SUMMARY: Rules for sketching Bode Plots

$$G(j\omega) = K_o (j\omega)^m (j\omega\tau + 1)^n \left[\left(\frac{j\omega}{\omega_n} \right)^2 + 2\varsigma \frac{j\omega}{\omega_n} + 1 \right]^q$$

1. The DC gain : K_o

   - slope of $|G(j\omega)| = 0$
   - value of $|G(j\omega)| = 20\log(|K_o|)\text{dB}$
   - phase of $G(j\omega) = \begin{cases} 0 & \text{if } K_o > 0 \\ \pm\pi & \text{if } K_o < 0 \end{cases}$

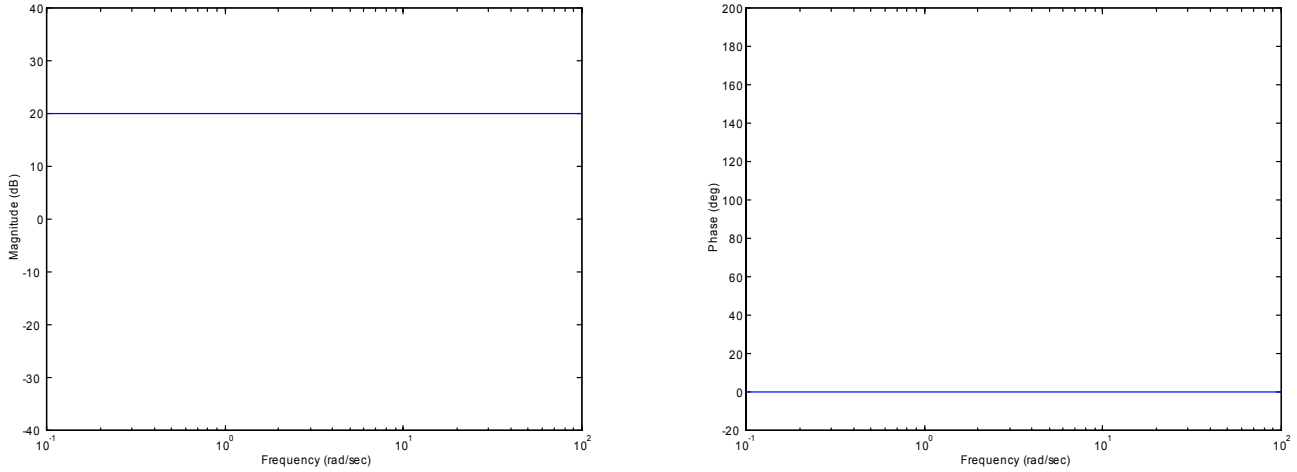

Figure 1-a: $G(j\omega) = 10$

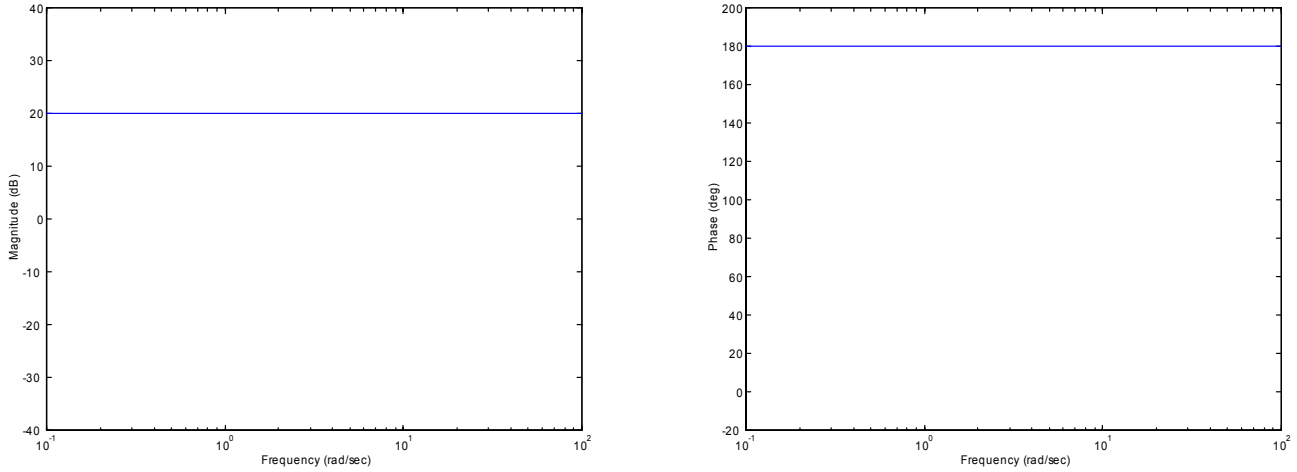

Figure 1-b: $G(j\omega) = -10$

2. The DC behavior $(\text{at } \omega \approx 0)$ - pole/zero at the origin : $(j\omega)^m$

- slope of $|G(j\omega)| = m \cdot 20\, dB/dec$
- phase of $G(j\omega) = m\frac{\pi}{2}$

