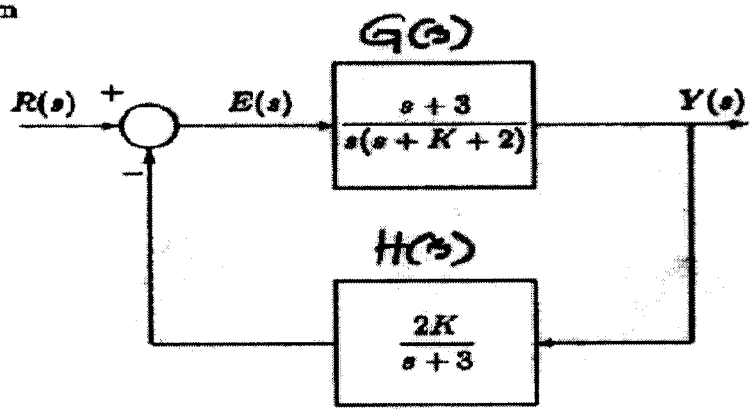
# Midterm Fall 98 - Solutions



**Problem 1.**

Consider the system




**(a)** <u>(2 points)</u> **Find the closed-loop and error transfer functions:**

$$\frac{Y(s)}{R(s)} \quad \text{and} \quad \frac{E(s)}{R(s)}.$$

**Reduce them to second order by performing cancellation where possible.**

$$\frac{Y(s)}{R(s)} = \frac{G}{1+HG} = \frac{\dfrac{s+3}{s(s+K+2)}}{1+\dfrac{2K}{\cancel{s+3}}\,\dfrac{\cancel{s+3}}{s(s+K+2)}}$$

$$= \frac{s+3}{s^2+(K+2)s+2K}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+HG} = \frac{s(s+K+2)}{s^2+(K+2)s+2K}$$

# Midterm Fall 98 - Solutions



**(b)** <u>(1 points)</u> **Let the input be a step function, that is, $r(t) = 1(t)$. Without calculating the complete response $e(t)$ but using only the final value theorem of the Laplace transform, find $y(\infty)$ and $e(\infty)$.**

Final Value Theorem:

$$f(\infty) = \lim_{s \to 0} \left[ s F(s) \right]$$

$$y(\infty) = \lim_{s \to \infty} \left[ s \frac{Y(s)}{R(s)} \frac{1}{s} \right] = \frac{3}{2K}$$

$$e(\infty) = \lim_{s \to 0} \left[ s \frac{E(s)}{R(s)} \frac{1}{s} \right] = 0$$

Next Page
Previous Page
Index

(c) (3 points) Find the step response of the output $y(t)$ for $K \neq 2$.

$$Y(s) = \frac{s+3}{s^2+(2+K)s+2K}\,\frac{1}{s} = \frac{s+3}{s(s+2)(s+K)}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+K}$$

$$A_1 = \left[sY(s)\right]\Big|_{s=0} = \frac{3}{2K}$$

$$A_2 = \left[(s+2)\,Y(s)\right]\Big|_{s=-2} = \frac{1}{4-2K}$$

$$A_3 = \left[(s+K)\,Y(s)\right]\Big|_{s=-K} = \frac{3-K}{K^2-2K}$$

$$Y(s) = \frac{3}{2Ks} + \frac{1}{(4-2K)(s+2)} + \frac{3-K}{(K^2-2K)(s+K)}$$

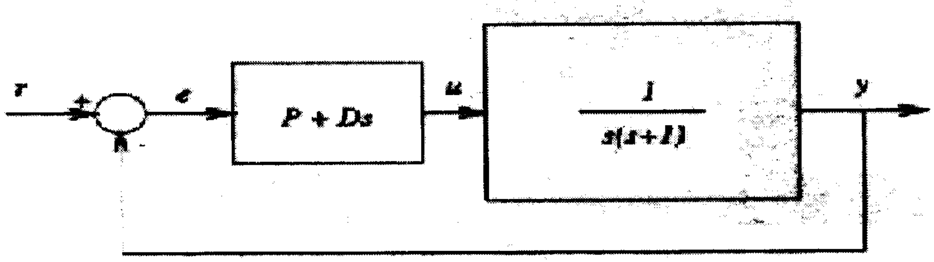
$$y(t) = \left[\frac{3}{2K} + \frac{1}{4-2K}e^{-2t} + \frac{3-K}{K^2-2K}e^{-Kt}\right]1(t)$$

---

Next Page
Previous Page

**Problem 2.** (4 points)

Consider the following feedback loop




Find the values of the proportional gain $P$ and the derivative gain $D$ so that the rise time be $t_r = 0.9$ seconds and the settling time be $t_s = 2.3$ seconds.

Closed-loop transfer function:

$$\frac{Y}{R} = \frac{P + Ds}{s^2 + \underbrace{(1+D)}_{2\zeta\omega_n}s + \underbrace{P}_{\omega_n^2}}$$

Thus $\omega_n = \sqrt{P}$ and $\zeta\omega_n = \frac{1+D}{2}$.

Recall that

$$t_r = \frac{1.8}{\omega_n}, \quad t_s = \frac{4.6}{\zeta\omega_n}.$$

Hence

$$P = \omega_n^2 = \left(\frac{1.8}{t_r}\right)^2 = 4$$

$$D = 2\zeta\omega_n - 1 = \frac{9.2}{t_s} - 1 = 3$$