AMES 207
NONLINEAR CONTROL — Prof. M. Krstic

Homework 4

Due May 19, 1998

1. Consider the model of the "kinematic" car:

$$\begin{aligned}\dot{x} &= u_1 \cos\theta \\ \dot{y} &= u_1 \sin\theta \\ \dot{\theta} &= u_1 \tan\phi \\ \dot{\phi} &= u_2\end{aligned}$$

   where u_1 and u_2 are control inputs ("drive " and "steer"). The linearization at the origin is obviously not completely controllable (check this!). By computing $[g_1, g_2]$ and $\big[g_1, [g_1, g_2]\big]$, show that the nonlinear system is completely controllable.

2. Show that the system from Problem 1 fails Brockett's condition, i.e., the origin is not C^1- stabilizable.

3. Consider the model of a spacecraft with thrusters generating torque around two axes:

$$\begin{aligned}J\dot{\omega} &= S(\omega)J\omega + Bu \\ \dot{\sigma} &= H(\sigma)\omega\end{aligned}$$

   where ω is the angular velocity, σ is the attitude (in modified Rodrigues parameters), $u = (u_1, u_2)$ are the torque inputs,

$$J = \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & J_3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

   the matrix $S(x)$ is a skew-symmetric matrix defined as

$$S(x) = \begin{bmatrix} 0 & x_3 & -x_2 \\ -x_3 & 0 & x_1 \\ x_2 & -x_1 & 0 \end{bmatrix},$$

   and the matrix $H(\sigma)$ is defined as

$$H(\sigma) = \frac{1}{4}I + \frac{1}{2}\left[\sigma\sigma^T - S(\sigma) - \frac{1}{2}|\sigma|^2 I\right].$$

   The linearization at the origin is obviously not completely controllable (check!). By computing $[g_1, f], [g_2, f], \big[g_2, [g_1, f]\big]$, and $\big[[g_2, [g_1, f]], f\big]$, show that the nonlinear system is completely controllable whenever $J_1 \neq J_2$.

4. Show that the system from Problem 3 fails Brockett's condition. Note that

$$H(\sigma)^{-1} = 4I + \frac{8}{(1+|\sigma|^2)^2}\left[\sigma\sigma^T + S(\sigma) - \frac{1}{2}(|\sigma|^4 + 3|\sigma|^2)I\right].$$