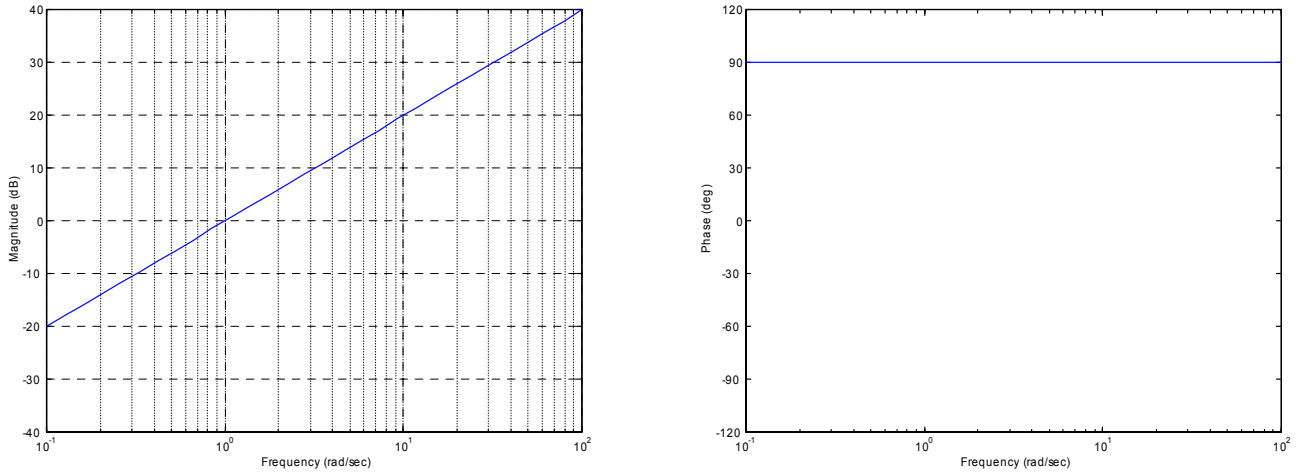

Figure 2-a: $G(j\omega) = j\omega$

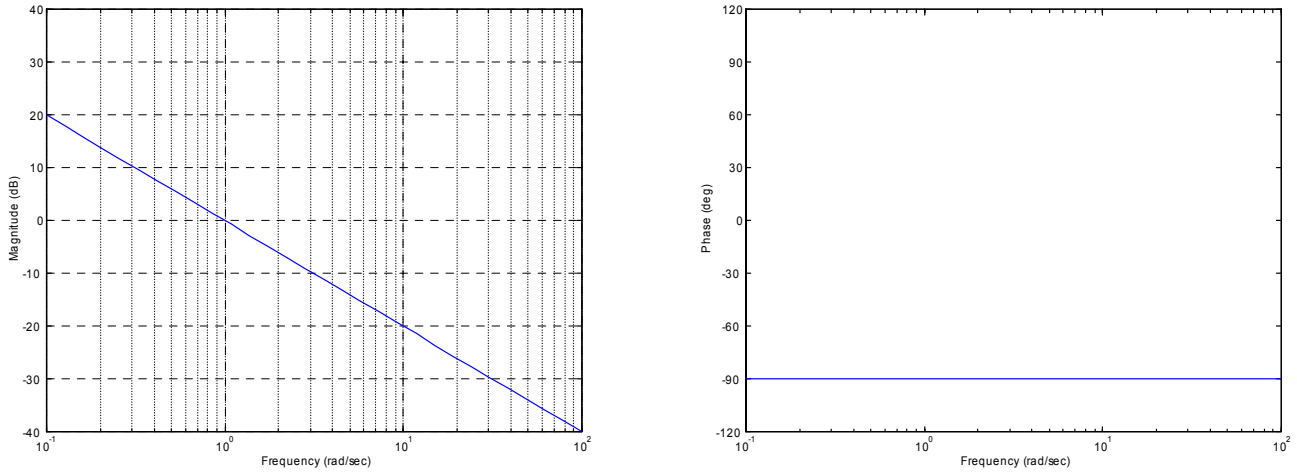

Figure 2-b: $G(j\omega) = \frac{1}{j\omega}$

3. Stable $(\tau > 0)$ real poles : $(j\omega\tau + 1)^n$ with $n < 0$

- Reduce the value of $|G(j\omega)|$ by $|n|20$ dB/dec (slope)
- Reduce the phase of $G(j\omega)$ by $|n|\frac{\pi}{2}$ (gradually)

