MAE 143B

Linear Control

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MIDTERM

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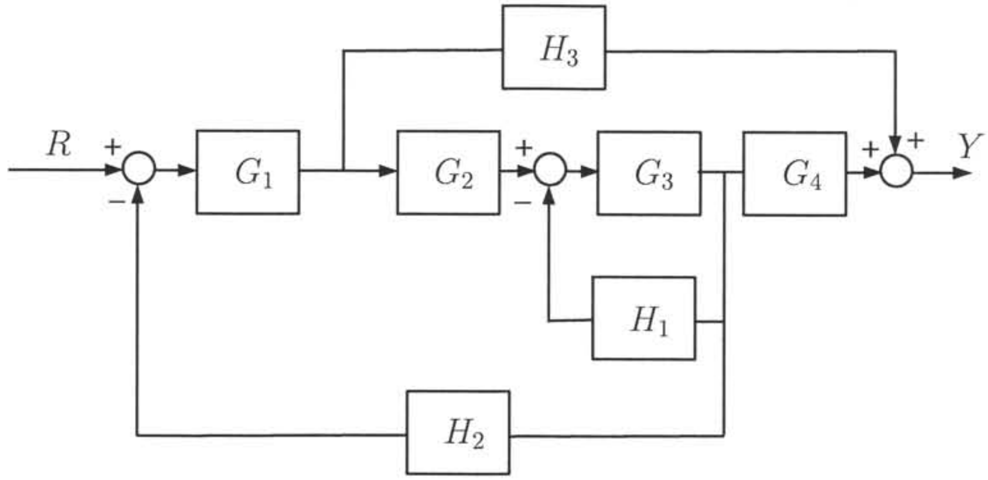
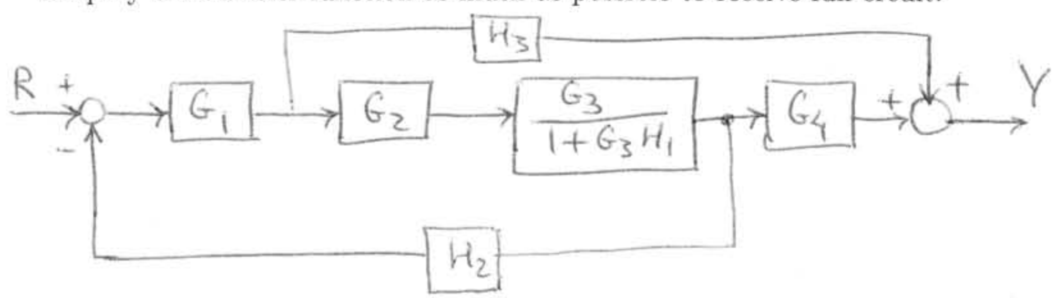
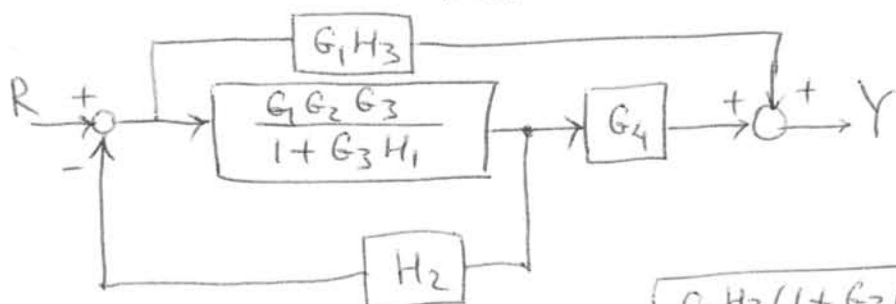
SOLUTIONS

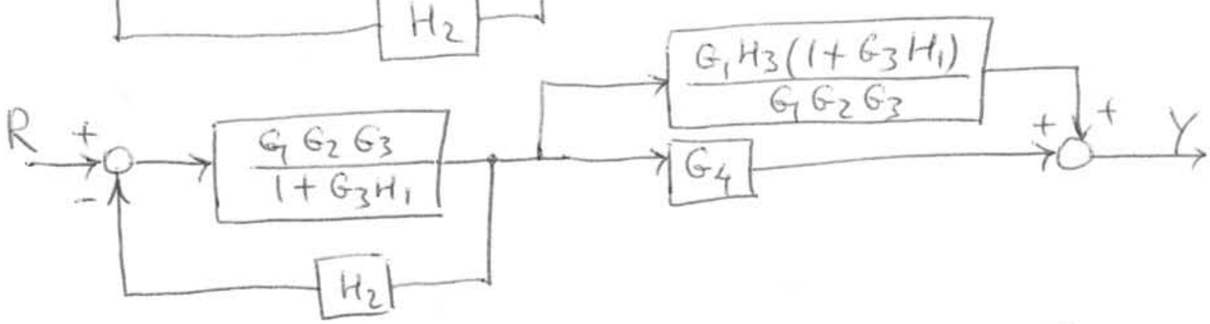
Problem 1. (5 points) Using block diagram reduction techniques, find $\frac{Y}{R}$.

