MAE 281A Prof. M. Krstic

NONLINEAR SYSTEMS February 15, 2000

MIDTERM EXAM

Take home. Open books and notes.
Total points: 25
Due February 16, 2000, at 5:00 p.m. in Professor Krstic's office.

Late submissions will not be accepted. Collaboration not allowed.

Problem 1.

(6 pts) Let g, h, and y be three positive functions on $(0, \infty)$ such that

$$\begin{aligned} \int_0^\infty g(t)dt &\leq C_1 \\ \int_0^\infty e^{\delta t} h(t)dt &\leq C_2 \\ \int_0^\infty e^{\delta t} y(t)dt &\leq C_3 , \end{aligned}$$

where δ, C_1, C_2, C_3 are positive constants. Assuming that

$$\dot{y} \leq g(t)y + h(t) , \qquad \forall t \geq 0$$

using Gronwall's lemma show that

$$y(t) \leq [C_2 + \delta C_3 + y(0)]e^{C_1 - \delta t} .$$

Problem 2.

(6 pts) Consider the system:

$$\begin{aligned} \dot{x} &= A(x,y)x + B(x)y \\ \dot{y} &= -GB(x)^{\mathrm{T}} x , \end{aligned}$$

where $x(t), y(t)$ are vectors of arbitrary dimensions, $A(x,y)$ is a matrix valued function that satisfies

$$A(x,y) + A(x,y)^{\mathrm{T}} \leq -qI , \qquad q > 0$$

and G is a positive definite symmetric matrix. Show that the equilibrium $x = 0, y = 0$ is globally stable, that $x(t)$ converges to zero, and that $y(t)$ converges to the null space of $B(0)$. If you can't solve the problem for general G, solve it for $G = I$ to receive partial credit.

Problem 3.

(6 pts) With Chetaev's theorem, show that the equilibrium at the origin of the following system is unstable:

$$\begin{aligned} \dot{x} &= |x|x + xy\sqrt{|y|} \\ \dot{y} &= -y + |x|\sqrt{|y|}\,. \end{aligned}$$

Don't worry about uniqueness of solutions.

Problem 4.

(7 pts) Calculate exactly (in closed form) the sensitivity function at $\lambda_0 = 1$ for the system

$$\dot{x} = -\lambda x^3.$$

What is the approximation $x(t,\lambda) \approx x(t,\lambda_0) + S(t)(\lambda - \lambda_0)$ for $\lambda = 7/2$?