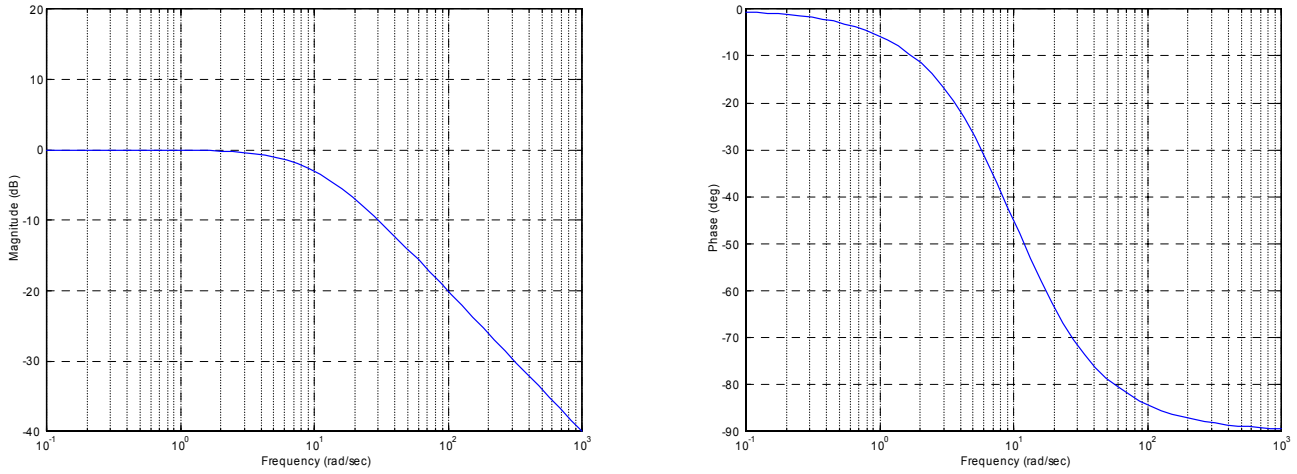

Figure 3: $G(j\omega) = \dfrac{1}{\left(j\omega\dfrac{1}{10} + 1\right)}$

4. Stable $(\tau > 0)$ real zeros : $(j\omega\tau + 1)^n$ with $n > 0$

- Increase the value of $|G(j\omega)|$ by $|n|20$ dB/dec (slope)
- Increase the phase of $G(j\omega)$ by $|n|\frac{\pi}{2}$ (gradually)