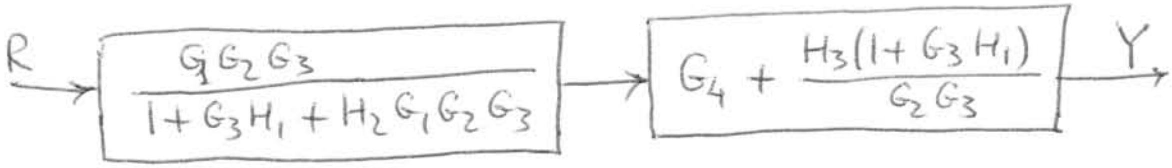


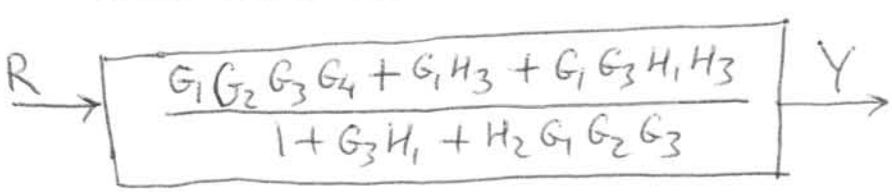
Simplify the transfer function as much as possible to receive full credit.

1)


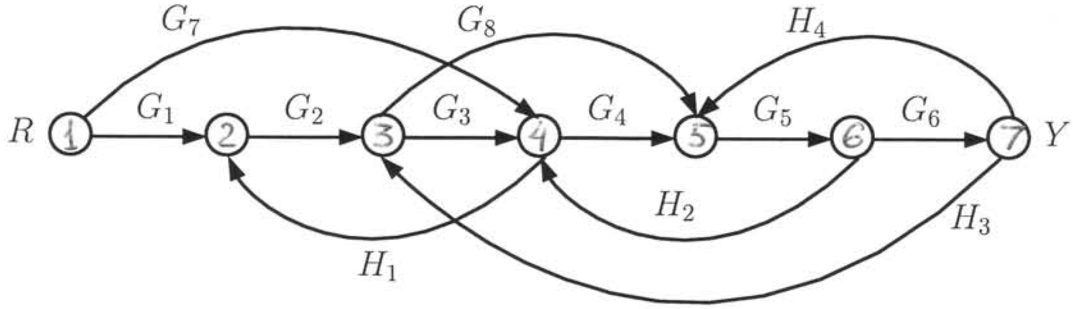
2)


3)


4)


5)


Problem 2. (7 points) Using Mason's rule, find the transfer function $\frac{Y}{R}$ in the following system:



	forward path	gain	
1)	1234567	$F_1 = G_1 G_2 G_3 G_4 G_5 G_6$	$\Delta_1 = 1$
2)	123567	$F_2 = G_1 G_2 G_8 G_5 G_6$	$\Delta_2 = 1$
3)	14567	$F_3 = G_7 G_4 G_5 G_6$	$\Delta_3 = 1$
4)	1423567	$F_4 = G_7 H_1 G_2 G_8 G_5 G_6$	$\Delta_4 = 1$

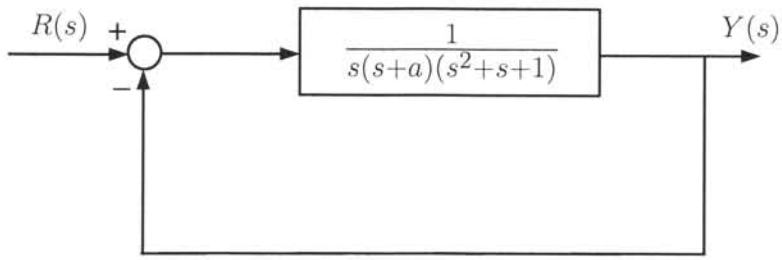
	loop	gain
1)	2342	$\ell_1 = G_2 G_3 H_1$
2)	4564	$\ell_2 = G_4 G_5 H_2$
3)	5675	$\ell_3 = G_5 G_6 H_4$
4)	345673	$\ell_4 = G_3 G_4 G_5 G_6 H_3$
5)	35673	$\ell_5 = G_8 G_5 G_6 H_3$
6)	235642	$\ell_6 = G_2 G_8 G_5 H_2 H_1$

