AMES 207: Prof. M. Krstić

NONLINEAR SYSTEMS Winter 1998

FINAL EXAM

Take home. Open books and notes.
Total points: 35
Due Saturday, March 14, 1998, at 12:00 noon in Professor Krstić's office.

Late submissions will not be accepted. Collaboration not allowed.

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1. (8 pts) Consider the system

$$\dot{x}_i = x_{i+1} - c_i x_i - k_i s_i(x) x_i + w_i(x) d\,,$$

$i = 1\,, \ldots\,, n\,, x_{n+1} = 0$, where c_i, $k_i > 0$ and $|w_i(x)| \leq s_i(x)$. Show that the system is ISS w.r.t. d. What is the type of the gain function (linear, quadratic, exponential, ...)?

2. (9pts) Using the center manifold theorem, determine whether the origin of the following system is asymptotically stable:

$$\begin{aligned}
\dot{x}_1 &= -x_2 + x_1 x_3 \\
\dot{x}_2 &= x_1 + x_2 x_3 \\
\dot{x}_3 &= -x_3 - \left(x_1^2 + x_2^2\right) + x_3^2\,.
\end{aligned}$$

3. (9 pts) Consider the system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_2 - \sin\omega t \left((x_1 + \sin\omega t)^2 + x_3\right) \\
\dot{x}_3 &= -x_3^n - (x_1 + \sin\omega t)^2 + \frac{1}{2}\,.
\end{aligned}$$

a) For $n = 1$, show that for sufficiently large ω there exists an exponentially stable periodic orbit in an $O\left(\frac{1}{\omega}\right)$ – neighborhood of the origin.
b) What can you claim for $n = 3$?

4. (9pts) Show that, for sufficiently small ϵ, the origin of the system

$$\begin{aligned}
\dot{x} &= x^2 + z + \cos(\epsilon y) - 1 \\
\epsilon \dot{y} &= -y + x^2 - x \\
\epsilon^2 \dot{z} &= -z + \sin y + \epsilon x^3
\end{aligned}$$

is exponentially stable. (Hint: treat $\mu = \epsilon^2$ as a seperate small parameter.) Since the system has three (rather than two) time scales, it has three levels of invariant manifolds – slow, medium, and fast. Without going into high accuracy, give the approximate expressions for these manifolds and discuss the trajectories of the system.