MAE 281A Prof. M. Krstic

NONLINEAR SYSTEMS Winter 2002

FINAL EXAM

Take home. Open books and notes.

Total points: 65

Due Friday, March 22, 2002, at 4:00 pm in Professor Krstic's office.

Late submissions will not be accepted. Collaboration not allowed.
Each problem is worth 13 points

Problem 1. Consider the system

$$\begin{aligned} \dot{x} &= -x + yx\sin x \\ \dot{y} &= -y + zy\sin y \\ \dot{z} &= -z\,. \end{aligned}$$

Using Gronwall's lemma (twice), show that

$$|x(t)| \le |x_0| \mathrm{e}^{|y_0| \mathrm{e}^{|z_0|}} \mathrm{e}^{-t}, \qquad \forall t \ge 0.$$

Problem 2. Analyze uniform stability of the origin of the linear time-varying system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -y - (2 + \sin t)x \end{aligned}$$

using the Lyapunov function

$$V = x^2 + \frac{y^2}{2 + \sin t}.$$

Does your analysis guarantee that $y(t) \to 0$ as $t \to \infty$.

Problem 3. Using the Lyapunov function candidate

$$V = \frac{x^4}{4} + \frac{y^2}{2} + \frac{z^4}{4},$$

study stability of the origin of the system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x^3 - y^3 - z^3 \\ \dot{z} &= -z + y\,. \end{aligned}$$

Problem 4. Using the Chetaev function

$$V = xz$$

prove that the origin of the system

$$\begin{aligned} \dot{x} &= yz + az \\ \dot{y} &= -xz \\ \dot{z} &= xy + ax, \end{aligned}$$

where $a > 0$ is a constant, is unstable. (This problem is related to instability of rigid body spinning motion around the "intermediate" axis.)

Problem 5. Show that the ISS gain function of the system

$$\begin{aligned} \dot{x} &= (3 + \cos(u))\mathrm{sgn}(x) \log \frac{1}{1 + |x|} + y \\ \dot{y} &= -(2 + x^2)|y|y + \frac{x^2}{1 + x^2} u \end{aligned}$$

from u to x is

$$\gamma(r) = \mathrm{e}^{\sqrt{r}} - 1.$$

Hint: Use Lyapunov functions of the form $V_1(x) = |x|$ and $V_2(y) = |y|$.