loops 1) and 3) do not touch

$$\frac{Y}{R} = \frac{F_1\Delta_1 + F_2\Delta_2 + F_3\Delta_3 + F_4\Delta_4}{1-(\ell_1+\ell_2+\ell_3+\ell_4+\ell_5+\ell_6)+\ell_1\ell_3}$$

$$= \frac{G_5 G_6 \left(G_1 G_2 G_3 G_4 + G_1 G_2 G_8 + G_7 G_4 + G_7 H_1 G_2 G_8 \right)}{1 - G_2 G_3 H_1 - G_4 G_5 H_2 - G_5 G_6 H_4 - G_3 G_4 G_5 G_6 H_3 - G_8 G_5 G_6 H_3 - G_2 G_8 G_5 H_2 H_1 + G_2 G_3 G_5 G_6 H_1 H_4}$$

Problem 3. (5 points) Consider the feedback system for controlling the yaw of a fighter jet:



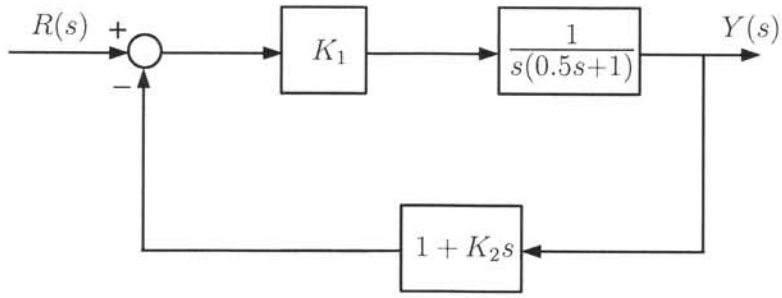
Find the sensitivity of the closed-loop transfer function $T(s)$ to a small change in the parameter a.

$$\frac{Y(s)}{R(s)} = T(s) = \frac{1}{s(s+a)(s^2+s+1)+1}$$

$$S_a^T = \frac{a}{T}\frac{dT}{da} = \frac{a}{T}\left(-\frac{s(s^2+s+1)}{\left(s(s+a)(s^2+s+1)+1\right)^2}\right)$$

$$= -\frac{as(s^2+s+1)}{s(s+a)(s^2+s+1)+1}$$

Problem 4. (7 points) Consider a DC motor with PD control:



(a) (4 points) Find K_1 and K_2 such that the peak time is 1 second and the settling time is 3 seconds.

(b) (3 points) What is the overshoot and the rise time for K_1 and K_2 determined in (a)?

$$\frac{Y}{R} = \frac{K_1}{0.5s^2+s+K_1(1+K_2s)} = \frac{2K_1}{s^2+2(1+K_1K_2)s+2K_1}$$

$$2K_1 = \omega_n^2, \quad 2(1+K_1K_2) = 2\zeta\omega_n$$

$$t_s = \frac{4.6}{\zeta\omega_n} \Rightarrow \zeta\omega_n = \frac{4.6}{3} = 1.533$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow \omega_n^2 = \frac{\pi^2}{t_p^2} + \omega_n^2\zeta^2 = \frac{\pi^2}{1^2} + 1.533^2 = 12.2197$$

$$\omega_n = \sqrt{12.2197} = 3.496$$

$$\zeta = \frac{1.533}{\omega_n} = 0.4385$$

$$K_1 = \frac{1}{2}\omega_n^2 = 6.11$$

$$K_2 = \frac{\zeta\omega_n - 1}{K_1} = 0.087$$

$$t_r = \frac{1.8}{\omega_n} = 0.515$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.216$$

Problem 5. (6 points) Determine the number of unstable poles for the following polynomials:

(a) (3 points) $p_1 = s^5 + 2s^4 + 2s^3 + 2s^2 + 3s + 4$

(b) (3 points) $p_2 = s^6 + s^5 + 3s^4 + 4s^3 + s^2 + s + 1$

(a)

s^5	1	2	3
s^4	2	2	4
s^3	1	1	
s^2	0	4	$\leftarrow$ zero element in first column
s^2	ε	4	
s^1	$1-\frac{4}{\varepsilon} \to -\infty$ as $\varepsilon \to 0^+$		
s^0	4		

Two sign changes $\rightarrow$ 2 unstable poles

(b)

s^6	1	3	1	1
s^5	1	4	1	
s^4	-1	0	1	
s^3	4	2		
s^2	$1/2$	1		
s^1	-6			
s^0	1			

Four sign changes $\Rightarrow$ 4 unstable poles