MAE 281A Prof. M. Krstic

NONLINEAR SYSTEMS Winter 2006

FINAL EXAM

Take home. Open books and notes.

Total points: 65

Due Tuesday, March 21, 2006, at 5:00 pm in Professor Krstic's office.

Collaboration not allowed.

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1. (20 pts) Show that the following system has an unstable equilibrium at the origin:

$$\begin{aligned} \dot{x}_1 &= x_1^3 + 2x_2^3 \\ \dot{x}_2 &= x_1 x_2^2 + x_2^3 \,. \end{aligned}$$

Hint: For second order systems one should always first try Chetaev functions of the form $V = x_1^2 - a x_2^2$, where a is some positive constant which you are free to choose in the analysis.

2. (25 pts) Show that the following system is ISS

$$\begin{aligned} \dot{x} &= -x + y^3 \\ \dot{y} &= -y - f(x) + p^2 \\ \dot{p} &= -p + u \,, \end{aligned}$$

where $f(x)$ is a function such that $\int_0^x f(\xi) d\xi$ and $x f(x)$ are positive definite and radially unbounded functions.

[An example of such a function is the function $f(x) = \mathrm{e}^x - 1$—convince yourself that this is so. A simpler, trivial example is the function is $f(x) = x$.]

This is the hardest problem on the exam. You can solve it either by showing that the (x, y)-system is ISS with respect to p and by noting that p is ISS with respect to u, or by directly building an ISS-Lyapunov function for the whole (x, y, p)-system (the latter is more difficult but more impressive if you can do it). In any case you will have to construct some Lyapunov functions. Hints: Use the terms $\int_0^x f(\xi) d\xi$ and $y^4/4$ in those Lyapunov functions. In showing that the (x, y)-system is ISS with respect to p you should use Young's inequality (the version with powers of 4 and 4/3).

3. (20 pts) Using averaging theory, analyze the following system:

$$\begin{aligned} \dot{x}_1 &= [(x_1 - \sin \omega t)^2 - x_2] \sin \omega t \\ \dot{x}_2 &= -(x_1 - \sin \omega t)^2 - x_2 \,. \end{aligned}$$