AMES 207 Prof. M. Krstic

NONLINEAR SYSTEMS Winter 1998

FINAL EXAM

Take home. Open books and notes.

Total points: 35

Due Saturday, March 14, 1998, at 12:00 noon in Professor Krstic's office.

Late submissions will not be accepted. Collaboration not allowed.

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1. (8 pts) Consider the system

$$\dot{x}_i = x_{i+1} - c_i x_i - k_i s_i(x) x_i + w_i(x) d,$$

$i = 1, \ldots, n, x_{n+1} = 0$, where $c_i, k_i > 0$ and $|w_i(x)| \leq s_i(x)$. Show that the system is ISS w.r.t. d. What is the type of the gain function (linear, quadratic, exponential, . . .)?

2. (9 pts) Using the center manifold theorem, determine whether the origin of the following system is asymptotically stable:

$$\begin{aligned} \dot{x}_1 &= -x_2 + x_1 x_3 \\ \dot{x}_2 &= x_1 + x_2 x_3 \\ \dot{x}_3 &= -x_3 - (x_1^2 + x_2^2) + x_3^2 \end{aligned}$$

3. (9 pts) Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_2 - \sin\omega t \left((x_1 + \sin\omega t)^2 + x_3 \right) \\ \dot{x}_3 &= -x_3^n - (x_1 + \sin\omega t)^2 + \frac{1}{2}. \end{aligned}$$

a) For $n = 1$, show that for sufficiently large ω there exists an exponentially stable periodic orbit in an $O\left(\frac{1}{\omega}\right) -$ neighborhood of the origin.

b) What can you claim for $n = 3$?

4. (9 pts) Show that, for sufficiently small ε, the origin of the system

$$\begin{aligned} \dot{x} &= x^2 + z + \cos(\varepsilon y) - 1 \\ \varepsilon \dot{y} &= -y + x^2 - x \\ \varepsilon^2 \dot{z} &= -z + \sin y + \varepsilon x^3 \end{aligned}$$

is exponentially stable. (Hint: treat $\mu = \varepsilon^2$ as a separate small parameter.) Since the system has three (rather than two) time scales, it has three levels of invariant manifolds – slow, medium, and fast. Without going into high accuracy, give the approximate expressions for these manifolds and discuss the trajectories of the system.