AMES 207 Prof. M. Krstic

NONLINEAR SYSTEMS February 12, 1998

MIDTERM EXAM

Take home. Open books and notes.
Total points: 30
Due February 13, 1998, at 5:00 p.m. in Professor Krstic's office.

Late submissions will not be accepted. Collaboration not allowed.

Problem 1.

Consider the system:

$$\begin{aligned}\dot{x} &= -cx + y^{2m} x \cos^2 x \\ \dot{y} &= -y^3\end{aligned}$$

Using either Gronwall's inequality or the comparison principle, show that

a) (5 pts) $x(t)$ is bounded for all $t \geq 0$ whenever $c = 0$ and $m > 1$.

b) (5 pts) $x(t) \to 0$ as $t \to \infty$ whenever $c > 0$ and $m = 1$.

Problem 2.

Consider the system:

$$\begin{aligned}\dot{x} &= -x + yx + z\cos x \\ \dot{y} &= -x^2 \\ \dot{z} &= -x\cos x\end{aligned}$$

a) (2 pts) Determine all the equilibria of the system.

b) (2 pts) Show that the equilibrium $x = y = z = 0$ is globally stable.

c) (2 pts) Show that $x(t) \to 0$ as $t \to \infty$.

d) (2 pts) Show that $z(t) \to 0$ as $t \to \infty$.

Problem 3.

With Chetaev's theorem, show that the equilibrium at the origin of the following two systems is unstable:

a)(4pts)

$$\begin{aligned} \dot{x} &= x^3 + xy^3 \\ \dot{y} &= -y + x^2 \end{aligned}$$

b)(4pts)

$$\begin{aligned} \dot{\xi} &= \eta + \xi^3 + 3\xi\eta^2 \\ \dot{\eta} &= -\xi + \eta^3 + 3\eta\xi^2 \end{aligned}$$

Problem 4.

(4 pts) Calculate exactly (in closed form) the sensitivity function at $\lambda_0 = 0$ for the system

$$\dot{x} = -x + \tan^{-1}(\lambda x).$$

How accurate is the approximation $x(t, \lambda) \approx x(t, \lambda_0) + S(t)(\lambda - \lambda_0)$ for large λ, say $\lambda > 1$?