta_2} u \,,$$

where θ_1 and θ_2 are unknown parameters.

(a) The design of polynomial pole placement controllers in discrete time follows closely the continuous time approach in Section 7.3 of the Ioannou-Sun book. Consider the objective of simply stabilizing the plant (in that case the internal model is just $Q_m = 1$). Find the coefficients of the feedback compensator $(p_1 z + p_0)/(z + l_0)$ that would place the closed loop poles at the roots of the polynomial $A^*(z) = (z - 1/2)(z^2 + 1/4)$. (Note the location of these poles, they are all inside the unit circle and distinct. Two of them are complex.) Remember, the compensator coefficients should be obtained by solving a Bezout equation by matching the coefficients of the like powers of z. The compensator coefficients should be expressed in terms of the general θ_1 and θ_2, not their particular values.

(b) While the non-adaptive pole placement design is the same like in the continuous-time case, discrete-time parameter estimators are a little different. First, you need a parametric model. Starting from the plant representation, one obtains the difference equation

$$y_t + \theta_1 y_{t-1} + \theta_2 y_{t-2} = u_{t-2} \,.$$

An advantage of discrete-time is that you don't need filters to design a parameter estimator because time derivatives do not appear. Instead of filters, one uses retarded values of y and u (stored in the computer memory). The parametric model is rewritten to be linear in the unknown parameter vector:

$$y_t - u_{t-2} = \phi_t^T \theta \,,$$

where $\theta = [\theta_1, \theta_2]^T$ and $\phi_t = [y_{t-1}, y_{t-2}]^T$. One of the most widely used parameter estimators in discrete time is the least-squares estimator. Its form for this parametrization would be

$$\begin{aligned} \hat{\theta}_t &= \hat{\theta}_{t-1} + \frac{P_{t-1}\phi_t}{1 + \phi_t^T P_{t-1}\phi_t}(y_t - u_{t-2} - \phi_t^T \hat{\theta}_{t-1}) \\ P_t &= P_{t-1} - \frac{P_{t-1}\phi_t \phi_t^T P_{t-1}}{1 + \phi_t^T P_{t-1}\phi_t}. \end{aligned}$$

Simulate the closed loop system (plant-controller-estimator) for plant parameter values $\theta_1 = -5$ and $\theta_2 = 6$. Note that these values make the plant open-loop unstable.

2. [30 pts] In this problem you will demonstrate your understanding of the role of the linear stability condition for extremum seeking schemes. Kartik's notes should be your main resource. Consider an extremum seeking scheme with a plant

$$\begin{aligned} f'' &= 2 \\ F_i(s) &= 1 \\ F_o(s) &= \frac{1}{s+2} \end{aligned}$$

and with

$$\begin{aligned} \omega &= 3 \\ \phi &= 0 \\ \Gamma_f(s) &= \frac{1}{s} \qquad \text{(step change is } \theta^*(t)\text{)} \\ \Gamma_\theta(s) &= \frac{1}{s^2} \qquad \text{(ramp change is } f^*(t)\text{)} \\ C_o(s) &= \frac{1}{s+1}. \end{aligned}$$

The only quantities not defined here are the probing amplitude a and the compensator $C_i(s)$. Determine the transfer function $L(s)$. Then, for $C_i(s) = 1$, using root locus, determine the interval of a for which extremum seeking should be locally exponentially stable. If this interval is the empty set, design a proper (relative degree zero) compensator $C_i(s)$ to ensure stability for some non-empty interval for $a > 0$, and state what this interval is.

Note:

(a) Calculations to bring $L(s)$ to the form where the Matlab root locus routine can be used are somewhat lengthy and are the main part of this problem. Be very careful with the algebra!

(b) In a real implementation it would be allowable to use an improper $C_i(s)$ because $\Gamma_\theta(s)$ is already relative degree two. However, in this problem you are asked to design a proper $C_i(s)$.