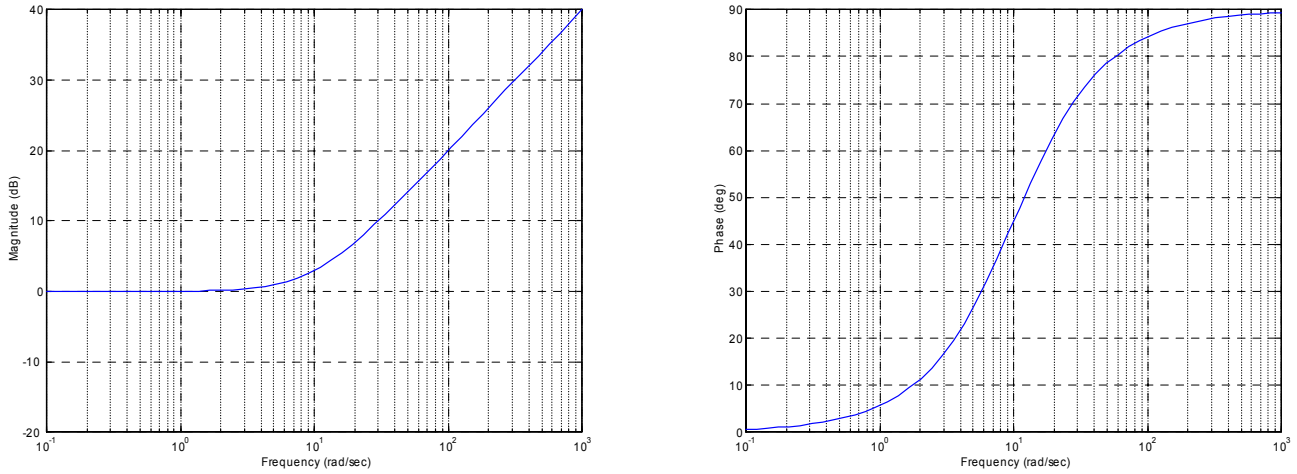

Figure 4: $G(j\omega) = \left(j\omega\dfrac{1}{10} + 1\right)$

5. Resonant stable $(\zeta > 0)$ pair of poles : $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^q$ with $q < 0$

- Reduce the value of $|G(j\omega)|$ by $2|q|20$ dB/dec (slope) - Peak in $|G(j\omega)|$
- Reduce the phase of $G(j\omega)$ by $2|q|\frac{\pi}{2}$ - Sharp change in phase

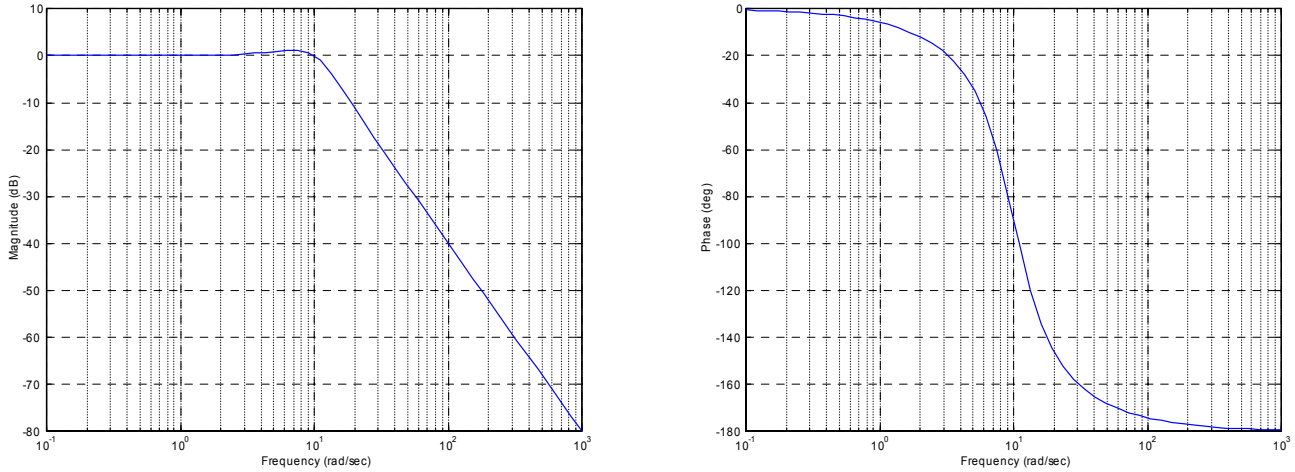

Figure 5: $G(j\omega) = \frac{1}{\left[\left(j\omega\frac{1}{10}\right)^2 + 2\cdot 0.5 j\omega\frac{1}{10} + 1\right]}$

