AMES 207
Nonlinear Control

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Homework 6

Due June 4, 1998

1. Consider the system

$$\begin{aligned} \dot{x} &= \eta + x^2\zeta \\ \dot{\zeta} &= -\zeta^3 + x \\ \dot{\eta} &= u\,. \end{aligned}$$

   Given a smooth known reference signal $x_r(t)$, design a control law that achieves asymptotic tracking, i.e., $\lim_{t\to\infty}[x(t) - x_r(t)] = 0$, while keeping $x(t), \zeta(t), \eta(t)$ bounded.
   Hint: Consider the system as a block-strict-fbk form with (x, ζ) as the first block. Start the backstepping procedure with $z_1 = x - x_r(t)$.

2. Design a (robust) controller based on nonlinear damping and backstepping which achieves input-to-state stability with respect to the perturbation Δ in the system

$$\begin{aligned} \dot{x} &= \xi + x + x^3\Delta \\ \dot{\xi} &= u\,. \end{aligned}$$

   Can you derive a *linear* controller that globally asymptotically stabilizes the equilibrium $x = \xi = 0$ in the case where Δ is *constant* and *nonpositive*?

3. Design a controller that achieves noise-to-state stability for the stochastic system

$$\begin{aligned} dx &= \xi dt + x^2 dw \\ d\xi &= u dt\,, \end{aligned}$$

   where w is an independent Wiener process with $E\{dw^2\} = \sigma(t)^2 dt$.

4. Design an output-feedback controller that achieves global asymptotic stability of the equilibrium at the origin of the system

$$\begin{aligned} \dot{x} &= \xi + x^2 \\ \dot{\xi} &= u \\ y &= x\,. \end{aligned}$$