MAE 281A Prof. M. Krstic

NONLINEAR SYSTEMS Winter 2001

FINAL EXAM

Take home. Open books and notes.

Total points: 50

Due Thursday, March 22, 2001, at 1:00 pm in Professor Krstic's office.

Late submissions will not be accepted. Collaboration not allowed.

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1. (9 pts) Using the Lyapunov function candidate $V = \frac{1}{2}(x^2 + y^2 + z^2)$, study stability of the origin of the system

$$\begin{aligned} \dot{x} &= -x + x^2 z \\ \dot{y} &= z \\ \dot{z} &= -y - z - x^3 \,. \end{aligned}$$

2. (9 pts) Show that the following system is ISS

$$\begin{aligned} \dot{x} &= -x + x^{1/3} y + p^2 \\ \dot{y} &= -y - x^{4/3} + p^3 \\ \dot{p} &= -p + u \,. \end{aligned}$$

3. (8 pts) Using the center manifold theorem, determine whether the origin of the following system is asymptotically stable:

$$\begin{aligned} \dot{y} &= yz + 2y^3 \\ \dot{z} &= -z - 2y^2 - 4y^4 - 2y^2 z \,. \end{aligned}$$

4. (8 pts) Using averaging theory, analyze the following system:

$$\begin{aligned} \dot{x} &= \epsilon[-x + 1 - 2(y + \sin t)^2] \\ \dot{y} &= \epsilon z \\ \dot{z} &= \epsilon \left\{ -z - \sin t \left[\frac{1}{2}x + (y + \sin t)^2 \right] \right\} \,. \end{aligned}$$

5. (8 pts) Using singular perturbation theory, study local exponential stability of the origin of the system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -z \\ \epsilon \dot{z} &= -z + \sin x + y \,. \end{aligned}$$

Is the origin globally exponentially stable?

6. (8 pts) Consider the feedback system with a linear block $\dfrac{1}{s(s+1)(s+2)}$ (like in class) and a nonlinearity $\mathrm{sgn}(y) + |y|y$ (note that it is an odd nonlinearity, so the describing functions method applies, and note that the first term was already studied in class). First, find the describing function for the nonlinearity. Then, determine if the feedback system is likely to have any periodic solutions.