6. Resonant stable $(\zeta > 0)$ pair of zeros : $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^q$ with $q > 0$

- Increase the value of $|G(j\omega)|$ by $2|q|20$ dB/dec (slope) - Dip in $|G(j\omega)|$
- Increase the phase of $G(j\omega)$ by $2|q|\frac{\pi}{2}$ - Sharp change in phase

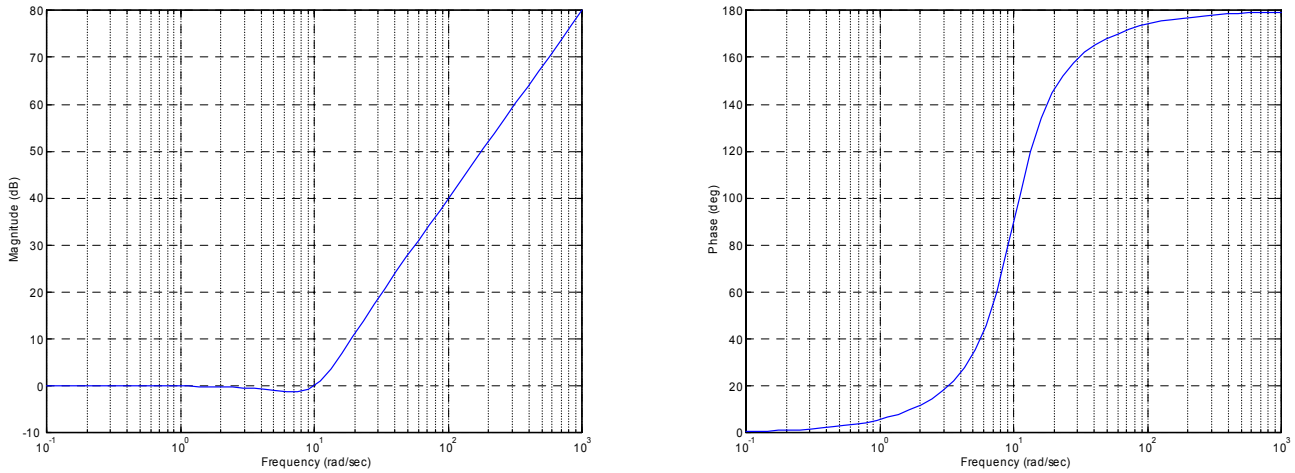

Figure 6: $G(j\omega) = \left[\left(j\omega\frac{1}{10}\right)^2 + 2\cdot 0.5 j\omega\frac{1}{10} + 1\right]$

7. Unstable poles/zeros (both real $(\tau < 0)$ and resonant $(\zeta < 0)$):

- Same $|G(j\omega)|$
- The effect on the phase of $G(j\omega)$ is opposite than the stable case.
  An unstable pole behaves like a stable zero.
  An unstable zero behaves like a stable pole.

Remember to normalize the terms of the transfer function. All the terms, except the DC gain, MUST be one at $\omega = 0$. Example:

$$\frac{1}{s-1} = -\frac{1}{-s+1} \Rightarrow \angle\left(\frac{1}{s-1}\right) = \pm\pi + \angle\left(\frac{1}{-s+1}\right)$$

