MAE 281A
Nonlinear Systems — Prof. M. Krstic

Homework 2

Due February 6, 2001

1. Khalil, Exercise 2.42.
2. Khalil, Exercise 2.46..
3. Using the comparison principle, show that if v, l_1, and l_2 are functions that satisfy

$$\dot{v} \leq -cv + l_1(t)v + l_2(t)\,, \qquad v(0) \geq 0$$

and if $c > 0$, then

$$v(t) \leq \left(v(0)\mathrm{e}^{-ct} + \|l_2\|_1\right)\mathrm{e}^{\|l_1\|_1}\,,$$

where $\|\cdot\|_1$ denotes the $\mathcal{L}_1$ norm defined as

$$\|f\|_1 = \int_0^\infty |f(t)|dt\,.$$

Using Gronwall's lemma show that

$$v(t) \leq \left(v(0)\mathrm{e}^{-ct} + \|l_2\|_1\right)\left(1 + \|l_1\|_1\mathrm{e}^{\|l_1\|_1}\right)\,.$$

Which of the two bounds is less conservative?