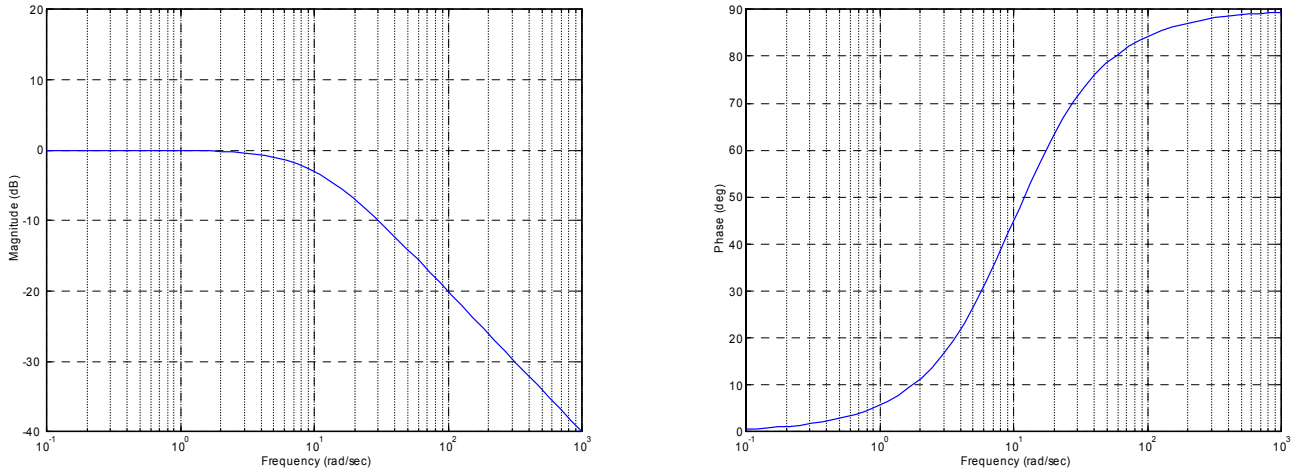

Figure 7-a: $G(j\omega) = \dfrac{1}{\left(-j\omega\dfrac{1}{10}+1\right)}$

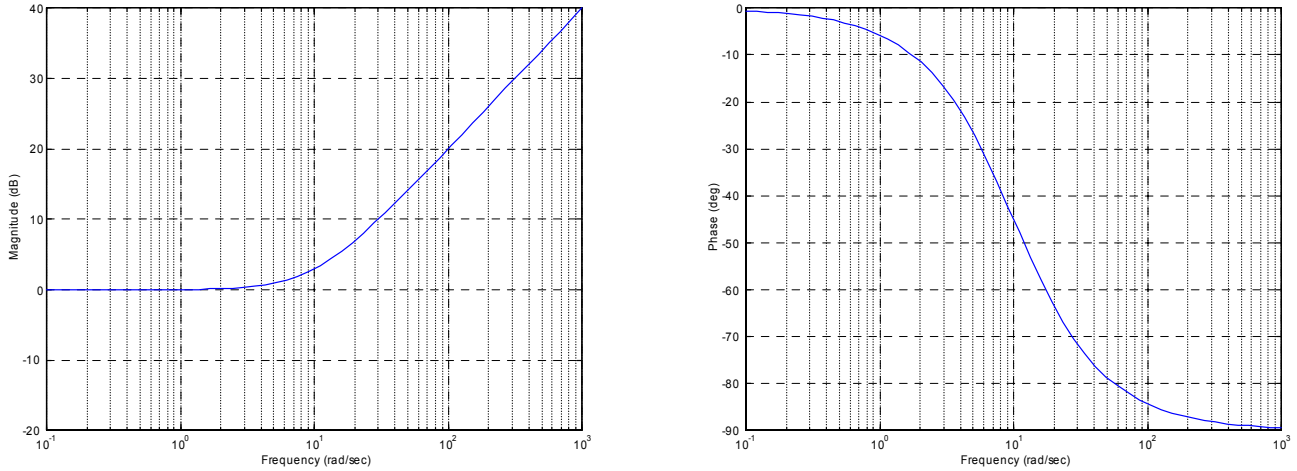

Figure 7-b: $G(j\omega) = \left(-j\omega\dfrac{1}{10}+1\right)$

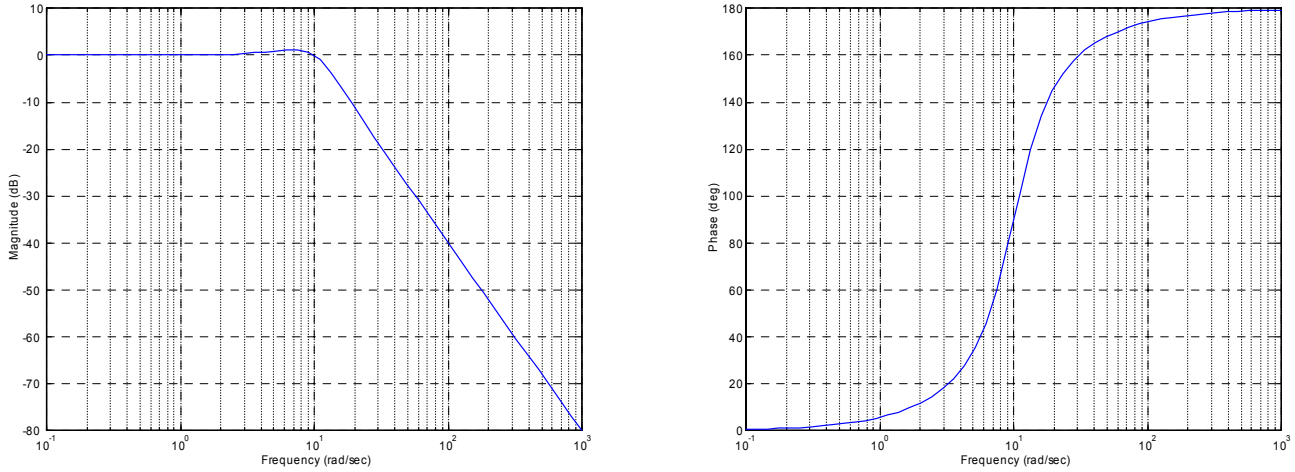

Figure 7-c: $G(j\omega)=\dfrac{1}{\left[\left(j\omega\dfrac{1}{10}\right)^2-2\cdot 0.5\,j\omega\dfrac{1}{10}+1\right]}$

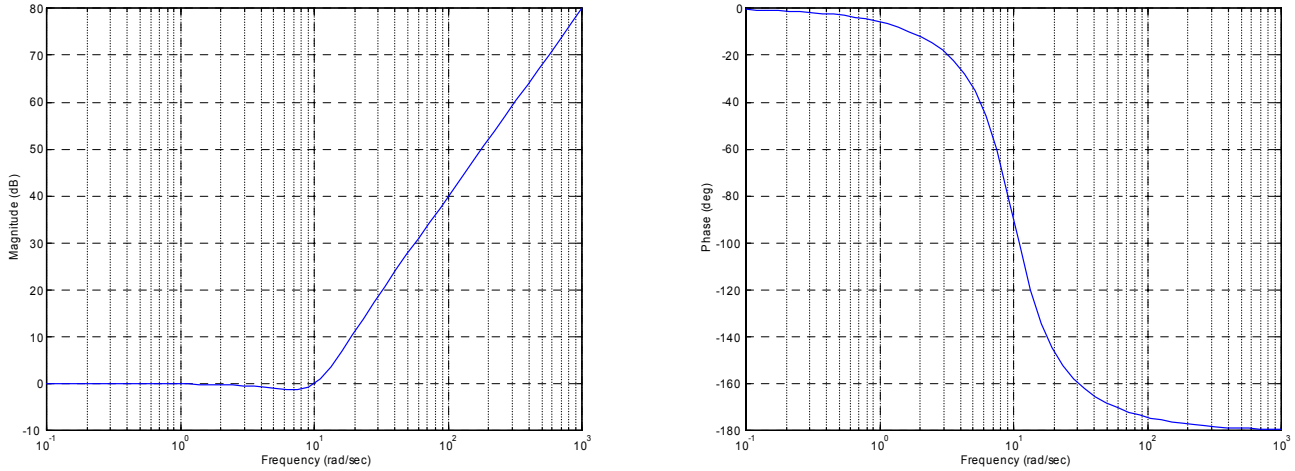

Figure 7-d: $G(j\omega)=\left[\left(j\omega\dfrac{1}{10}\right)^2-2\cdot 0.5\,j\omega\dfrac{1}{10}+1\right]$